

# TECHNICAL DETAILS OF THE IPHC MSE FRAMEWORK

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# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Management Strategy Evaluation . . . . .	3
<b>2</b>	<b>Operating model</b>	<b>6</b>
2.1	The state object . . . . .	7
2.2	Notation . . . . .	8
2.3	Population dynamics . . . . .	9
2.3.1	Age increment . . . . .	10
2.3.2	Recruitment . . . . .	10
2.3.3	Movement . . . . .	13
2.3.4	Mortality . . . . .	15
2.3.5	Maturity . . . . .	23
2.3.6	Spawning biomass . . . . .	23
2.3.7	Size-at-age . . . . .	24
2.3.8	Initial population . . . . .	24
2.3.9	Parameter evolution through time . . . . .	28
2.4	Conditioning the Operating Models . . . . .	28
2.4.1	Observations and Data . . . . .	28
2.4.2	Predictions from the stock assessment . . . . .	30
2.5	Projecting the Operating Models . . . . .	30

2.5.1	Recruitment . . . . .	30
2.5.2	Fishing mortality . . . . .	30
2.5.3	Movement . . . . .	30
2.5.4	Weight-at-age . . . . .	31
2.6	Reference Points . . . . .	31
<b>3</b>	<b>Specifications for the Pacific halibut MSE framework</b>	<b>32</b>
3.1	Management Procedures . . . . .	33
3.1.1	Data Generation . . . . .	33
3.1.2	Modelling length-at-age . . . . .	36
3.1.3	Estimation Model . . . . .	36
3.1.4	Harvest Rule . . . . .	38
3.2	Population structure . . . . .	38
3.2.1	Input population parameters . . . . .	38
3.2.2	Derived population parameters . . . . .	38
3.3	Structure of the fisheries . . . . .	38
3.3.1	Fishery parameters . . . . .	39
	<b>Appendix A Parameters and notation</b>	<b>41</b>
	<b>Appendix B Fishing mortality using exploitation rates: two approaches</b>	<b>44</b>
B.1	Fisheries are independent and do not effect each other . . . . .	44
B.2	Fisheries are sequential and earlier fisheries effect later ones . . . . .	47
B.3	Comparison . . . . .	49
B.4	A more complex example with three fisheries and discard mortality . . . . .	51

# Chapter 1

## Introduction

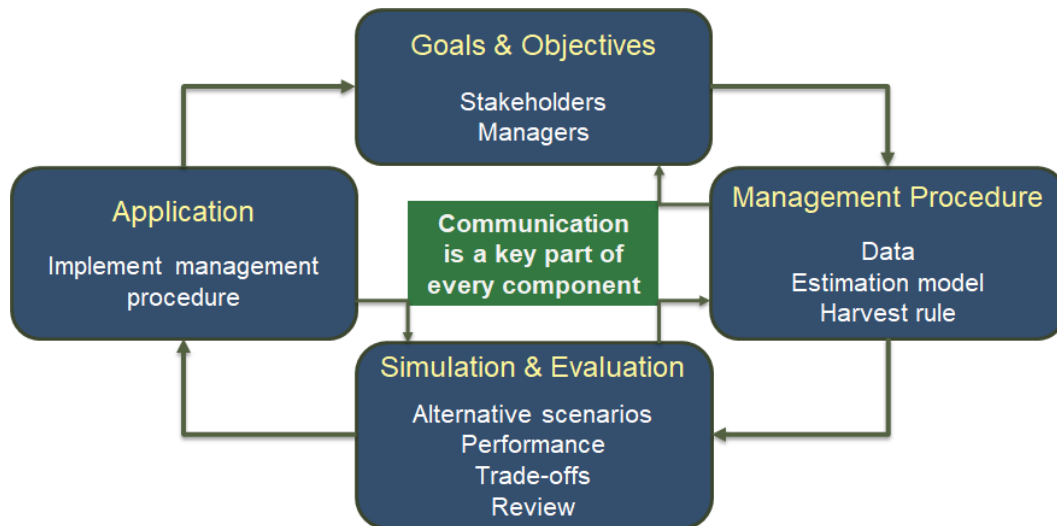
This technical document describes the Management Strategy Evaluation (MSE) framework and its elements, details specifications of the framework for the evaluation of scale and distribution management procedures, provides definition of terms used, and defines the technical details of the models and equations used within the framework. This is a working document that will be revised often as development of the MSE framework progresses. Therefore, this document is currently incomplete and will have occasional revisions.

### 1.1 Management Strategy Evaluation

MSE is a process to evaluate harvest strategies and develop a management procedure that is robust to uncertainty and meets defined objectives, and can be partitioned into four separate components that interact with each other (Figure 1.1). Management Procedures (MPs) are defined, often with input from stakeholders and managers but not necessarily, and evaluated against objectives which are determined with input from stakeholders and managers. Simulations of the various MPs are performed and evaluated against the objectives to identify the best performing MP to apply within a harvest strategy policy.

A harvest strategy policy can be implemented in a number of ways. Many fisheries are managed by applying the chosen management procedure each management cycle and implementing the results as management. Other agencies use the outcomes of the management procedure as a reference from which other considerations (e.g., socio-economic) are taken into account when determining a tactical decision of the management outcomes. This variability around the management procedure is called implementation variability and should be a part of the simulations and evaluation.

The four boxes shown in Figure 1.1 are all important component of an MSE. The objectives are the connection to stakeholders and managers. Performance metrics are derived from well defined objectives that are used in the evaluation. Management procedures are the link to a transparent management process and need to be clearly defined so that they are formulaic and can be written as computer code for the closed-loop simulations. The closed-loop simulations also consist of an operating model which simulates the population and produces the observations needed for the management procedure. Applying the best performing MP is the goal of MSE but is not the



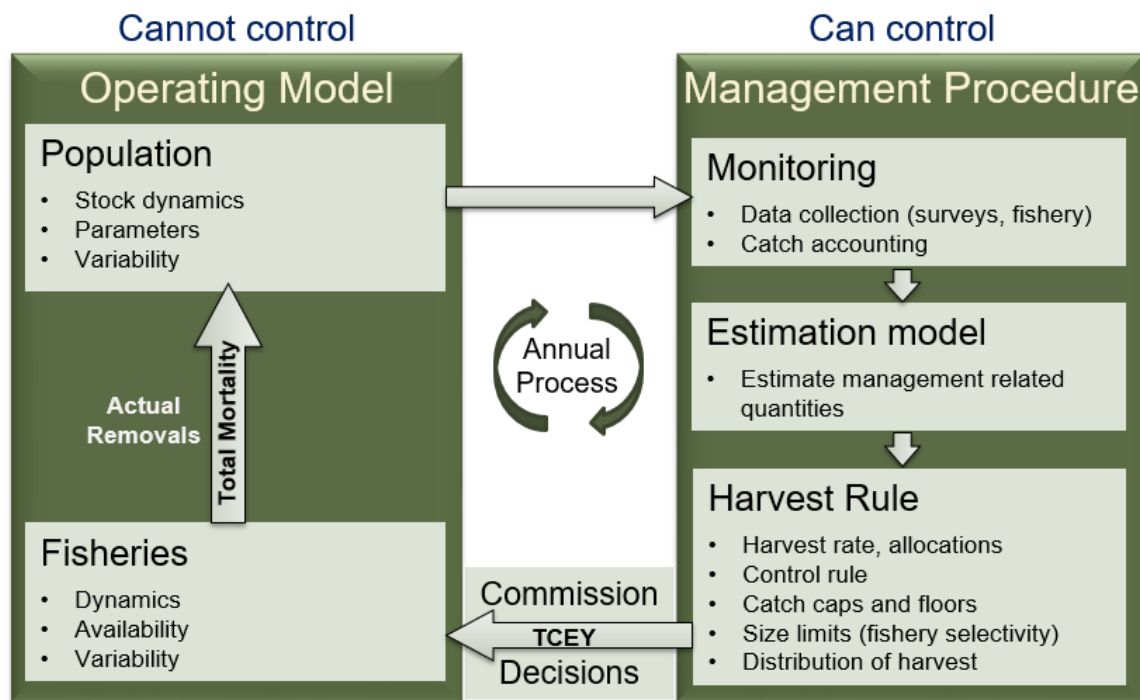
**Figure 1.1:** An illustration of the closed-loop simulation within the MSE framework consisting of an operating model and a management procedure.

end. The MSE should be updated as additional observations and knowledge is gained from the population, fishery, or management process.

The engine of the MSE framework is the closed-loop simulation with the operating model (OM) and management procedure (Figure 1.2). The OM simulates the dynamics of the population and the fisheries that interact with it. The processes simulated by the OM can be thought of as processes that management does not, or chooses not, to control. For example, natural mortality is not a process that is not managed, and some aspects of the fisheries are not managed (e.g., specific daily decisions). These unmanaged processes result in variability that is normal to the system, referred to as ‘natural variability’ in this document, and is simulated by the OM.

The MP consists of elements that are managed and may include data collection and monitoring, estimation models, and the harvest rules that determine how the fisheries are managed. MSE can evaluate any of these elements including how changes in monitoring, and different estimation model, or various harvest rules affect the outcomes. This elements may be simple or complex.

The chapters in this document begin with a generalized operating model that can be specified for any fish population. The following chapter presents the specifications of the MSE for Pacific halibut fisheries, and the sections within that chapter follow the three boxes in Figure 1.1 labeled Goals & Objectives, Management Procedure, and Simulation.



**Figure 1.2:** An illustration of the closed-loop simulation within the MSE framework consisting of an operating model and a management procedure.

## Chapter 2

# Operating model

In a management strategy evaluation (MSE), operating models (OM) simulate the population and fishery dynamics. It incorporates life-history processes such as recruitment, growth, migration, maturation, and mortality of the fish population, as well as fishery processes such as selectivity, availability, and catchability. Descriptions of the various processes are provided below along with the mathematical equations used to simulate those processes. Many of the details are drawn from the Hilborn and Walters (1992), Quinn and Deriso (1999), the CASAL manual Bull *et al.* (2012), Stock Synthesis technical details Methot and Wetzel (2013), and the Coleraine manual Hilborn *et al.* (2000).

There is uncertainty in the parameterization of the processes, natural variability in the processes, and multiple hypotheses about the mechanisms of the processes. These three sources of variability are introduced in three different ways.

1. Parameter uncertainty is introduced by conditioning the operating model to data, and determining the distribution of uncertainty for each parameter as well as correlation with other parameters. Parameter values for an individual simulated trajectory are randomly drawn from the multivariate estimated probability distribution. Therefore, each simulated trajectory uses a different set of parameters, thus including variability that represents the uncertainty in the parameters. This is described in Section 2.4.
2. Natural variability is introduced by defining a random process associated with various concepts. For example, recruitment varies naturally and is modelled by including random deviates applied annually to average recruitment. Other processes may have specific patterns such as changes in weight-at-age. This is described in Section 2.5.
3. Structural uncertainty is included by defining multiple hypotheses and implementing them as separate operating models. For example, growth may occur in different ways between models. Or, data may be structured in a different way when conditioning the model. Structural uncertainty captures the variability that can not be captured by the two methods above.

Parameters that will have uncertainty are defined (and those that are fixed are given fixed values), methods to include natural variability are defined, and potential areas of structural uncertainty are noted.

## 2.1 The state object

The state is the accounting of the population in numbers within an operating model and is contained in a state object with many dimensions. The state represents the intrinsic characteristics of the modeled population: age, maturity, and sex. This state is then evolved on a computational domain parameterized by time and space, which are extrinsic variables. Furthermore, sectors (fisheries and survey) interact with the state. Clarified this way, the state object contains a representation of the stock at a place and time and can be subsetted along any of these axes as needed to determine the state for any combination of these dimensions.

The dimensions are fixed inputs that are defined by the user, thus may be unique to any operating model. The different dimensions, and maximum ranges, are shown in Table 2.1. Maturity state (immature or mature) is not included as a dimension here (specifically for the Pacific halibut operating model) but may be a useful characteristic to track for some stocks, depending on fishing intensity and the proportion maturing at age. Instead, the mature population is determined using the proportion mature at age, which can be applied to various dimensions of the population state (see Section 2.3).

**Table 2.1:** Partitions of the state object that are fixed inputs and the likely minimum and maximum input for each partition.

Variable	Dimension		Description
	Min	Max	
Age ( $a$ )	1	251	Age classes ranging from 0 to 250. Halibut will likely use 0 to 30 and age always starts at zero. A capital $A$ indicates the maximum age
Sex ( $s$ )	1	3	Sex, which includes female, male, and unsexed, in that order, labeled 1, 2, and 3.
Time ( $t$ )	0	$\infty$	A minimum and maximum time-step (e.g., year) is input by user. The difference $+1$ determines the number of time-steps. These are not projected time-steps, but time-steps modelled to condition the OM. A time-step will typically be a year, but specific points in time (e.g., beginning, middle, or end of the year) may be noted in a superscript (see below).
Region ( $r$ )	1	$\infty$	Number of spatial regions with migration between
Area within Region ( $r_l$ )	1	$\infty$	Number of areas within a region. Migration is not modelled between areas.
Sector ( $f$ )	1	$\infty$	Number of fishing-related sectors, which includes fisheries and surveys. Sectors typically will operate at the region level or a finer scale, but there may be a case where a sector operates across regions (which is unlikely for the Pacific halibut operating model).

The state object is the key component of the population dynamics and must contain sufficient information to determine the population dynamics as well as any intermediate calculations, such as fishery catches. In this implementation, there are always six partitions, but some operating models may have a partition with only one element, effectively eliminating that dimension. For example, a single-sex, single-area model with no maturity partition would simply be a matrix of years and ages.



## 2.2 Notation

Notation of the variables in the operating model uses the concept of defining a quantity of the population (such as numbers or biomass), subscripted by various characteristics (intrinsic and extrinsic) and superscripted by specific concepts (such as spawning or exploitable). The subscripts reflect the intrinsic and extrinsic characteristics of the population by listing the intrinsic characteristics first (age and sex), followed by the extrinsic characteristics (time, region/area, and fishing sector). The possible subscripts are defined in Table 2.1 and are always subscripted in the order presented in those tables. For example, the numbers for age and sex in a year and region is  $N_{a,s|y,r}$ . When a subscript is not included, it is implied that the quantity is a summation over that index (or the index doesn't apply, as in the case of fishing sector) and ambiguity will be alleviated using the letter associated with the index when necessary (e.g.,  $N_{s=1|y=1}$  is the number of females in year 1 summed over all ages and regions).

Variables specific to a fishing sector ( $f$ ) include a subscript for that sector at the end. For example, the catch-at-age for females from sector  $f$  in year 1 and region 2 would be notated as  $C_{a,1|1,2,f}$ . Fishery sectors typically will operate at the region level or a finer scale, but the region subscript is retained for clarity and in case a sector does operate across regions (which is unlikely for the Pacific halibut operating model).

