

# TECHNICAL DETAILS OF THE IPHC MSE FRAMEWORK

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# Chapter 1

## Introduction

This technical document describes the Management Strategy Evaluation (MSE) framework and its elements, details specifications of the framework for the evaluation of scale and distribution management procedures, provides definition of terms used, and defines the technical details of the models and equations used within the framework. This is a working document that will be revised often as development of the MSE framework progresses. Therefore, this document is currently incomplete and will have occasional revisions.

The engine of the MSE framework consists of a closed-loop simulation with an operating model and a management procedure (Figure 1.1).

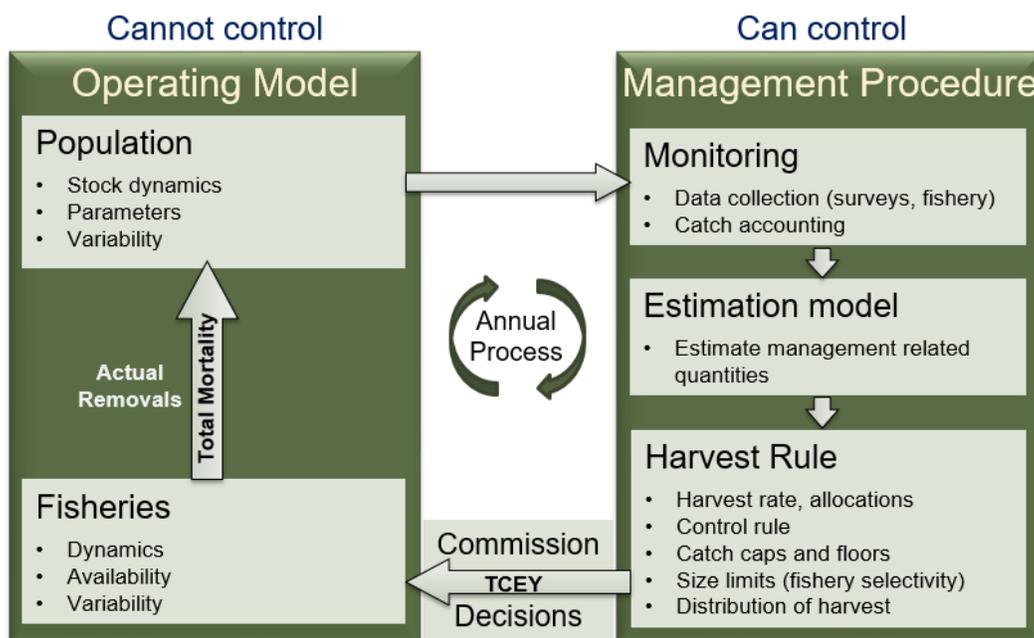


Figure 1.1: An illustration of the closed-loop simulation within the MSE framework consisting of an operating model and a management procedure.

## Chapter 2

# Operating model

In a management strategy evaluation (MSE), operating models (OM) simulate the population and fishery dynamics. It incorporates life-history processes such as recruitment, growth, migration, maturation, and mortality of the fish population, as well as fishery processes such as selectivity, availability, and catchability. Descriptions of the various processes are provided below along with the mathematical equations used to simulate those processes. Many of the details are drawn from the Hilborn and Walters (1992), Quinn and Deriso (1999), the CASAL manual Bull *et al.* (2012), Stock Synthesis technical details Methot and Wetzel (2013), and the Coleraine manual Hilborn *et al.* (2000).

There is uncertainty in the parameterization of the processes, natural variability in the processes, and multiple hypotheses about the mechanisms of the processes. These three sources of variability are introduced in three different ways.

1. Parameter uncertainty is introduced by conditioning the operating model to data, and determining the distribution of uncertainty for each parameter as well as correlation with other parameters. Parameter values for an individual simulated trajectory are randomly drawn from the multivariate estimated probability distribution. Therefore, each simulated trajectory uses a different set of parameters, thus including variability that represents the uncertainty in the parameters. This is described in Section 2.4.
2. Natural variability is introduced by defining a random process associated with various concepts. For example, recruitment varies naturally and is modelled by including random deviates applied annually to average recruitment. Other processes may have specific patterns such as changes in weight-at-age. This is described in Section 2.5.
3. Structural uncertainty is included by defining multiple hypotheses and implementing them as separate operating models. For example, growth may occur in different ways between models. Or, data may be structured in a different way when conditioning the model. Structural uncertainty captures the variability that can not be captured by the two methods above.

Parameters that will have uncertainty are defined (and those that are fixed are given fixed values), methods to include natural variability are defined, and potential areas of structural uncertainty are noted.

## 2.1 The state object

The state is the accounting of the population in numbers within an operating model and is contained in a state object with many dimensions. The state represents the intrinsic characteristics of the modeled population: age, maturity, and sex. This state is then evolved on a computational domain parameterized by time and space, which are extrinsic variables. Furthermore, sectors (fisheries and survey) interact with the state. Clarified this way, the state object contains a representation of the stock at a place and time and can be subsetted along any of these axes as needed to determine the state for any combination of these dimensions.

The dimensions are fixed inputs that are defined by the user, thus may be unique to any operating model. The different dimensions, and maximum ranges, are shown in Table 2.1. Maturity state (immature or mature) is not included as a dimension here (specifically for the Pacific halibut operating model) but may be a useful characteristic to track for some stocks, depending on fishing intensity and the proportion maturing at age. Instead, the mature population is determined using the proportion mature at age, which can be applied to various dimensions of the population state (see Section 2.3).

Table 2.1: Partitions of the state object that are fixed inputs and the likely minimum and maximum input for each partition.

Variable	Dimension		Description
	Min	Max	
Age ( $a$ )	1	251	Age classes ranging from 0 to 250. Halibut will likely use 0 to 30 and age always starts at zero. A capital $A$ indicates the maximum age
Sex ( $s$ )	1	3	Sex, which includes female, male, and unsexed, in that order, labeled 1, 2, and 3.
Time ( $t$ )	0	$\infty$	A minimum and maximum time-step (e.g., year) is input by user. The difference +1 determines the number of time-steps. These are not projected time-steps, but time-steps modelled to condition the OM. A time-step will typically be a year, but specific points in time (e.g., beginning, middle, or end of the year) may be noted in a superscript (see below).
Region ( $r$ )	1	$\infty$	Number of spatial regions with migration between
Area within Region ( $r_l$ )	1	$\infty$	Number of areas within a region. Migration