Finally, superscripts are used to notate specific concepts such as spawning biomass, which would be notated as  $B_{s=1|y=1}^{sp}$  to represent the spawning biomass for females in year 1 over all regions. Additionally, a superscript that is a number between 0 and 1 indicates the time in the year that the quantity is calculated. For example,  $B^{sp,0}$ ,  $B^{sp,0.5}$ ,  $B^{sp,1}$  would be the spawning biomass calculated at the beginning of the year, middle of the year, and end of the year, respectively. Possible superscripts and their definition are shown in Table 2.2.

**Table 2.2:** Superscripts for variable and their meaning.

Variable	Description
ma	Mature
sp	Spawning. Most often used with $B^{sp}$ to represent spawning biomass.
sr	Selected and retained referring to the fish that are landed by a fishery sector.
n	Numbers. Indicates that a quantity, such as catch, is in numbers ( $C^n$ ). Note that if a superscript is not used on catch, it is in weight.
'	Denotes update made to numbers-at-age after partial timestep is complete, which includes the effect of movement but not mortality.
''	Denotes update made to numbers-at-age at the end of the timestep.
Number 0-1	A number between zero and 1 (inclusive) indicates the time within the year. For example, 0 indicates beginning of the year, 0.5 indicates middle of the year, and 1 indicates end of the year.

## 2.3 Population dynamics

The population dynamics are modelled as an age-structured annual process accounting for changes in the numbers-at-age for each partition within the state (e.g., age and sex).

The sequence of processes from the start of the time-step (typically annual) is

1. *age increment*,
2. *recruitment (based on spawning biomass calculated at end of previous time-step or at the beginning of the current time-step)*,
3. *movement*,
4. *mortality*. The sequence of mortality from all sources is theoretically described below, but does not need to be specifically modelled as such because the mortality calculations will appropriately account for the sequence, as described in Section 2.3.4 and a later Appendix.
  - (a) portion of natural mortality,
  - (b) fishing mortality for one or more sectors,
  - (c) portion of natural mortality,
  - (d) fishing mortality for one or more sectors,
  - (e) etc., until a full time-step of natural mortality has been applied
5. *spawning*.

The state object ( $N_{a,s|t,r}$ ) is updated at three different points in the annual process, and superscripts note the time point.

- $N$  : Beginning of the time-step after age increment and recruitment of age 0.
- $N'$  : After movement before mortality
- $N''$  : End of the time-step, after all natural and fishing mortality

At any point in time, the biomass may be desired and can be calculated from numbers-at-age ( $N_{a,s|t,r}$ ) and weight-at-age ( $W_{a,s|t,r}$ ).

$$(2.1) \quad B_{a,s|t,r} = N_{a,s|t,r} W_{a,s|t,r}$$

Various partitions of biomass may be desired. For example, spawning biomass is the weight of spawning fish, and exploitable biomass is the weight of fish available to a specific sector. Biomass can also be calculated at specific points of time in the time-step. These various types of biomass will be defined in the sections below, and will be noted with a superscript. For example, spawning biomass is  $B_y^{sp}$ .

This section describes the technical specifications of the general population dynamics and how the historical population can be modelled given inputs such as catch and weight-at-age, as well as parameters that may be fixed or estimated from data. Conditioning the operating model is the process of determining the range of parameters and hypotheses that describe the observations, and is covered in Section 2.4. Projecting the population forward in time is discussed in Section 2.5, and involves defining random and fixed processes such as recruitment and changes in weight-at-age.

### 2.3.1 Age increment

The numbers-at-age at the beginning of the time-step with an annual time-step is obtained by incrementing the previous time-step's age class to one time-step older and calculating recruitment for age 0.

$$(2.2) \quad N_{a,s|t,r} = \begin{cases} R_{s|t,r} & a = 0 \\ N_{a-1,s|t-1,r} & 1 \leq a < A \\ N_{a-1,s|t-1,r} + N_{A,s|t-1,r} & a = A \end{cases}$$

### 2.3.2 Recruitment

Recruitment is a function of the spawning biomass calculated from the end of the previous time-step after all of the processes (movement and mortality) have occurred. See Section 2.3.6 for a description of spawning biomass.

$$(2.3) \quad R_{s|t,r} = p_{y,r}^R \times p_s^R \times f(B_{s=1|t-1}^{sp,1}) \times e^{(\varepsilon_{y,r} - b_y \frac{\sigma_R^2}{2})} \times e^{I_y \delta}$$

where  $p_{y,r}$  is the proportion recruiting to region  $r$  in time-step  $t$ ,  $p_s$  is the proportion of sex  $s$  (typically 0.50),  $f(B_{s=1|t-1}^{sp,1})$  is the equilibrium stock-recruit relationship using the end of the time-step spawning biomass (superscripts) for females from the previous time-step (subscripts),  $e^\varepsilon$  is the annual deviation in recruitment for time-step  $t$ ,  $b$  is a bias-correction multiplier, and  $e^{I_y \delta}$  is an overall adjustment for recruitment regime shift.

### Density-dependent Recruitment

Density-dependence in the spawner-recruit relationship is modelled using a Beverton-Holt formulation.

$$(2.4) \quad f(B_{s=1|t-1}^{sp,1}) = \frac{B_{s=1|t-1}^{sp,1}}{a + bB_{s=1|t-1}^{sp,1}}$$

where the parameters  $a$  and  $b$  are determined from steepness ( $h$ ), unfished equilibrium recruitment ( $R_0$ ), and unfished equilibrium female spawning biomass ( $B_0^{sp}$ ).

$$(2.5) \quad a = \frac{(1-h)B_0^{sp}}{4hR_0}$$

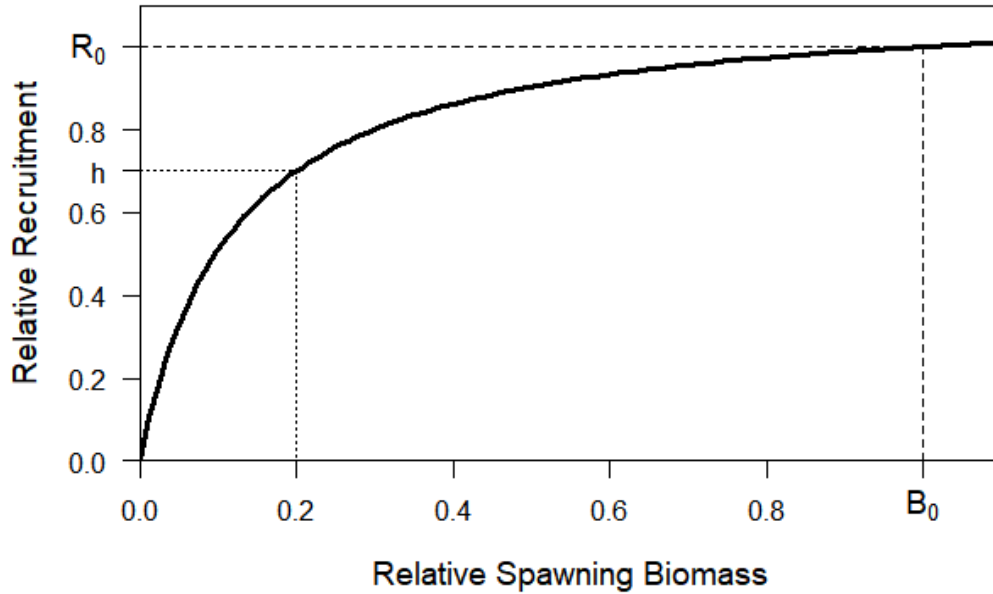
$$(2.6) \quad b = \frac{5h-1}{4hR_0}$$

Steepness ( $h$ ) is a parameter noting the percentage of unfished equilibrium recruitment ( $R_0$ ) that occurs when the female spawning biomass is 20% of unfished equilibrium female spawning biomass

$(B_0^{sp})$ . This can be shown using equations 2.4 and 2.5, and assuming that female spawning biomass is  $\frac{1}{5}^{th}$  of unfished equilibrium female spawning biomass.

$$\begin{aligned}
 & \frac{1/5 B_0^{sp}}{\frac{(1-h)B_0^{sp}}{4hR_0} + \frac{5h-1}{4hR_0}(1/5 B_0^{sp})} \\
 &= \frac{1/5}{\frac{1-h+1/5(5h-1)}{4hR_0}} \\
 &= \frac{4/5hR_0}{4/5} \\
 &= hR_0
 \end{aligned}$$

The same method can be used to show that  $B_0$  results in  $R_0$ . An example Beverton-Holt stock-recruit curve with a steepness of 0.70 is shown in Figure 2.1.



**Figure 2.1:** An example Beverton-Holt stock-recruit curve with a steepness ( $h$ ) of 0.70.

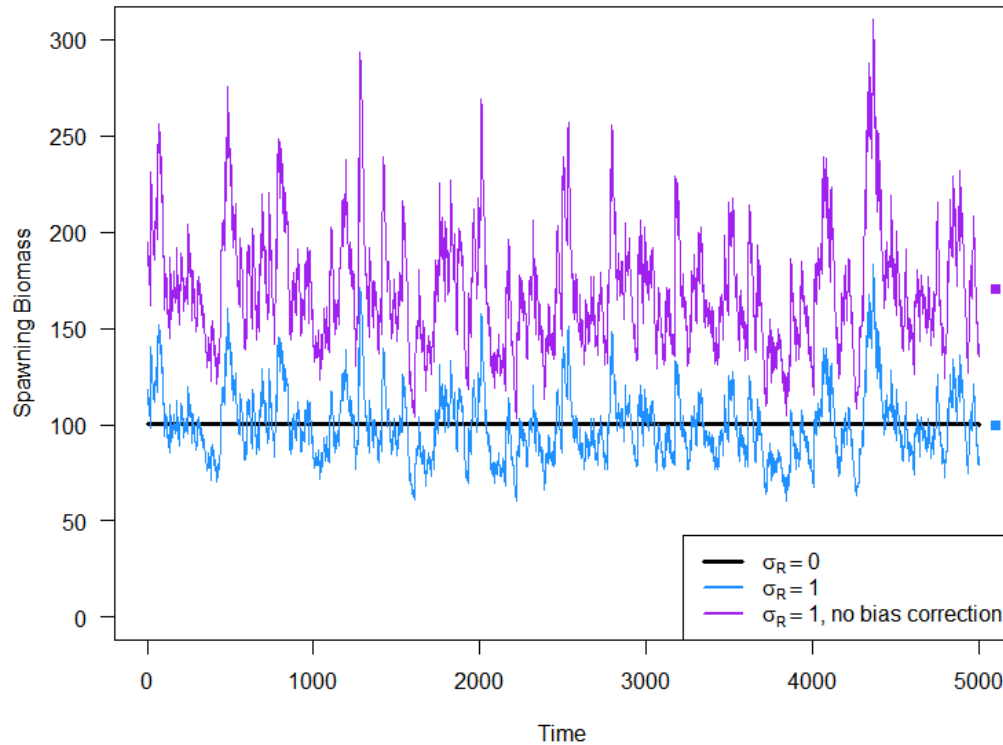
The parameter for steepness is typically fixed because accurate estimation requires data informative of recruitment at low biomass levels and variability in recruitment often reduces the information content. The parameter  $R_0$  is often estimated, and  $B_0^{sp}$  can be calculated from  $R_0$  and other life history parameters. Given those three parameters,  $a$  and  $b$  can be calculated.

## Recruitment Deviation

Recruitment varies around the stock-recruit curve, which is defined as mean recruitment. The distribution of recruitment is assumed to be lognormal and is parameterized using a Gaussian distributed deviate with an exponentiated mean of one and a variance notated as  $\sigma_R^2$ .

$$(2.7) \quad \varepsilon_{y,r} \sim N(\mu = 0, \sigma_R^2)$$

The arithmetic mean of the lognormal distribution is  $e^{\mu + \sigma^2/2}$ , and because  $R_0$  is unfished equilibrium mean recruitment, a bias correction must be applied when simulating log deviates from a normal distribution with a mean/median equal to zero. As shown in equation 2.3,  $e^{(\varepsilon_{y,r} - \sigma_R^2/2)}$  is used where the bias correction is  $-\sigma_R^2/2$ . This ensures that unfished equilibrium recruitment is, on average,  $R_0$  (e.g., the mean of the biased corrected exponentiated deviate is equal to 1). Figure 2.2 shows an example simulation without fishing when bias correcting and not bias correcting, and (Methot and Taylor 2011) present an analytical proof why bias correction is necessary.



**Figure 2.2:** An example simulated projection of spawning biomass with no fishing mortality from an age-structured model with  $\sigma_R = 1$  and bias-corrected recruitment deviates (blue line) and recruitment deviates not bias-corrected (purple line). A simulated trajectory with  $\sigma_R = 0$  is shown by the flat black line, and the means of each simulated trajectory are shown by the appropriate colored square to the right.

Full bias-correction is necessary when simulating the fish population because the full lognormal distribution is used to simulate deviates, as shown in Figure 2.2. However, during estimation, information is reducing the uncertainty (i.e., distribution) around a deviation, and pulling it away from a value of zero. Therefore, a deviate without any information during estimation will be zero and not need bias correction, but a deviate that is fully informed (i.e., known exactly as in a simulation) will need full bias correction. In most estimation models, deviates are not often fully informed and a partial bias-correction is necessary. (Methot and Taylor 2011) provide a much more detailed discussion of this phenomenon. The parameter  $b_y$  is included in equation 2.3 to allow for bias-correction if needed during estimation.

The recruitment process is a coastwide process with age-0 recruits distributed to regions. Therefore, the deviates may be region-specific, but it may be more appropriate to use a single coastwide deviate for each year and simulate region-specific variability across time-steps with the parameters

representing the proportion recruiting to each region ( $p_{y,r}^R$ ). Using region-specific deviates and proportion of recruits may be confounding.

## Recruitment Distribution

Recruitment of age-0 fish to the population is determined from spawning biomass, and depending on the settlement process for a fish species and ocean dynamics, an age-0 fish may recruit a considerable distance from where spawning occurred. Furthermore, fish may migrate to spawning regions that are far from regions they occupied when not spawning. Therefore, the recruitment process is modelled assuming a coastwide spawning population (e.g., fish may spawn in regions where they are not present during the time of fishing) producing age-0 fish (recruits) throughout specified regions. The proportion of recruits in each region in each year is represented with the parameter  $p_{y,r}^R$  as shown in equation 2.3, and  $r - 1$  parameters need to be specified for each time-step because the  $r^{th}$  parameter is one minus the sum of the specified parameters (ie.,  $\sum_r p_{y,r}^R = 1$ ).

## Recruitment Link to an Environmental Variable

Recruitment is modelled (equation 2.3) using a stock-recruit relationship (equation 2.4) that produces an average level of recruitment given current spawning biomass. Changes in the environment may change that average level of spawning biomass and is modelled using an environmental index ( $I_y$ ) in equation 2.3 with the function  $e^{I_y * \delta}$ . The parameter  $\delta$  is a covariate determining how the average recruitment is affected by the environmental index.

### 2.3.3 Movement

In its most simple form, movement (also called migration) is the proportion of individuals that move from region  $j$  to region  $k$  (individuals can only move among regions and movement among areas within regions is not explicitly modelled). The probability that the individual stays in its current region is equal to one minus the sum of the probabilities of moving out of the current region. Movement is specific to the partitions age, sex, time-step, and region.

One of the most common ways to model movement is using a transition matrix. Let  $\Psi_{j \rightarrow k}$  be the instantaneous movement from region  $j$  to region  $k$  expressed as the proportion of the population in region  $j$  moving to region  $k$ . The diagonal of the transition matrix will be the proportion that stay in region  $j$  and the off-diagonals of  $\Psi_{j \rightarrow k}$  will represent the proportions that move out of region  $j$ . The row of the matrix corresponds to  $j$  (from) and  $k$  corresponds to the column of the matrix (to). Each row of the transition matrix should sum to 1. Each dimension of the transition matrix will be equal to the number of regions. For example, let there be  $n$  regions ( $R = 1 \dots n$ ), then the transition matrix for each age, sex, and time-step will look like:

$$\begin{bmatrix} \Psi_{R=1} & \Psi_{1 \rightarrow 2} & \cdots & \Psi_{1 \rightarrow n} \\ \Psi_{2 \rightarrow 1} & \Psi_{R=2} & \cdots & \Psi_{2 \rightarrow n} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{n \rightarrow 1} & \Psi_{n \rightarrow 2} & \cdots & \Psi_{R=n} \end{bmatrix}$$

The numbers-at-age in region  $j$  after movement, for a given age, sex, and time-step, is determined from the following equation.

$$\begin{aligned}
N'_{a,s|t,r=j} &= N_{|r=j} - N_{|r=j} \sum_{k \neq j} \Psi_{a,s|t,j \rightarrow k} + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\
&= N_{|r=j} \left( 1 - \sum_{k \neq j} \Psi_{a,s|t,j \rightarrow k} \right) + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\
&= N_{|r=j} \Psi_{a,s|t,j \rightarrow j} + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\
(2.8) \qquad &= \sum_{k \in r} N_{|r=k} \Psi_{a,s|t,k \rightarrow j}
\end{aligned}$$

### Movement parameters

There are two options for the construction of the transition matrix:

1. entered as simple proportions in an array by time-step, age, sex, and region of origin, or
2. parameterize the proportions-at-age as a function of age and modify the parameters of the function for each time-step and sex.

A parameterized approach is implemented using functions called *constant* (Equation 2.9), *exponential* (Equation 2.10), or *double exponential* (Equation 2.11). Additionally, specific values for defined ages can be entered *Values* (Equation ??)

$$(2.9) \qquad \Psi_{a,s|t,k \rightarrow j} = \begin{cases} 0 & a \leq \psi_0 \\ \psi_c & a > \psi_0 \end{cases}$$

where,  $\psi_0$  is the oldest age with a movement probability of zero before the first non-zero movement probability, and  $\psi_c$  is a constant proportion for all ages greater than  $\psi_0$ .

$$(2.10) \qquad \Psi_{a,s|t,k \rightarrow j} = \begin{cases} 0 & a \leq \psi_0 \\ \frac{e^{\psi_\lambda(a-\psi_0+1)}}{\max(\Psi_{a,s|t,k \rightarrow j})} \times (\psi_{max} - \psi_{min}) & a > \psi_0 \end{cases}$$

where,  $\psi_0$  is the oldest age with a movement probability of zero before the first non-zero movement probability,  $\psi_\lambda$  is the slope parameter of the exponential function,  $\psi_{min}$  is the minimum non-zero probability, and  $\psi_{max}$  is the maximum probability.

$$(2.11) \qquad \Psi_{a,s|t,k \rightarrow j} = \begin{cases} 0 & a \leq \psi_0 \\ \frac{e^{\psi_\lambda L(a-\psi_0)-1}}{\max(\Psi_{a,s|t,k \rightarrow j})} \psi_{maxL} & \psi_0 < a \leq \psi_{peakL} \\ (\psi_{maxR} - \psi_a) e^{-\psi_\lambda R(a-\psi_0+1)} + \psi_a & a > \psi_{peakL} \end{cases}$$

where,  $\psi_0$  is the oldest age with a movement probability of zero before the first non-zero movement probability,  $\psi_{\lambda_L}$  is the slope parameter of the exponential function for the left side of the function,  $\psi_{\lambda_R}$  is the slope parameter of the exponential function for the right side of the function,  $\psi_{max_L}$  is the maximum non-zero probability on the left side of the curve,  $\psi_{max_R}$  is the maximum non-zero probability on the right side of the curve,  $\psi_{peak_L}$  is the age associated with the peak of the left curve.

$$(2.12) \quad \Psi_{a,s|t,k \rightarrow j} = \begin{cases} 0 & a \leq \psi_0 \\ \psi_{v_a} & a > \psi_0 \end{cases}$$

Overall, the following are the possible parameters in the four functions described by equations 2.9 to 2.12 that may be specific to a sex, time, and region.

1.  $\psi_0$ : The oldest age with a movement probability of zero before the first non-zero movement probability. Therefore, all ages from age 0 to age  $\psi_0$  do not move out the region they are in.
2.  $\psi_c$ : The constant non-zero probability of movement.
3.  $\psi_\lambda$ : The ‘slope’ of the exponential function in either the exponential function or associated with the left ( $\lambda_L$ ) or right ( $\lambda_R$ ) side of the double exponential.
4.  $\psi_{min}$ : The minimum non-zero probability of movement-at-age in the exponential function.
5.  $\psi_{max}$ : The maximum probability of the movement-at-age in the exponential function or for the left ( $max_L$ ) or right ( $max_R$ ) side of the double exponential function.
6.  $\psi_{peak_L}$ : The age at which the peak of the left ( $peak_L$ ) side of the double exponential occurs. This is the transition between the left and right sides of the double exponential function. This parameter is an integer and the peak of the right side is one greater than  $\psi_{peak_L}$ .
7.  $\psi_a$ : The asymptote of the right side of the double exponential function.
8.  $\psi_{v_a}$ : Specific probability-at-age. Subsequent values after the last entered age are set to the last entered age.

### 2.3.4 Mortality

These operating models contain two types of mortality: natural mortality and fishing mortality. These are described below with definitions and mathematical equations. Total mortality, the summation of natural and fishing mortality, is noted as  $Z$  and is often modelled using a differential equation describing the instantaneous change with respect to a short period of time.

$$\frac{dN}{dt} = -Z \times N$$

Expanding to a single annual time step, the numbers in the next time-step, if mortality was the only process, are

$$(2.13) \quad N_{y+1} = N_y e^{-Z}$$



However, fishing mortality is often assumed to occur at a specific point in time.

$$(2.14) \quad N_{y+1} = N_y e^{-M} (1 - U_y)$$

where  $U_y$  is an annual exploitation rate. This formulation makes the calculations simpler, faster, and easier to interpret. These equations are general mortality equations, and the specific equations related to natural and fishing mortality for the operating models are described below. We follow the FAO definition (<http://www.fao.org/3/a0212e/a0212e12.htm>) and call the fishing mortality process in Equation 2.13 instantaneous and that in Equation 2.14 finite.

## Natural Mortality

Natural mortality represents mortality from all sources other than fishing (e.g., natural causes, predation, and emigration out of the area being modelled) and may reflect some processes that are not specifically accounted for in the model. Many fisheries models assume that natural mortality is constant over time, which will likely capture the general trend in abundance, but natural mortality likely varies from time-step to time-step. Therefore, the operating model allows for natural mortality that is age, sex, time-step, and region specific, but will likely assume a single value for natural mortality for each sex.

## Fishing Mortality

Fishing mortality can be modelled using the Baranov catch equation (an instantaneous formulation as with natural mortality), but it is simpler, faster, and more interpretable to model fishing mortality as a finite exploitation rate (also called Pope’s approximation). This assumes that fishing occurs at a specific point in time, which will be an important assumption to consider when the fishery operates year round and at high mortality rates. For most applications, especially Pacific halibut with relatively low fishing mortality rates and a defined season, the exploitation rate is a useful approximation.

$$(2.15) \quad U_{|t,r,f} = \frac{C_{|t,r,f}}{B_{|t,r,f}^{sr,p_f}}$$

where  $C_{|t,r,f}$  is the catch in time-step  $t$  and region  $r$  for sector  $f$ , and  $B_{|t,r,f}^{sr}$  is the selected-and-retained biomass for that fishery. The time-point is the proportion of the time-step,  $p_f$ , at which the fishery occurs, and is commonly defined as 0.5. The fishing sectors typically operate at a scale finer than region, but region is used in the equations for fishing mortality because a single Pacific halibut may be available to any of the sectors in a region throughout an annual time step. Therefore, sectors are tracked at the region level, but may represent fishing in a particular area within a region (through selectivity and fishery timing,  $p_f$ ). If a sector operates at a greater scale than region, that sector should be divided into region-specific sectors. In other words, a sector only operates within a region. Therefore, the region and sector subscripts are redundant, but retained.

Selectivity represents the probability that a fish of a particular age will be caught by the sector. This is a combination of gear selection (e.g., the size of the hook or the width of mesh in a net) and availability (are fish of that age in the area being fished). We do not separate these components and instead model them as a single probability. The selected proportions at age generally increase

from young ages to older ages, but may also decline at the oldest ages. This is referred to as dome-shaped selectivity and may occur because older fish move out of the fishing area and become less available to the fishery, older fish may be able to avoid or escape the fishing gear, etc. Selectivity in this model is forced to asymptote at one (and not greater) for at least one age. Therefore, the exploitation rate refers to the proportion of a fully selected age-class of fish removed from the population.

The proportion selected at age can be entered specifically for each age, modelled using a logistic formulation to asymptote at one (equation 2.16),

$$(2.16) \quad S_{a,s|t,r,f} = \frac{\zeta_{max,s|t,r,f}}{1 + 19^{(\zeta_{a50,s|t,r,f} - age)/(\zeta_{a95,s|t,r,f} - \zeta_{a50,s|t,r,f})}}$$

or modelled using a double-normal function to allow for dome-shaped selectivity (equation 2.17).

$$(2.17) \quad S_{a,s|t,r,f} = \begin{cases} \zeta_{max,s|t,r,f} \frac{e^{-(a - \zeta_{peak,s|t,r,f})^2}}{2\zeta_{\sigma_L,s|t,r,f}} & age < \zeta_{peak,s|t,r,f} \\ \text{Max} \left[ \zeta_{max,s|t,r,f} \frac{e^{-(a - \zeta_{peak,s|t,r,f})^2}}{2\zeta_{\sigma_R,s|t,r,f}}, \zeta_{final,s|t,r,f} \right] & age \geq \zeta_{peak,s|t,r,f} \end{cases}$$

Parameters are described below. Examples of these two parameterizations are shown in Figure 2.3. Additional parameterizations may be introduced in the future. An option for all selectivity parameterizations is to define the age at which selectivity is zero for that age and all lower ages. This parameter is  $\zeta_{zero,s|t,r,f}$ .

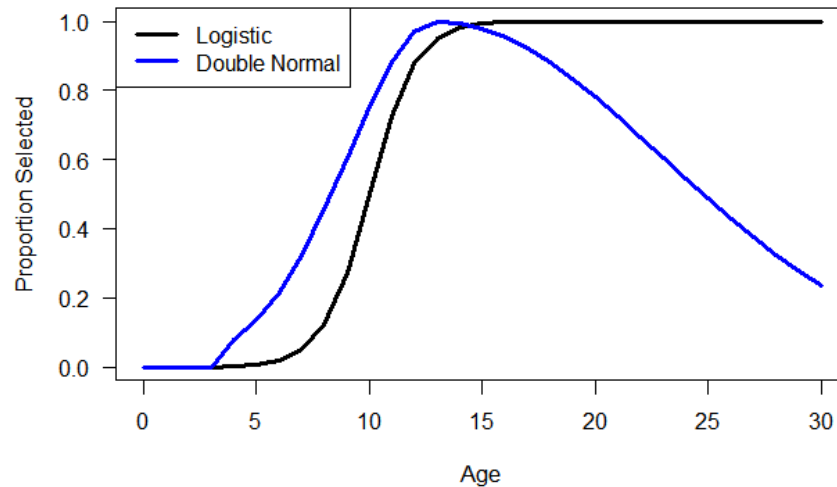
There are three parameters in the logistic function for selectivity.

1.  $\zeta_{a50,s|t,r}$ : The age at which the probability of selection is 50% for sex  $s$ , at time  $t$ , for sector  $f$ .
2.  $\zeta_{a95,s|t,r}$ : The age at which the probability of selection is 95% for sex  $s$ , at time  $t$ , for sector  $f$ .
3.  $\zeta_{max,s|t,r}$ : The maximum probability of selection (asymptote) for sex  $s$ , at time  $t$ , for sector  $f$ .

There are five parameters in the double normal function for selectivity.

1.  $\zeta_{peak,s|t,r}$ : The age at which the probability of selection is at its maximum for sex  $s$ , at time  $t$ , for sector  $f$ . This is the division between the left and right sides of the function.
2.  $\zeta_{\sigma_L,s|t,r}$ : The standard deviation of the normal distribution for ages younger than the peak age ( $\zeta_{peak,s|t,r}$ ). This side of the selection curve is referred to as the left side or ascending limb.

3.  $\zeta_{\sigma_R,s|t,r}$ : The standard deviation of the normal distribution for ages older than the peak age ( $\zeta_{peak,s|t,r}$ ). This side of the selection curve is referred to as the right side or descending limb. To create an asymptotic selectivity ogive with the double normal this parameter would be fixed at a sufficiently large value.
4.  $\zeta_{max,s|t,r}$ : The probability at the age associated with the peak. This must be set at 1.0 for one of the sexes, but may be less than 1.0 for the other sex.
5.  $\zeta_{final,s|t,r}$ : The lowest value of the function on the right side of the function. The probability for ages that would be calculated less than this value are fixed at this value.



**Figure 2.3:** Examples of the logistic and double-normal parameterizations for selectivity.

The availability of fish to a sector changes from year to year and changes may be made to gear for efficiency or to meet changes in regulation. Therefore, selectivity likely varies over time, hence the time subscript on the parameters and selectivity-at-age. Time-varying selectivity-at-age can be implemented by adjusting the parameters across time according to the methods described in section 2.3.9.

Specific terms are used to refer to fishery related quantities. Landings are the fish that are landed and quantified. These include commercial landings of O32 Pacific halibut at processing plants and Pacific halibut kept in the recreational fishery. Captured fish refers to fish that are captured by fishing gear, of which some may subsequently survive if released. Some sources may refer to that as catch, but **the term catch in this document is synonymous with landings**. Of the captured fish that are subsequently released, some may die; this is called discard mortality. The sum of catch (i.e., landings) and discard mortality is the total fishing mortality. To model total fishing mortality an exploitation rate, selectivity curve, and retention curve are needed.

Retention-at-age represents the probability that a captured fish is retained. This curve typically increases from lower probabilities at younger ages and nearing one at older ages, but often does not reach exactly one at its peak to represent the occasional discarding or loss of older/larger fish in that fishery. Low retention of young fish may represent a minimum size limit or high-grading for larger/older fish. Low retentions of older fish may represent a maximum size limit or high-grading for smaller/younger fish. It is important to use retention because it can be used to calculate the proportion of fish-at-age that are released and may suffer discard mortality.

Retention is parameterized using the same options as selectivity (direct input by age, logistic, or double-normal) The retention parameters are

1.  $\eta_{a50,s|t,r}$ : The age at which the probability of retention is 50% for sex  $s$ , at time  $t$ , for sector  $f$ .
2.  $\eta_{a95,s|t,r}$ : The age at which the probability of retention is 95% for sex  $s$ , at time  $t$ , for sector  $f$ .
3.  $\eta_{max,s|t,r}$ : The asymptote or maximum probability of retention at any age (ranges from 0 to 1).

There are five parameters in the double normal function for selectivity and one optional parameter.

1.  $\eta_{peak,s|t,r}$ : (Required) The age at which the probability of retention is at its maximum for sex  $s$ , at time  $t$ , for sector  $f$ .
2.  $\eta_{\sigma_L,s|t,r}$ : (Required) The standard deviation of the normal distribution for ages younger than the peak age ( $\eta_{peak,s|t,r}$ ). This side of the retention curve is referred to as the left side or ascending limb.
3.  $\eta_{\sigma_R,s|t,r}$ : (Required) The standard deviation of the normal distribution for ages older than the peak age ( $\eta_{peak,s|t,r}$ ). This side of the retention curve is referred to as the right side or descending limb. To create an asymptotic retention ogive with the double normal this parameter would be fixed at a sufficiently large value.
4.  $\eta_{max,s|t,r}$ : The asymptote or maximum probability of retention at any age (ranges from 0 to 1).
5.  $\eta_{final,s|t,r}$ : The lowest value of the function on the right side of the function. The probability for ages that would be calculated less than this value are fixed at this value.

The logistic function for retention is

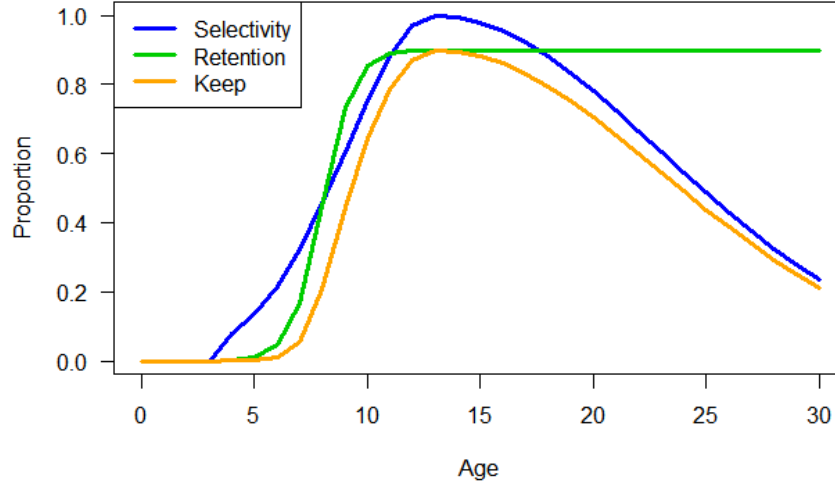
$$(2.18) \quad R_{a,s|t,r,f} = \frac{\eta_{max,s|t,r}}{1 + 19^{(\eta_{a50,s|t,r,f} - age)/(\eta_{a50,s|t,r,f} - \eta_{a95,s|t,r,f})}}$$

and the double-normal function for retention is

$$(2.19) \quad R_{a,s|t,r,f} = \begin{cases} \eta_{max,s|t,r,f} \frac{e^{-\frac{(a-\eta_{peak,s|t,r,f})^2}{2\eta_{\sigma_L,s|t,r,f}^2}}}{2\eta_{\sigma_L,s|t,r,f}} & age < \eta_{peak,s|t,r,f} \\ \text{Max} \left[ \eta_{max,s|t,r,f} \frac{e^{-\frac{(a-\eta_{peak,s|t,r,f})^2}{2\eta_{\sigma_R,s|t,r,f}^2}}}{2\eta_{\sigma_R,s|t,r,f}}, \eta_{final,s|t,r,f} \right] & age \geq \eta_{peak,s|t,r,f} \end{cases}$$

It is important to not confuse retention with selectivity. For fisheries that retain all fish, the retention curve would be one across all ages because selectivity accounts for the sector not catching

younger fish. The resulting product of retention and selectivity is called the keep curve and represents the probabilities-at-age of fish that are captured and retained, thus kept and landed. Figure 2.4 shows examples of selectivity and retention curves, and the resulting keep curve.



**Figure 2.4:** Examples of the double-normal parameterization for selectivity, a logistic parameterization for retention, and the resulting keep curve.

The sectors are assumed to operate at a very specific point of time defined as the proportion of the time-step ( $p_f$ ). Some sectors will operate before others and will affect the abundance available later in the time-step. Therefore, the sequential operation of the sectors is accounted for by applying the probability that a fish survives sectors that occurred previously (catch and discard mortality). This necessitates determining only those sectors that occurred before the sector of interest.

Catch at age and sex in numbers ( $C^n$ ) or weight ( $C$ ) for a sector in an area can be determined from the numbers-at-age ( $N_a$ ), natural mortality rate ( $M$ ), exploitation rate ( $U_f$ ), selectivity ( $S$ ), proportion that are subsequently retained ( $R$ ), mean weight-at-age ( $W$ ), and the mortality from fisheries that occurred before the sector of interest. Appendix B presents details of this method as well as a method that does not take the sequence into account.

We denote the numbers-at-age and the biomass-at-age at a particular point in time in a time-step as  $N_{a,s|t,r}^p$  and  $B_{a,s|t,r}^p$ , respectively, where the superscript indicates the proportion of the time-step. Given this notation, the catch for a particular sector is

$$\begin{aligned} C_{a,s|t,r,f}^n &= N_{a,s|t,r}^p U_{|t,r,f} S_{a,s|t,r,f} R_{a,s|t,r,f} \\ C_{a,s|t,r,f} &= N_{a,s|t,r}^p W_{a,s|t,r,f} U_{|t,r,f} S_{a,s|t,r,f} R_{a,s|t,r,f} \end{aligned}$$

and the total catch for a sector is

$$(2.20) \quad C_{|t,r,f}^n = \sum_{a=0}^A \sum_s C_{a,s|t,r,f}^n$$

$$(2.21) \quad C_{|t,r,f} = \sum_{a=0}^A \sum_s C_{a,s|t,r,f} = U_{|t,r,f} B_{|t,r,f}^{sr,p}$$

where  $B_{|t,r,f}^{sr,p}$  is the selected and retained biomass for sector  $f$ , and will be discussed later.

Discarded fish-at-age (fish caught but not retained) that suffer mortality after release (discard mortality) are an additional source of fishing mortality not accounted for in the retained catch. Discard mortality is calculated as

$$\begin{aligned} D_{a,s|t,r,f}^n &= N_{a,s|t,r}^p U_{|t,r,f} S_{a,s|t,r,f} (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f} \\ D_{a,s|t,r,f} &= N_{a,s|t,r}^p W_{a,s|t,r,f} U_{|t,r,f} S_{a,s|t,r,f} (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f} \end{aligned}$$

where  $d_{a,s|t,r,f}$  is the discard mortality rate (DMR) and  $(1 - R_{y,a,s,f}) d_{y,a,s,r,f}$  is the proportion of selected fish that are released and do not survive. The summation of catch ( $C_{|t,r,f}$ ) and discarded fish that die ( $D_{|t,r,f}$ ) is the total mortality for sector  $f$ .

$$(2.22) \quad TM_{|t,r,f} = C_{|t,r,f} + D_{|t,r,f}$$

When modelling multiple fisheries occurring at different times, the calculation of  $N^p$  incorporates the mortality from fisheries that occurred previous to the sector of interest. The code may divide a time-step into sub-time-steps, but a more efficient method can be done using the probability that a fish survives an earlier sector in that time-step. The reader is referred to Appendix B for the details. The catch for a sector is

$$(2.23) \quad \begin{aligned} C_{a,s|t,r,f}^n &= N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \times \\ &\quad \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\} \times \\ &\quad U_{|t,r,f} S_{a,s|t,r,f} R_{a,s|t,r,f} \end{aligned}$$

$$(2.24) \quad C_{a,s|t,r,f} = C_{a,s|t,r,f}^n W_{a,s|t,r}$$

where

$$N_{a,s|t,r}^p = N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\}$$

and incorporates the product of the probabilities-at-age of surviving fisheries that occurred prior to the sector of interest ( $f$ ). The total predicted catch for a sector in a region is shown in equation B.3.

Discarded fish-at-age (fish caught but not retained) that suffer mortality after release (discard mortality) is calculated in a similar manner.

$$(2.25) \quad \begin{aligned} D_{a,s|t,r,f}^n &= N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \times \\ &\quad \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\} \times \\ &\quad U_{|t,r,f} S_{a,s|t,r,f} (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f} \end{aligned}$$

$$(2.26) \quad D_{a,s|t,r,f} = D_{a,s|t,r,f}^n W_{a,s|t,r,f}$$

where  $d_{a,s|t,r,f}$  is the discard mortality rate (DMR) and  $(1 - R_{y,a,s,f}) d_{y,a,s,r,f}$  is the proportion of selected fish that are released and do not survive. The summation of catch ( $C_{|t,r,f}$ ) and discarded fish that die ( $D_{|t,r,f}$ ) is the total mortality for sector  $f$  (equation 2.22) and can be written as

$$(2.27) \quad TM_{|t,r,f}^n = \sum_{a=0}^A \sum_s N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \times \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\} \times S_{a,s|t,r,f} [R_{a,s|t,r,f} + (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f}]$$

$$(2.28) \quad TM_{t,r,f} = \sum_{a=0}^A \sum_s N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \times \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\} \times S_{a,s|t,r,f} [R_{a,s|t,r,f} + (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f}] W_{a,s|t,r,f}$$

The selected-and-retained biomass for sector  $f$  in the population is simply the catch divided by the exploitation rate, but catch is an input and selected-at-retained biomass must be calculated from selectivity ( $S$ ), proportion retained ( $R$ ), and mean weight-at-age ( $W$ ) to determine the exploitation rate  $U_f$  (equation B.11).

$$(2.29) \quad B_{|t,r,f}^{sr,p} = \frac{C_{a,s|t,r,f}}{U_{|t,r,f}} = \sum_{a=0}^A \sum_s N_{a,s|t,r} e^{-p_f M_{a,s|t,r}} \times \prod_{i \in p_j < p_f} \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\} \times S_{a,s|t,r,f} R_{a,s|t,r,f} W_{a,s|t,r,f}$$

Therefore, natural and fishing mortality can be accounted for simultaneously, and the numbers-at-age in the next time-step, accounting for all mortality, are

$$(2.30) \quad N_{a,s|t,r}'' = N_{a,s|t,r}' e^{-M_{a,s|t,r}} \prod_f \{1 - U_{|t,r,i} S_{a,s|t,r,i} [R_{a,s|t,r,i} + (1 - R_{a,s|t,r,i}) d_{a,s|t,r,i}]\}$$

See Appendix B for details.

The exploitation rate is defined to be between zero and one, but it is possible that the exploitation rate may exceed one if the calculated exploitable biomass is less than the fixed input catch for a sector. If the exploitation rate for a specific sector exceeds 1, a negative population size may occur. Therefore, a maximum exploitation rate ( $U_{max}$ ) must be specified, which is realistically less than one. If the exploitation rate for a sector exceeds the defined maximum, the exploitation rate for that sector should be set to the defined maximum. When this adjustment occurs, the predicted catch will be different than the input catch, and a penalty should be applied since catches are considered observed inputs (not data with error). This penalty will be discussed in a later section on conditioning.

### 2.3.5 Maturity

Maturity and spawning may be separated into two separate states with maturity being a part of the state object, and the numbers of mature fish specifically tracked as part of the state. However, this operating model does not partition maturity in the state, but instead simply determines the numbers of mature fish from mature proportions-at-age (called the maturity curve).

$$(2.31) \quad N_{a,s|t,r}^{ma} = N_{a,s|t,r}'' \Omega_{a,s|t,r}$$

#### Maturity functional forms and parameters

The maturity curve ( $\Omega$ ) may be an empirical vector of proportions input by the user from externally estimated data. Alternatively, the vector or proportions may be determined from a functional form, such as a logistic equation, with appropriate parameters defined.

$$(2.32) \quad \Omega_{a,s|t,r} = \frac{\omega_{max,s|t,r}}{1 + 19^{(\omega_{a50,s|t,r} - age)/(\omega_{a95,s|t,r} - \omega_{a50,s|t,r})}}$$

There are three parameters in this asymmetric logistic function.

1.  $\omega_{max,s|t,r}$ : The asymptote or maximum proportion mature at any age (ranges from 0 to 1).
2.  $\omega_{a50,s|t,r}$ : The age at which the proportion mature equals 50% of the asymptote for sex  $s$ , at time  $t$ , and in region  $r$ .
3.  $\omega_{a95,s|t,r}$ : The age at which the proportion mature is 95% of the asymptote for sex  $s$ , at time  $t$ , and in region  $r$ . Must be greater than  $\omega_{a50,s|t,r}$ .

### 2.3.6 Spawning biomass

The number of spawning individuals is the number that are mature at age times the proportion spawning at age. This allows for the accounting of individuals that are mature (able to produce gametes) but are not actively spawning in that time-step (e.g., skip spawning), and those that are mature and actively spawning in that time-step.

$$(2.33) \quad N_{a,s|t,r}^{sp} = N_{a,s|t,r}^{ma} \Phi_{a,s|t,r}$$

The spawning biomass ( $B^{sp}$ ) is calculated as follows.

$$(2.34) \quad B_{|t,r}^{sp} = \sum_{a=0}^A \sum_s N_{a,s|t,r}^{sp} W_{a,s|t,r}^1$$

where  $W_{a,s|t,r}^1$  is the weight-at-age for that age, sex, time-step, and region at the end of the time-step.



Most sex-specific stock assessments account for only the female spawning biomass, which would simply be

$$(2.35) \quad B_{s=1|t,r}^{sp} = \sum_{a=0}^A N_{a,s=1|t,r}^{sp} W_{a,s=1|t,r}^1$$

### Spawning proportion parameters

The proportions spawning ( $\Phi$ ) is a vector of proportions-at-age input by the user from externally estimated data. Typically, this vector contains a value of 1 for all ages because there is currently a paucity of information for many fish stocks.

#### 2.3.7 Size-at-age

Growth is not modelled specifically (e.g, length-at-age), but weight-at-age is used to calculate biomass-at-age from numbers-at-age. Mean weight-at-age, sex, and region for a particular time-step and sector ( $W_{a,s|t,r,f}$ , which will simply be referred to as weight-at-age regardless of the various partitions), is input by the user. Sector-specific weight-at-age are used because selectivity may operate on the larger fish of a certain age, resulting in a larger weight-at-age than in the population.

Projecting variability in weight-at-age is discussed in Section 2.5.

#### 2.3.8 Initial population

The initial population is the partitioned population numbers at the start of the first modelled time-step, and is based on unfished equilibrium recruitment ( $R_0$ ) with three potential adjustments.

1. An overall adjustment ( $e^{\delta|I}$ ) that changes  $R_0$  (i.e., the overall scale of recruitment) and could mimic a different regime that influenced the initial population.
2. Cohort ( $a$ ) specific adjustments to account for recruitment variability ( $\varepsilon_{a|I}$ ).
3. Adjustments by age to account for an average level of fishing that occurred before the initial time-step:  $\prod_f \{1 - U_{|t,r,f} S_{a,s|t,r,f} [R_{a,s|t,r,f} + (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f}]\}$ .

Calculating the initial equilibrium population size is not a simple calculation when region is in the partition and initial recruitment deviations are used. It is easiest to build up the population sequentially by each cohort that makes up the initial population using the sequence of processes described in Section 2.3: recruitment, movement, and mortality. This will be time-consuming, but only has to be done while an operating model is being conditioned.

The numbers at age 0 for the cohort that is age  $a$  in the initial time-step ( $I$ ) for each sex and region

is

$$(2.36) \quad N_{0,s|I-a,r} = p_s p_{a|I,r} e^{\delta|I} R_0 e^{\left(\varepsilon_a|I - \frac{\sigma_R^2}{2}\right)} \prod_f \{1 - U_{|I,r,f} S_{0,s|I,r,f} [R_{0,s|I,r,f} + (1 - R_{0,s|I,r,f}) d_{0,s|I,r,f}]\}$$

where  $p_s$  is the proportion of sex  $s$  at birth ( $\sum_s p_s = 1$ ) and  $p_{a|I,r}$  is the proportion of cohort  $a$  recruiting to region  $r$  in the initial time-step ( $\sum_r p_{a|I,r} = 1$ ). It is assumed that there is a single selectivity curve for the initial time period that applies equally to all cohorts and a constant exploitation rate for all cohorts over their life-span before the initial time-step.

Equation 2.36 calculates the number for each cohort when they were age 0 prior to the initial time-step. To calculate the numbers-at-age in the initial time-step, the annual process for each cohort is iterated up to the age that each cohort would be in the initial time-step. For example, to calculate the cohort that is age 3 in the initial population, Equation 2.36 would first determine the number of that cohort that were born into the population three time-steps prior. Then, the population dynamics would apply to that cohort for three iterations (0 to 1, 1 to 2, and 2 to 3) to determine the numbers in that cohort at age 3 in the initial time-step. Therefore, the annual process for the cohort that is age  $a$  in the initial time-step is iterated from  $i = a \dots 1$  in the following equations. It begins by incrementing the annual process.

$$(2.37) \quad N_{a-i+1,s|I-i+1,r} = N_{a-i,s|I-i,r}$$

Then, movement from region  $j$  to  $k$  is applied. The subscripts for age, sex, and time-step are dropped for clarity in the derivation below, but are noted in the final equation. Also note that the movement-at-age,  $Psi$ , does not change in time-steps prior to the initial time-step.

$$(2.38) \quad \begin{aligned} N'_{a-i+1,s|I-i+1,j} &= N_{|j} - N_{|j} \sum_{k \neq j} \Psi_{|j \rightarrow k} + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\ &= N_{|j} \left(1 - \sum_{k \neq j} \Psi_{|j \rightarrow k}\right) + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\ &= N_{|j} \Psi_{|j \rightarrow j} + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\ &= \sum_{k \in r} N_{a-i+1,s|I-i+1,k} \Psi_{a-i+1,s|I,k \rightarrow j} \end{aligned}$$

Finally, natural and fishing mortality is applied. Subscripts for age, sex, and time-step are dropped for clarity, but are the same as in the left side of the equation unless indicated (e.g.,  $M$ ,  $U$ ,  $S$ ,  $R$ , and  $d$ ).

$$(2.39) \quad N''_{a-i+1,s|I-i+1,r} = N'_{|r} e^{(-M|I,r)} \prod_f \{1 - U_{|I,r,f} S_{|I,r,f} [R_{|I,r,f} + (1 - R_{|I,r,f}) d_{|I,r,f}]\}$$

Graphically, the process will look like:

$U_{t=I-\{5\dots 0\}}$							
<i>Sel</i>		<i>I</i> - 5	<i>I</i> - 4	<i>I</i> - 3	<i>I</i> - 2	<i>I</i> - 1	<i>I</i>
<i>S</i> <sub>0</sub>		<i>coh</i> <sub>5<i>a</i>=0</sub>	<i>coh</i> <sub>4<i>a</i>=0</sub>	<i>coh</i> <sub>3<i>a</i>=0</sub>	<i>coh</i> <sub>2<i>a</i>=0</sub>	<i>coh</i> <sub>1<i>a</i>=0</sub>	<i>coh</i> <sub>0<i>a</i>=0</sub>
<i>S</i> <sub>1</sub>			<i>coh</i> <sub>5<i>a</i>=1</sub>	...	...	...	<i>coh</i> <sub>1<i>a</i>=1</sub>
<i>S</i> <sub>2</sub>				<i>coh</i> <sub>5<i>a</i>=2</sub>	...	...	<i>coh</i> <sub>2<i>a</i>=2</sub>
<i>S</i> <sub>3</sub>					<i>coh</i> <sub>5<i>a</i>=3</sub>	...	<i>coh</i> <sub>3<i>a</i>=3</sub>
<i>S</i> <sub>4</sub>						<i>coh</i> <sub>5<i>a</i>=4</sub>	<i>coh</i> <sub>4<i>a</i>=4</sub>
<i>S</i> <sub>5</sub>							<i>coh</i> <sub>5<i>a</i>=5</sub>

**Table 2.3**

		Years before Initial						
		...	I-5	I-4	I-3	I-2	I-1	I
Cohort							<i>coh</i> <sub>1<i>a</i>=0</sub>	<i>coh</i> <sub>0<i>a</i>=0</sub>
						<i>coh</i> <sub>2<i>a</i>=0</sub>	<i>coh</i> <sub>2<i>a</i>=1</sub>	<i>coh</i> <sub>1<i>a</i>=1</sub>
				<i>coh</i> <sub>3<i>a</i>=0</sub>	<i>coh</i> <sub>3<i>a</i>=1</sub>	<i>coh</i> <sub>3<i>a</i>=2</sub>	<i>coh</i> <sub>3<i>a</i>=3</sub>	<i>coh</i> <sub>2<i>a</i>=2</sub>
			<i>coh</i> <sub>4<i>a</i>=0</sub>	<i>coh</i> <sub>4<i>a</i>=1</sub>	<i>coh</i> <sub>4<i>a</i>=2</sub>	<i>coh</i> <sub>4<i>a</i>=3</sub>	<i>coh</i> <sub>4<i>a</i>=4</sub>	<i>coh</i> <sub>3<i>a</i>=3</sub>
		<i>coh</i> <sub>5<i>a</i>=0</sub>	<i>coh</i> <sub>5<i>a</i>=1</sub>	<i>coh</i> <sub>5<i>a</i>=2</sub>	<i>coh</i> <sub>5<i>a</i>=3</sub>	<i>coh</i> <sub>5<i>a</i>=4</sub>	<i>coh</i> <sub>5<i>a</i>=5</sub>	<i>coh</i> <sub>4<i>a</i>=4</sub>
		...	...	...	...	...	...	...

**Summary of initial population numbers**

To summarize how the initial numbers in the partitions are completed, psuedo code is provided below.

1. Determine the initial number of age zero ( $i = 0$ ) fish for the cohort (“*coh*” in the schematic representation above) of age  $a$  from Equation 2.36, where  $a = 0, 1, 2, \dots, nA$ . The  $n$  is a multiplier on the plus group age  $A$  to simulate beyond the plus group to ensure the dynamics of the plus group are correct.

2. Loop over  $a = 1, 2, \dots, nA$ . For each  $a$ ,
  - (a) loop over  $i = a, a - 1, \dots, 2, 1$  applying Equations 2.37, 2.38, and 2.39 to build up the initial numbers for each cohort at age  $a$ .
3. sum numbers over  $a = A, A + 1, \dots, nA$  to create the plus group ( $A$ ).

A value of 3 is typically used for  $n$ , but it depends on the plus group age and the time willing to spend iterating over ages.

### Calculation of other initial population values

Initial spawning biomass is calculated after the initial numbers-at-age are completed.

$$(2.40) \quad B_{|I,r}^{sp} = \sum_{a=0}^A \sum_s N_{a,s|I,r}'' \Omega_{a,s|I,r} \Phi_{a,s|I,r} W_{y,a,s,r}^1$$

### Population Dynamics

The sequence

1. *age increment*,
2. *spawning (or may occur at end)*,
3. *recruitment*,
4. *movement*,
5. *mortality*, and
6. *spawning (or may occur at beginning)*.

The notation

- $N$  : Beginning of the time-step after age increment and recruitment.  
 $N'$  : After movement before mortality  
 $N''$  : End of the time-step, after natural and fishing mortality

The equations

$$N_{y,a,s,r,m} = \begin{cases} R_{s|t,r} & a = 0 \\ N_{a-1,s|t-1,r} & 1 \leq a < A \\ N_{a-1,s|t-1,r} + N_{a,s|t-1,r} & a = A \end{cases}$$

$$N'_{a,s|t,r} = \sum_{k \in r} N_{a,s|t,k} \Psi_{a,s|t,k \rightarrow r}$$

$$N''_{a,s|t,r} = N'_{a,s|t,r} e^{-M_{a,s|t,r}} \prod_f \{1 - U_{|I,r,f} S_{|I,r,f} [R_{|I,r,f} + (1 - R_{|I,r,f}) d_{|I,r,f}]\}$$

### 2.3.9 Parameter evolution through time

A description of deviations applied to parameters.

## 2.4 Conditioning the Operating Models

### 2.4.1 Observations and Data

The current assessment used eight categories of observations and data in the modelling and fitting process (IPHC-2020-SA-01, <https://iphc.int/uploads/pdf/sa/2020/iphc-2020-sa-01.pdf>). A detailed description of the various data used in the stock assessment and related data sets are provided in IPHC-2020-SA-02 (<https://iphc.int/uploads/pdf/sa/2020/iphc-2020-sa-02.pdf>). Below is a description of the categories of data/observations and what may be available for use in conditioning the multi-area operating model, including data that may not currently be used in the stock assessment.

#### **Fishing mortality**

The mortality of Pacific halibut due to fishing (i.e., landings and discard mortality) are not treated as data in the stock assessment because they are entered as fixed, known values without error (empirical observations). Fishing mortality is a very important driver of the population dynamics.

Sectors

#### **FISS Indices**

These data are an annual relative index of abundance or biomass. They represent changes in abundance or biomass from year to year, but do not represent the absolute scale. Fishery-Independent Setline Survey (FISS) numbers-per-unit-effort (NPUE), survey weight-per-unit-effort (WPUE), and fishery catch-per-unit-effort in weight (CPUE) are available for Pacific halibut. The survey index in numbers-per-unit-effort (NPUE) is available coastwide, by Biological Region, or by IPHC Regulatory Area.

#### **FISS Age Compositions**

These data are the numbers-at-age of Pacific halibut (commonly in proportions) in the survey catches and fishery catches. Survey observations are available for each sex and two years of separate sex data are currently available for fishery landings (2017 and 2018). Differences in sex ratios inform differences in selectivity, availability, and potentially movement between areas.

## **Fishery CPUE**

These data are an annual relative index of abundance or biomass. They represent changes in abundance or biomass from year to year, but do not represent the absolute scale. Fishery-Independent Setline Survey (FISS) numbers-per-unit-effort (NPUE), survey weight-per-unit-effort (WPUE), and fishery catch-per-unit-effort in weight (CPUE) are available for Pacific halibut.

## **Fishery Age Compositions**

These data are the numbers-at-age of Pacific halibut (commonly in proportions) in the survey catches and fishery catches. Survey observations are available for each sex and two years of separate sex data are currently available for fishery landings (2017 and 2018). Differences in sex ratios inform differences in selectivity, availability, and potentially movement between areas.

## **Weight-at-age**

These data are the weight-at-age of Pacific halibut observed from various sources and are commonly summarized as the average weight-at-age. They are entered as empirical data in the stock assessment and are not involved in the fitting process.

## **Maturity-at-age**

There are limited data on maturity-at-age for Pacific halibut and a single ogive representing the probability of being mature at each age is used to calculate fecundity and entered as empirical data.

## **Environmental Observations**

The Pacific halibut stock assessment uses an index linked to average recruitment (high or low) that is developed from the Pacific Decadal Oscillation (PDO).

Additional environmental observations may be useful to condition the multi-area operating model. These may be ocean temperatures or prey abundance, for example.

## **Other Surveys**

There are many other surveys that catch Pacific halibut, including NMFS trawl and longline surveys. These data may inform abundances of various cohorts in specific areas.

## **Lengths**

Length data are not used in the age-structured stock assessment model but more samples are available coastwide than age data. They may be useful to investigate differences between areas.

## **Stock distribution**

The stock distribution estimated from FISS data are available by IPHC Regulatory Area and by Biological Region. Changes in this distribution over time may indicate differences in fishing pressure between areas and may also inform movement.

## **Tag returns**

Tagging data are useful to inform movement, migration, and mortality. There are many years of tag releases and returns for Pacific halibut, which are mostly informative of movement between specific areas. The synthesis of this information over many years can provide an insight into the movement of Pacific halibut.

### **2.4.2 Predictions from the stock assessment**

The stock assessment integrates various data sources to predict population quantities as well as uncertainty in those quantities. Four individual models are combined in an ensemble to account for structural and parameter uncertainty. Two of the individual models incorporate a short time-series starting in 1993, thus only predictions from 1993 onward can be supplied by the ensemble.

## **2.5 Projecting the Operating Models**

### **2.5.1 Recruitment**

See IPHC-2018-MSAB012-07. Discuss regimes, PDO, and recruitment variability.

### **2.5.2 Fishing mortality**

Based on the management procedure. Mimic catch sharing plans by IPHC Regulatory Area to determine allocation across sectors. Selectivity deviations for commercial fishery and survey.

### **2.5.3 Movement**

Annual variability in movement yet to be determined.

### 2.5.4 Weight-at-age

Projected values of weight-at-age are modelled using a random walk to introduce inter-annual variability. Done by region with some synchrony.

## 2.6 Reference Points

Unfished, equilibrium spawning biomass at the start of the year was found using weight-at-age, maturity-at-age, natural mortality, and unfished, equilibrium recruitment ( $R_0$ ).

$$(2.41) \quad B_0 = R_0 \left[ \sum_{a=1}^{A-1} e^{-M(a-1)} \bar{w}_a m_a + \frac{e^{-M(A-1)} \bar{w}_A m_A}{1 - e^{-M}} \right]$$

Unfished equilibrium spawning biomass (i.e.,  $B_0$ ) requires the definition of many parameters (e.g., weight-at-age) that are likely time-varying, thus the spawning biomass will fluctuate without fishing. Therefore, dynamic calculations of unfished equilibrium spawning biomass are calculated using information from recent years. This is a measure of the stock size if fishing had not occurred and is useful to calculate a stock status that is reflective of the effect of fishing and not the environment.

Dynamic unfished quantities are calculated by simulating a ‘shadow’ state alongside the fished state. All parameters (including deviations) are the same, except that the ‘shadow’ state does not have any fishing mortality. The additional processing time is minimal because the ‘shadow’ state is entirely processed in memory with almost negligible additional reading or writing to disk and the fishing processes are not called, which are typically a large part of the processing time.

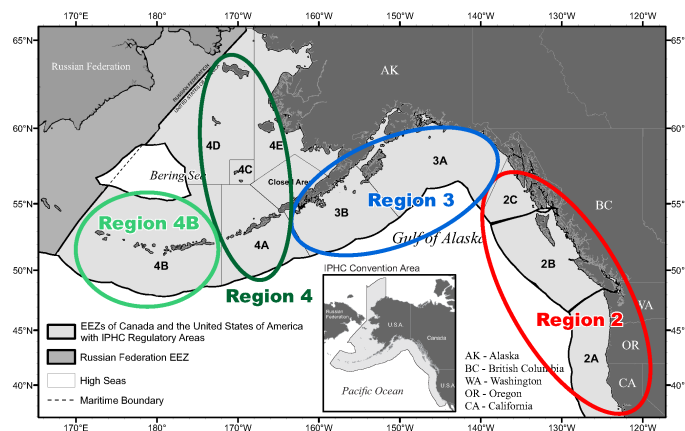
The calculation of Total Mortality based on SPR...



## Chapter 3

# Specifications for the Pacific halibut MSE framework

The Management Strategy Evaluation for Pacific halibut is currently being used to investigate management procedures to set the coastwide mortality limit (coastwide scale) and determine catch limits for each IPHC Regulatory Area (distribution of mortality limits, Figure 3.1). This requires a multi-area operating model with fleets represented within IPHC Regulatory Areas. The specifications for this MSE are provided below.



**Figure 3.1:** IPHC Regulatory Areas (grey shaded areas) and Biological Regions (colored circles).

There are many parameters that make up the Operating Model including population parameters and fishery parameters. Some of the parameters are simply a set parameter that drives the population, and are called input parameters. Derived parameters are calculated from parameters, inputs, and outputs. An example is that natural mortality ( $M$ ) is an input parameter that in combination with other parameters results in the spawning biomass (a derived parameter). Parameters are described below.

## 3.1 Management Procedures

### 3.1.1 Data Generation

The OM provides outputs that can be used to generate data from the Pacific halibut stock. In particular, for each year in the simulation the model provides numbers-at-age in the stock by sex and region, numbers-at-age in the catch by sex, region and fishery sector, numbers-at-age available to each fishery at a specific point in time, selectivity-at-age by fishery sector, and weight-at-age by sex, region and fishery sector. From these outputs it would be possible to obtain all the inputs currently used in the stock assessment. To maintain consistency between the various elements needed by the Estimation Model (EM) and with the way data are collected in reality, all quantities are first generated at an IPhC Regulatory Area level and then aggregated to a coastwide level.

**Coastwide total mortality** The total mortality (TM) for a fishery is set equal to the TCEY resulting from the application of the harvest control rule, but may be modified in the OM to account for implementation error. Therefore, the coastwide total mortality required by the assessment is obtained by summing the total mortality from each fishery.

**Proportion at age in the catch and in the survey.** Proportion-at-age in the catch by fishery sector are derived from the numbers-at-age available to each fishery sector at a specific point in time times the selectivity-at-age for each specific fishery sector. From this exploitable abundance-at-age the proportions-at-age are calculated.

$$(3.1) \quad pNAA_{a,s|t,r} = \frac{(N_{a,s|t,r} \times S_{a,s|t,f})}{\sum_{a=0}^A N_{a,s|t,r}}$$

where  $N_{a,s|t,r}$  are the numbers-at-age in the population available to a specific fishery sector  $f$ ,  $S_{a,s|t,f}$  is the selectivity by age and sex for each year and fishery sector  $f$ . Observation error is implemented by means of a Dirichlet distribution, using a sample size (3.1.1) as scale parameter. The generated proportion-at-age by fishery sector are then multiplied by the fishery sector total mortality in number, to calculate catch-at-age in numbers per each fishery sector. These catch-at-age are aggregated to a coastwide level, and the proportions-at-age by sector are re-calculated to be used in the EM.

**Sample size** The number of fish aged for each fishery (sample size) is used as the scale parameter for the Dirichlet distribution. The total coastwide number of fish aged in a specific year is also an input of the estimation model. Two options are suggested for the calculation of the sample size:

1. the sample size by Biological Region is generated from the sample size by Biological Region used in the long coastwide assessment model: in particular, the sample size is randomly drawn from the sample size available historically for each fishery sector.

2. the sample size by Biological Region is generated directly using a fixed proportion of the total mortality by fishery sector or total abundance by Biological Region:
  - sample size for the directed commercial, non-directed commercial discard mortality, recreational and subsistence sectors: for each sector, the sample size is calculated using a fixed proportion of the total mortality by Biological Region and sector.
  - sample size for survey and directed commercial discard mortality sectors: for each sector, the sample size is calculated using a fixed proportion of the available numbers-at-age by region.

The proportion chosen could be an average derived from observations on the historical data.

**Survey NPUE and commercial WPUE.** The Fishery Independent Setline Survey (FISS) NPUE ( $I_{t,r}^{sur}$ ) and the commercial WPUE ( $I_{t,r}^{comm}$ ) are needed for the EM at a coastwide level, and for the MP at a Biological Region or IPHC Regulatory Area level. The OM produce the numbers-at-age available to the survey and the numbers-at-age available to the directed commercial sectors: from these values, the exploitable abundance and biomass are calculated at a Biological Region level. These are then multiplied by the catchability for each specific Biological Region.

$$(3.2) \quad I_{t,r}^{sur} = q_{t,r}^{sur} \sum_{a=0}^A N_{a,s|t,r} \times S_{a,s|t,f}$$

$$(3.3) \quad I_{t,r}^{comm} = q_{t,r}^{comm} \times \sum_{a=0}^A N_{a,s|t,r} \times S_{a,s|t,f} \times W_{a,s|t,f}$$

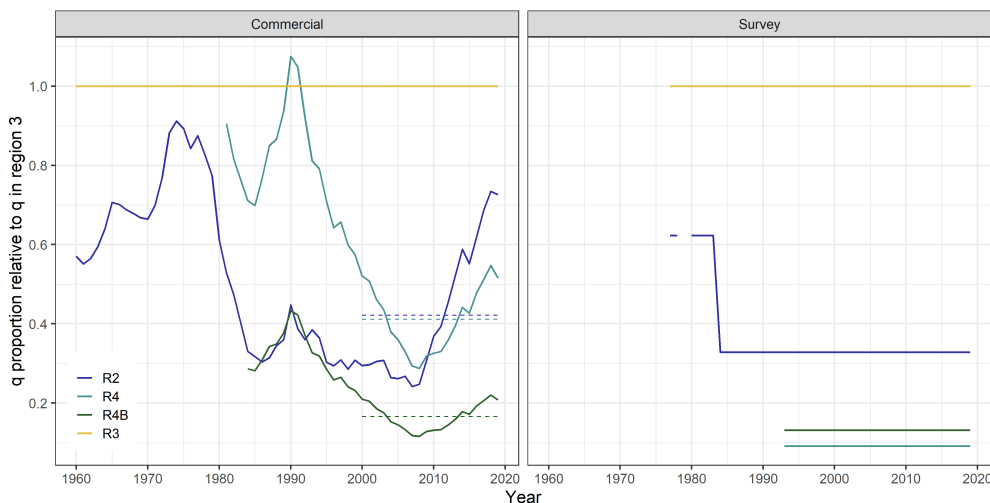
Some of the harvest control rules require the provision of the over 32 inches (O32) stock distribution at biological region and regulatory area level. In this case, the numbers-at-age are multiplied by the probability of each age to be 32 inches or bigger (see section ??):

$$(3.4) \quad I_{t,r}^{sur} = q_{t,r}^{sur} \sum_{a=0}^A N_{a,s|t,r,f} \times O32prob_{a,s} \times S_{a,s|t,f}$$

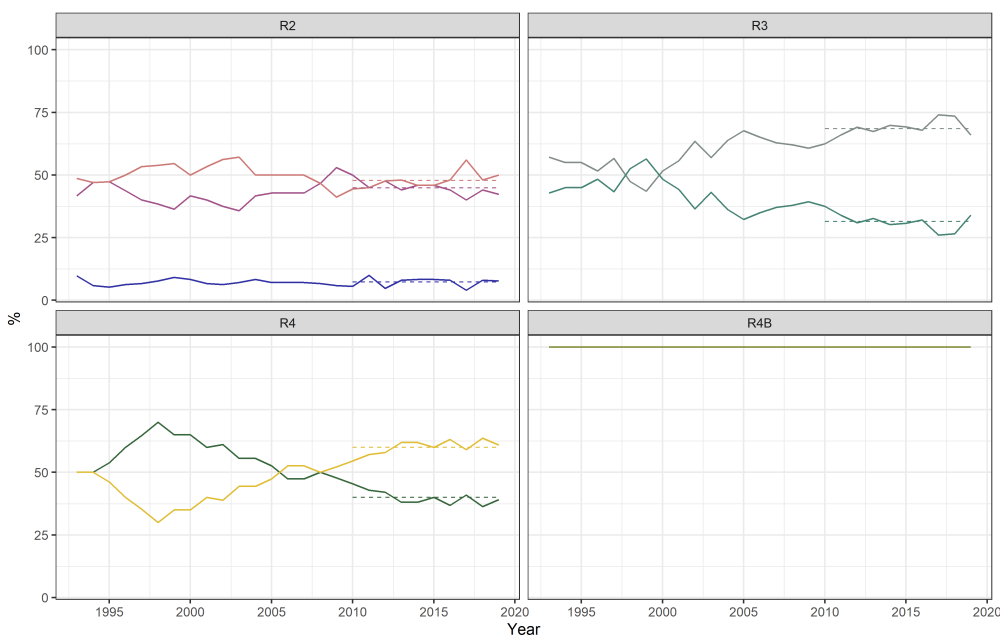
(3.5)

Regional  $q$  is specified as a relative  $q$  by region (i.e. Biological Region 3 will have  $q = 1$ , and the other regional  $q$  will be relative to this one) whose weighted mean (where the weight is the survey bottom area and the commercial catch for the NPUE and WPUE respectively) will equal the coastwide  $q$ . The proportion of each regional  $q$  relative to  $q$  in Biological Region 3 was arbitrarily fixed to the average of the last 20 years (Fig 3.2)

To derive the indices at the IPHC Regulatory Area level, as required by some of the MPs, the exploitable abundance and biomass are partitioned to each IPHC Regulatory Area using an average of the historical stock distribution from the modelled FISS survey (Fig 3.3). The catchability is assumed to be equal for each IPHC Regulatory Area in a Biological Region.



**Figure 3.2:** Proportion of each regional  $q$  relative to Biological Region 3 for the commercial WPUE (left) and the survey NPUE (right). The average of the years 2000 to 2019 is shown as a dotted line.



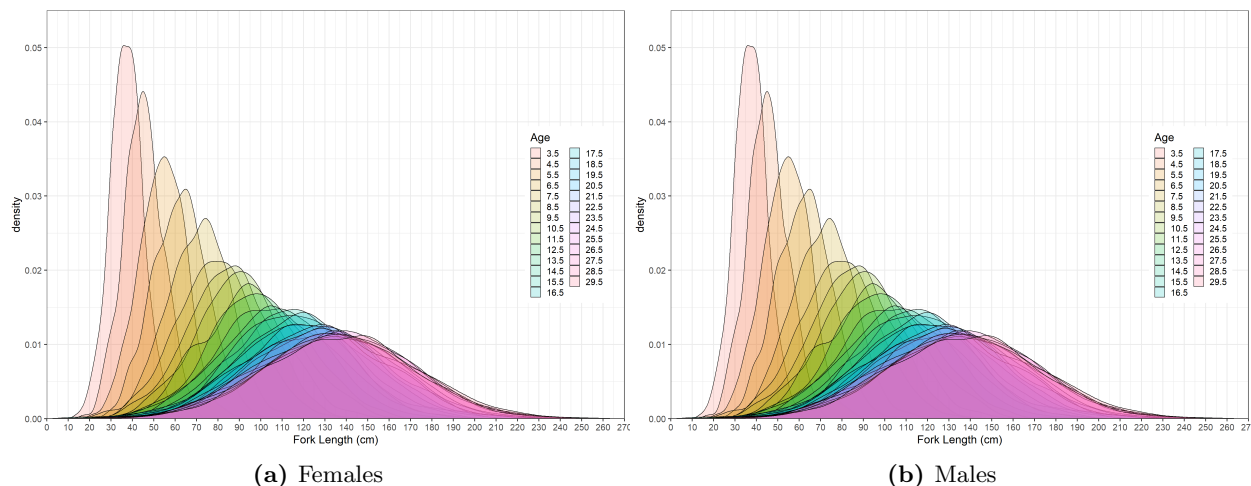
**Figure 3.3:** Proportion of stock distribution in each IPHC Regulatory Area by Biological Region from the modelled FISS survey. The 10 year average is shown as a dotted line.

In the base case the catchability coefficient is time invariant. Alternative scenarios will test a time variant catchability parameter modeled as a random walk of the coastwide  $q$  or of the regional  $q$  as a function of the abundance in each Biological Region.

The coastwide NPUE and WPUE will be calculated as the weighted average of the indices for each Biological Region. The weights are the survey bottom area and the commercial catch for the NPUE and WPUE, respectively.

### 3.1.2 Modelling length-at-age

Fish have different lengths within age groups and this variability was modelled using probability distributions. The data used to inform the model are the length-at-age distribution resulting from the long coastwide SS model used in the ensemble for Pacific halibut (section `Biology_at_age_in_endyr_with_CV` of the report file). One-thousand randomly generated values with mean equal to the average length-at-age and standard deviation of length-at-age were drawn from a normal distribution for males and females separately (Figure 3.4).



**Figure 3.4:** Length distribution at ages 3 to 25 for females and males of Pacific halibut

For each age group, the proportion of fish within each length bin  $l$  for age  $a$  and gender  $s$  is equal to:

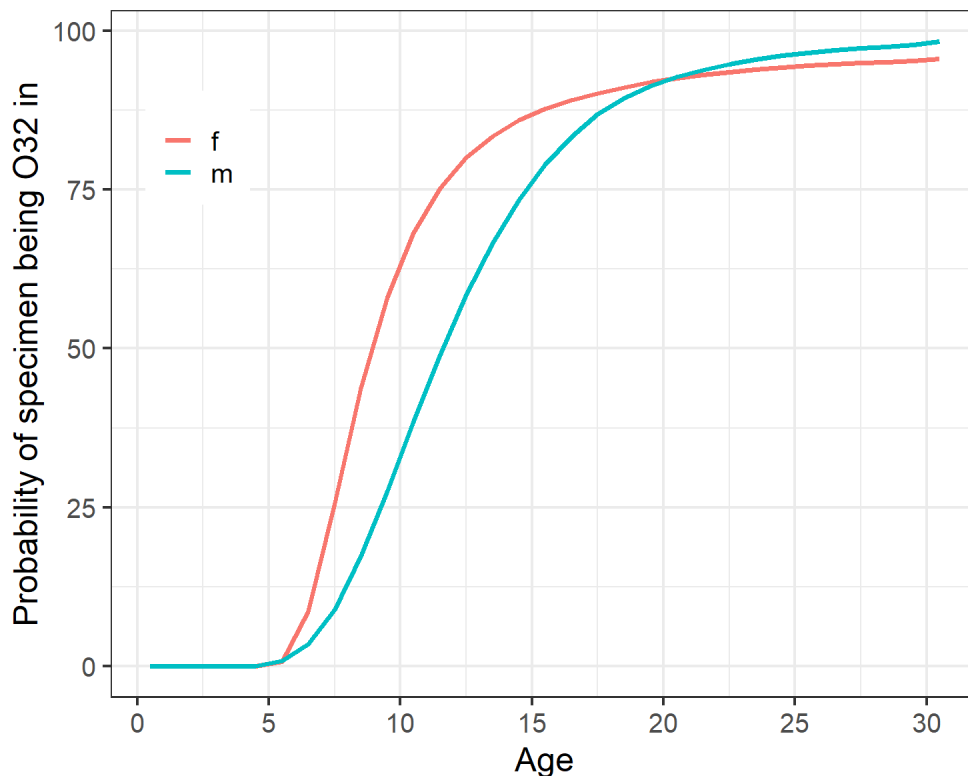
$$(3.6) \quad \Phi_{s,a,l} = \begin{cases} \Phi \frac{L'_m \text{in} - \tilde{L}_{s,a}}{\sigma_{s,a}} & l = 1 \\ \Phi \frac{L'_{l+1} - \tilde{L}_{s,a}}{\sigma_{s,a}} - \Phi \frac{L'_l - \tilde{L}_{s,a}}{\sigma_{s,a}} & 1 < l < A_l \\ 1 - \Phi \frac{L'_m \text{ax} - \tilde{L}_{s,a}}{\sigma_{s,a}} & l = A_l \end{cases}$$

where  $\Phi$  is the standard normal cumulative density function,  $L'_l$  is the lower limit of the smallest bin,  $L'_l$  is the lower limit of the length bin  $l$ ,  $L'_m \text{ax}$  is the lower limit of the largest bin.

The proportion of fish in each age class above 32 inches are then summed up (Fig 3.5).

### 3.1.3 Estimation Model

The estimation model chosen for Pacific halibut uses two stock synthesis (v. 3.30.13) models mimicking the short coastwide and long coastwide models in the stock assessment. This approach



**Figure 3.5:** Proportion of fish above 32 inches by sex

aims at capturing the correlated error and potential biases in the estimated management quantities. The two stock synthesis models are averaged to represent a simplified version of the stock assessment ensemble currently used for Pacific halibut. The coastwide models (long and short coastwide) were chosen from the four currently used in the stock assessment ensemble and were streamlined to increase efficiency and to reduce the time of the MSE simulations, yet retain the complexity and uncertainty captured by the full stock assessment ensemble. The short and long coastwide models represent the uncertainty in natural mortality rates (estimated in the long time-series but fixed for females in the short time-series), the environmental effect on recruitment (estimated only in the long time-series), as well as other structural and parameter assumptions.

The streamlining of the coastwide models consisted of:

- Reducing the amount of data included (e.g. fewer years with age composition, long coastwide model starting from 1935, etc.)
- Using the optimized version of stock Synthesis (SS 3.30.13)
- Fixing annual deviations in selectivity parameters for the historical time series, and estimate only the deviations for the most recent 10 years.
- Using the ss.par file from the original assessment model for the starting parameters values

To speed up the estimation time, the hessian is not estimated, the estimation of parameters start after the last phase (so all parameters are estimated at the same time), and screen outputs are reduced to a minimum.

See IPHC-2020-SRB-08 for more description.

### **3.1.4 Harvest Rule**

The harvest rule is the defined procedure that uses outputs from data generation and the estimation models to determine mortality limits for each fishery. Currently, this uses a fishing mortality rate based on SPR, a control rule to reduce the fishing intensity below specified stock sizes, and a distribution procedure to distribute the coastwide TCEY to each IPHC Regulatory Areas and then to each fishery. Different specifications of the harvest rule are the main focus for investigation of management procedures for Pacific halibut.

## **3.2 Population structure**

To simulate the distribution of mortality and determine how management procedures meet objectives specific to IPHC Regulatory Areas and fisheries, the operating model will have to include multiple regions with migration between them. Biological Regions (Figure 3.1) have been defined based on current knowledge of movement as well as biological understanding. A Biological Region is larger than an IPHC Regulatory Area (Figure 3.1), but fisheries operate at the level of the IPHC Regulatory Area or finer. Movement will not be specifically modelled between areas within a region, but movement will always be modelled between regions. Even though the computer program for the operating model allows flexibility to define any arrangement of regions and areas, with movement modelled between Biological Regions, it would be moot to model movement within a Biological Region on an annual time-step because it is assumed that a fish may be anywhere within the region within a year. The modelling of fisheries in separate areas is described below. Additionally, the detailed understanding of movement within a Biological Region is not well understood and would be difficult to parameterize.

### **3.2.1 Input population parameters**

### **3.2.2 Derived population parameters**

## **3.3 Structure of the fisheries**

The annual mortality limits determined for various fishery sectors occur at the level of IPHC Regulatory Areas (Figure 3.1). However, some fisheries for Pacific halibut may operate at a finer scale than an IPHC Regulatory Area. The best definition of fishery areas was determined from the objectives defined for the MSE, input from stakeholders, as well as the availability of knowledge to parameterize the fisheries for simulation, which is likely at the IPHC Regulatory Area level. This can be done when modelling the population at larger Biological Regions by defining separate sectors within a region, and using separate selectivity curves and exploitation rates to account for the availability of Pacific halibut to a particular sector (see Section ??). This assumes that each

fishery within a Biological Region operates on the same pool of fish, but each fishery encounters those fish differently.

See IPHC-2020-SRB016-08 for a description of the fisheries included in the OM.

### **3.3.1 Fishery parameters**

Parameters for each fishery were determined from the 2019 long AAF assessment model. Selectivity for the directed commercial and survey fisheries were made asymptotic because movement should account for at least some of the differences in availability between Biological Regions.



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# Appendix A

## Parameters and notation

**Table A.1:** Dimension and partition notation in the operating models (used as subscripts).

Parameter	Description
$y$	Year.
$a$	Age. A capital $A$ indicates the maximum age.
$s$	Sex, which includes female, male, and unsexed, in that order, labeled 1, 2, and 3.
$r$	Region with movement occurring between regions.
$r_l$	Area within region. Movement is not modelled between areas.
$im$	Immature state of the maturity partition.
$ma$	Mature state of the maturity partition.
$sp$	Spawning state calculated from the maturity partition. Note that this is not a specific partition in the state object.
$f$	Fishery sector.
$I$	Initial, meaning the starting time-step.

**Table A.2:** Parameters in the operating models.

Parameter	Description
<i>Population dynamics parameters</i>	
$N_{y,a,s,r,m}$	Numbers for year, age, sex, region, and maturity
$B_0$	Unfished equilibrium biomass
$R_0$	Unfished equilibrium recruitment
$B_{y,a,s,r,m}$	Biomass at the beginning of year $y$ (and possibly other partitions as noted)
$B_{y,a,s,r,m}^M$	Mature biomass at the beginning of year $y$
$B_{y,a,s,r,m}^S$	Spawning biomass at the beginning of year $y$
$B_{y,r,f}^R$	Biomass selected and retained by sector $f$ in year $y$
$M_{y,a,s,r}$	Natural mortality
$W_{y,a,s,r}$	Mean weight-at-age in year $y$ (and possibly other partitions)
$\Omega$	Proportion of mature individual at age, or the proportion transitioning from immature to mature at age, depending on partitioning maturity in the state object.
$\Phi$	Proportion of spawning individuals at age
$\Psi_{j \rightarrow k}$	Movement rate from area $j$ to area $k$
$\rho_1$	Parameter for Type II functional response of movement from area 1 to area 2
$\rho_2$	Parameter for Type II functional response of movement from area 1 to area 2
$p_s$	Proportion of females at birth.
$p_{y,r}$	Proportion of recruitment in region $r$ in year $y$ . Sums to one over regions.
$p_f$	Proportion of natural mortality that occurs before exploitation from sector $f$ occurs.

Table A.2 continued.

Parameter	Description
<i>Fishing mortality related parameters</i>	
$C_{y,a,s,r,f}$	Catch (in weight) for year, age, sex, region, and sector. Catch summed over age and sex ( $C_{y,r,f}$ ) is typically an input to the model.
$C_{y,a,s,r,f}^N$	Catch (in numbers) for year, region, and sector.
$U_{y,r,f}$	Exploitation rate for sector $f$
$S_{a,f}$	Selectivity-at-age for sector $f$
$R_{a,f}$	Proportion retained-at-age for sector $f$
<i>Survey parameters</i>	
$Z_j$	Survey index for year $j$
$q$	Survey catchability
$\tau_j$	Error in year $j$ for the survey series
$\sigma_{\tau_j}^2$	Total variability of the survey in year $j$
<i>CPUE parameters</i>	
$U_{u,i}$	CPUE for year $i$
$\alpha$	Multiplier in relationship between CPUE and abundance for the CPUE series
$\beta$	Nonlinearity parameter in relationship between CPUE and abundance
$\nu_i$	Error in year $i$ for CPUE series
$\sigma_{\nu_i}^2$	Total variability of the CPUE series in year $i$

## Appendix B

# Fishing mortality using exploitation rates: two approaches

This appendix presents two methods to determine catch and exploitation rates when modelling the fisheries with an exploitation rate (finite or Pope's approximation) instead of the instantaneous formulation (i.e. Baranov equation). The benefit of using an exploitation rate is that the code does not have to iterate to find the fishing mortality rate (i.e., there is not closed-form solution for the Baranov formulation). This will speed up the simulation time.

These equations have been simplified to show the concept. For example, retention and discard mortality are not considered, and sex and region subscripts have been omitted. The subscripts remaining are  $a$  for age,  $t$  for time-step, and  $f$  for fleet/fishery. Superscripts are used to indicate specific about the quantity, such as  $n$  for numbers and  $sr$  for selected-retained (i.e., exploitable by a particular fleet), as well as the timing within the time-step (e.g., 0.25 would be one-quarter of the natural mortality in that time-step occurred).

### B.1 Fisheries are independent and do not effect each other

To make things simpler, in a sense, we may make the assumption that the fisheries are independent of each other or they occur at exactly the same time. That means that the sequential nature of the fisheries does not have to be tracked, making the equations and accounting simpler. However, an

additional complication is introduced in making sure that the total exploitation rate (sum over all fisheries) does not exceed a defined maximum (or a value of one).

Fishing mortality is parameterized with an exploitation rate and is assumed to occur at a specific point in time after a proportion of the mortality ( $p_{|f}$ ) has occurred. The proportion may be equal for all fleets, and if not then it is assumed that the removals from a fleet operating before other fleets does not affect the biomass available to subsequent fleets (may be OK with small exploitation rates).

Catch at age and sex in numbers ( $C^n$ ) or weight ( $C$ ) for a sector in an area can be determined from the exploitation rate ( $U$ ), selectivity ( $S$ ), and mean weight-at-age ( $W$ ). Note that retention is not listed to simplify the examples shown here.

$$(B.1) \quad C_{a|t,f}^n = U_{|t,f} N_{a|t} S_{a|t,f} e^{-p_f M_{a|t}}$$

$$(B.2) \quad C_{a|t,f} = U_{|t,f} N_{a|t} S_{a|t,f} W_{a|t,f} e^{-p_f M_{a|t}}$$

and the total predicted catch for a sector in a region is

$$(B.3) \quad C_{|t,f}^n = \sum_{a=0}^A C_{a|t,f}^n$$

$$(B.4) \quad C_{|t,f} = \sum_{a=0}^A C_{a|t,f} = U_{|t,f} B_{|t,f}^{sr,p}$$

Natural and fishing mortality can be accounted for simultaneously, and the numbers-at-age in the next time-step, after all mortality, can be simply determined with a single equation. Let's assume there are two fleets with the mortality from fleet 1 occurring after three fifths of the natural mortality ( $p_{|1} = 0.6$ ) and the mortality from fleet 2 occurring after one-quarter of the natural mortality ( $p_{|2} = 0.25$ ). The catch (in numbers) for each fleet is

$$(B.5) \quad C_{a|t,f=1}^n = N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6 M_a}$$

$$(B.6) \quad C_{a|t,f=2}^n = N_{a|t} U_{|t,f=2} S_{a|t,f=2} e^{-0.25 M_a}$$

The numbers-at-age in the next year, accounting for fishing mortality by removing the catch at the appropriate time, is

$$(B.7) \quad N_{a|t+1} = \left[ \left[ N_{t,a} e^{-0.25M_a} - C_{a|f=2}^N \right] e^{-(0.6-0.25)M_a} - C_{a|f=1}^N \right] e^{-(1-0.6)M_a}$$

Converting the catch ( $C^n$ ) to exploitation rates using equation B.5 and simplifying produces the equation for N in the next time-step.

$$(B.8) \quad \begin{aligned} N_{a|t+1} &= \left[ \left[ N_{t,a} e^{-0.25M_a} - N_{a|t} U_{|t,f=2} S_{a|t,f=2} e^{-0.25M_a} \right] e^{-(0.6-0.25)M_a} - \right. \\ &\quad \left. N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} \right] e^{-(1-0.6)M_a} \\ &= \left[ N_{t,a} e^{-0.25M_a} e^{-0.35M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) - N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} \right] e^{-0.4M_a} \\ &= \left[ N_{t,a} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) - N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} \right] e^{-0.4M_a} \\ &= \left[ N_{t,a} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2} - U_{|t,f=1} S_{a|t,f=1}) \right] e^{-0.4M_a} \\ &= N_{t,a} e^{-M_a} \left( 1 - \sum_f U_{|t,f} S_{a|t,f} \right) \end{aligned}$$

This can be generalized to any set of  $p_{|f}$  as long as the proportions are sorted from smallest to largest in the derivation. The sequential nature of the fleets does not need to be accounted for in the calculations.

However, a potential problem is that the sum of the exploitation rates in equation B.14 may exceed a value of one (or some defined maximum), which is theoretically impossible. Therefore, a maximum exploitation rate ( $U_{max}$ ) must be specified, which is realistically less than one. To determine if the overall exploitation rate is greater than  $U_{max}$ , the partition-specific exploitation rates (e.g., age, sex, region, and fleet) for a time-step are summed across fleets within a region, and the maximum rate within a region over the partitions are determined. This is called  $U_y^{maxObs}$ .

$$(B.9) \quad U_{|t}^{maxObs} = \max_a \left( \sum_f S_{a|t,f} U_{|t,f} \right)$$

If  $U_y^{maxObs} > U_{max}$ , then

$$(B.10) \quad U_{|t,f} = \frac{U_{max}}{U_{|t}^{maxObs}} \frac{C_{|t,f}}{B_{|t,f}^{st,p}}$$

which is simply an adjustment to the original exploitation rate ( $U_{|t,f}$ ). When this adjustment occurs, the predicted catch will be different than the input catch, and a penalty should be applied since catches are considered observed inputs (not data with error).

Catch is an input and biomass is calculated as part of the modelling process, so the exploitation rate is calculated as the ratio between catch and exploitable biomass for a particular fleet.

$$(B.11) \quad U_{|t,f} = \frac{C_{|t,f}}{B_{|t,f}^{sr,p}}$$

where  $C_{|t,r,f}$  is the catch in time-step  $t$  and region  $r$  for sector  $f$ , and  $B_{|t,r,f}^{sr,p}$  is the selected-and-retained (exploitable) biomass for that fishery.

The exploitable biomass is calculated from the numbers-at-age ( $N$ ), selectivity ( $S$ ), and mean weight-at-age ( $W$ ).

$$(B.12) \quad B_{|t,f}^{sr,p} = \sum_{a=0}^A N_{a|t} S_{a|t,f} W_{a|t,f} e^{-p|f M_{a|t}}$$

## B.2 Fisheries are sequential and earlier fisheries effect later ones

The more appropriate way to model the fisheries, but more complex in terms of accounting, is to account for the decline in the population from fisheries occurring before later fisheries. For example, as above, let's assume there are two fleets with the mortality from fleet 1 occurring after three fifths of the natural mortality ( $p_{|1} = 0.6$ ) and the mortality from fleet 2 occurring after one-quarter of the natural mortality ( $p_{|2} = 0.25$ ). The "pulse" fishing activity of fleet 2 causes a reduction in the population by the time fleet 1 operates its fishery, and the catch (in numbers) for each fleet would be calculated as follows.

$$\begin{aligned} C_{a|t,f=2}^n &= N_{a|t} e^{-p_2 M_{a|t}} U_{|t,2} S_{a|t,2} \\ C_{a|t,f=1}^n &= \left( N_{a|t} e^{-p_2 M_{a|t}} - C_{a|t,f=2}^n \right) U_{|t,1} S_{a|t,1} e^{-(p_1-p_2) M_{a|t}} \\ &= \left( N_{a|t} e^{-p_2 M_{a|t}} - U_{|t,2} S_{a|t,2} e^{-p_2 M_{a|t}} N_{a|t} \right) U_{|t,1} S_{a|t,1} e^{-(p_1-p_2) M_{a|t}} \\ &= N_{a|t} e^{-p_2 M_{a|t}} \left( 1 - U_{|t,2} S_{a|t,2} \right) U_{|t,1} S_{a|t,1} e^{-(p_1-p_2) M_{a|t}} \\ &= N_{a|t} e^{-p_1 M_{a|t}} \left( 1 - U_{|t,2} S_{a|t,2} \right) U_{|t,1} S_{a|t,1} \end{aligned}$$

Generally,

$$(B.13) \quad C_{a|t,f}^n = U_{|t,f} S_{a|t,f} e^{-p_f M_{a|t}} N_{a|t} \prod_{f' \in p_j \neq f < p_f} (1 - U_j S_{a|t,f'})$$



where  $(1 - U_j S_{a|t,f'})$  is the probability of surviving a fishery that occurs before the fishery for fleet  $f$ . This requires some additional logic to determine which fisheries have occurred before the fishery of interest to properly account for that preceding mortality.

The numbers-at-age in the next time-step can be derived in a similar manner as above, except using the newly defined catch equations.

$$(B.14) \quad N_{a|t+1} = \left[ \left[ N_{t,a} e^{-0.25M_a} - C_{a|f=2}^N \right] e^{-(0.6-0.25)M_a} - C_{a|f=1}^N \right] e^{-(1-0.6)M_a}$$

Converting the catch ( $C^N$ ) to exploitation rates using equation B.13 and simplifying produces the equation for  $N$  in the next time-step.

$$\begin{aligned} N_{a|t+1} &= \left[ \left[ N_{a|t} e^{-0.25M_a} - N_{a|t} U_{|t,f=2} S_{a|t,f=2} e^{-0.25M_a} \right] e^{-(0.6-0.25)M_a} - \right. \\ &\quad \left. N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) \right] e^{-(1-0.6)M_a} \\ &= \left[ N_{a|t} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) - N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) \right] e^{-0.4M_a} \\ &= \left[ N_{a|t} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) (1 - U_{|t,f=1} S_{a|t,f=1}) \right] e^{-0.4M_a} \\ (B.15) \quad & N_{a|t} e^{-M_a} \prod_f (1 - U_{|t,f} S_{a|t,f}) \end{aligned}$$

This is simply the numbers-at-age in the current time-step times the survival from natural causes times the survival from each fishery. With this formulation, adjusting for a maximum exploitation rate is not necessary, other than ensuring that each fleet-specific exploitation rate does not exceed a value of one (or a defined maximum).

Catch is an input and biomass is calculated as part of the modelling process, so the exploitation rate is calculated as the ratio between catch and exploitable biomass for a particular fleet, as shown in B.11.

The exploitable biomass is calculated from the numbers-at-age ( $N$ ), selectivity ( $S$ ), and mean weight-at-age ( $W$ ), and accounts for the decrease in abundance due to fisheries that occurred

previously.

$$(B.16) \quad B_{|t,f=2}^{sr,p} = \sum_{a=0}^A N_{a|t} S_{a|t,2} W_{a|t,2} e^{-p_{|2} M_{a|t}}$$

$$(B.17) \quad B_{|t,f=1}^{sr,p} = \sum_{a=0}^A \left( N_{a|t} e^{-p_{|2} M_{a|t}} - C_{a|t,2}^n \right) S_{a|t,1} W_{a|t,1} e^{-(p_{|1}-p_{|2}) M_{a|t}}$$

$$(B.18) \quad = \sum_{a=0}^A \left( N_{a|t} e^{-p_{|2} M_{a|t}} - U_{|t,2} S_{a|t,2} e^{-p_{|2} M_{a|t}} N_{a|t} \right) S_{a|t,1} W_{a|t,1} e^{-(p_{|1}-p_{|2}) M_{a|t}}$$

$$(B.19) \quad = \sum_{a=0}^A N_{a|t} e^{-p_{|2} M_{a|t}} (1 - U_{|t,2} S_{a|t,2}) S_{a|t,1} W_{a|t,1} e^{-(p_{|1}-p_{|2}) M_{a|t}}$$

$$(B.20) \quad = \sum_{a=0}^A N_{a|t} e^{-p_{|1} M_{a|t}} (1 - U_{|t,2} S_{a|t,2}) S_{a|t,1} W_{a|t,1}$$

$$(B.21)$$

which is the same result if using the equation  $C = U \times B$ .

### B.3 Comparison

It can be shown that these two assumptions produce the exact same results when only one fishery is considered. When two or more fisheries occur at exactly the same time, the catch is exactly the same (i.e., one fishery does not occur before another, thus they operate on the same biomass), but the equation for  $N_{t+1}$  is slightly different between the two formulations. Say that two fisheries operate  $\frac{3}{5}^{ths}$  of the way through the time-step, each with an exploitation rate of 0.5. If they operated independently and each took half of the exploitable biomass, then all of the exploitable biomass would be removed since the sum of the two exploitation rates is 1. The real issue comes in when the sum of the exploitation rates is greater than 1, which is theoretically impossible.

Using the above two fisheries occurring at exactly the same time, let's assume that the exploitation rates were 0.3 and 0.4, which is still quite high for a marine commercial fishery. That is a combined exploitation rate 0.7 (assuming selectivity equals 1) and the equations for abundance in the next year (equations B.14 and B.15) are

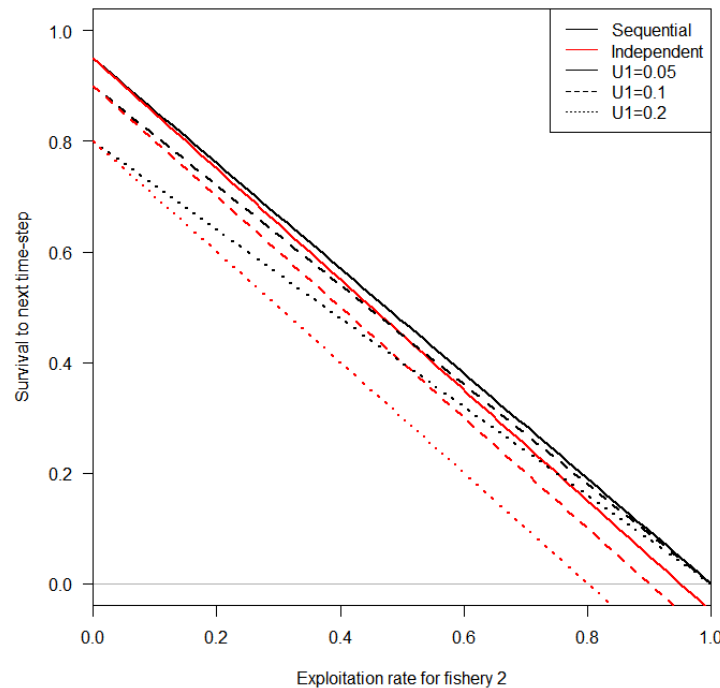
$$\text{Independent fisheries} = N_{a|t} e^{-M_a} [1 - (0.3 + 0.4)] = N_{a|t} e^{-M_a} (0.3)$$

$$\text{Sequential fisheries} = N_{a|t} e^{-M_a} (1 - 0.3)(1 - 0.4) = N_{a|t} e^{-M_a} (0.42)$$

Therefore, the number-at-age is 1.4 times greater for the sequential fishery compared to the independent fisheries. This occurs because of the product of the two exploitation rates is added back in, when expanded.

$$\begin{aligned}\text{Independent fisheries} &= [1 - (U_1 + U_2)] = (1 - U_1 - U_2) \\ \text{Sequential fisheries} &= (1 - U_1)(1 - U_2) = (1 - U_1 - U_2 + U_1U_2)\end{aligned}$$

This is especially useful when the exploitation rate is high because it never allows the exploitation rate to exceed one. For example, when  $U=1$  (the theoretical maximum) the independent fisheries equation results in a multiplier of -1, while the sequential fisheries equation results in a multiplier of 0. When exploitation rates are 0.2 for two fisheries, the difference between the two methods is small (i.e.,  $0.2 \times 0.2 = 0.04$  and 1.07 times greater for the sequential fisheries). Additionally, exploitation rates in the sequential method are more interpretable as exploitation rates and do not need adjustments to make them remain below the theoretical maximum of one. Figure B.1 shows that at small exploitation rates, the difference in survival between the two methods is small.



**Figure B.1:** A comparison of survival to the next-time step for the sequential (black) and independent (red) methods assuming exploitation rates ( $U$ ) of 0.05, 0.1, and 0.2 for fishery 1, and a range of exploitation rates for fishery 2.

It is unlikely that two fisheries operate independently when exploitation rates are high and it may be more prudent to treat them in a sequential nature, calculating the exploitation rates for each from the sequential exploitable biomass (or splitting them up into many catch events and switching back and forth between the two, which is overly complicated). We propose to use the equations under the sequential fisheries, and when two or more fisheries operate at the exact same time, the fishery operates in a sequence in order of the size of their catch for that time-step (smallest to largest).

## B.4 A more complex example with three fisheries and discard mortality

It is worth working through the concept of sequential fisheries when three fisheries occur and each has discard mortality. Let's assume there are three fleets with the mortality from fleet 1 occurring after one-fifth of the time-step ( $p_{|1} = 0.2$ ), the mortality from fleet 2 occurring after two-fifths of the time-step ( $p_{|2} = 0.4$ ), and the mortality from fleet 3 occurring after four-fifths of the time-step ( $p_{|3} = 0.8$ ). The total mortality (in numbers) includes catch and discard mortality for each fleet and would be calculated as follows (with the current time-step subscript ( $t$ ) removed for simplicity).

$$\begin{aligned}
TM_{a|f=1}^n &= C_{a|1}^n + D_{a|1}^n \\
&= N_a e^{-p_1 M_a} U_{|1} S_{a|1} R_{a|1} + N_a e^{-p_1 M_a} U_{|1} S_{a|1} (1 - R_{a|1}) d_{a|1} \\
&= N_a e^{-p_1 M_a} U_{|1} S_{a|1} [R_{a|1} + (1 - R_{a|1}) d_{a|1}] \\
TM_{a|f=2}^n &= \left( N_a e^{-p_1 M_a} - TM_{a|1}^n \right) U_2 S_{a|2} [R_{a|2} + (1 - R_{a|2}) d_{a|2}] e^{-(p_2 - p_1) M_a} \\
&= N_a e^{-p_2 M_a} U_2 S_{a|2} [R_{a|2} + (1 - R_{a|2}) d_{a|2}] [1 - U_1 S_{a|1} [R_{a|1} + (1 - R_{a|1}) d_{a|1}]] \\
TM_{a|f=3}^n &= \left[ \left( N_a e^{-p_1 M_a} - TM_{a|1}^n \right) e^{-(p_2 - p_1)} - TM_{a|2}^n \right] U_3 S_{a|3} [R_{a|3} + (1 - R_{a|3}) d_{a|3}] e^{-(p_3 - p_2) M_a} \\
&= N_a e^{-p_3 M_a} U_3 S_{a|3} [R_{a|3} + (1 - R_{a|3}) d_{a|3}] \prod_{i=1}^2 \{1 - U_i S_{a|i} [R_{a|i} + (1 - R_{a|i}) d_{a|i}]\}
\end{aligned}$$

Generally,

$$(B.22) \quad TM_{a|f}^n = N_a e^{-p_f M_a} U_f S_{a|f} [R_{a|f} + (1 - R_{a|f})d_{a|f}] \times \prod_{i \in p_j < p_f} \{1 - U_i S_{a|i} [R_{a|i} + (1 - R_{a|i})d_{a|i}]\}$$

where  $j$  is over all fleets. In essence, the total mortality is determined from the numbers-at-age that survived naturally to that point and survived the probability of fishing mortality (retained or discarded and died) from all fleets up to that point in the time-step.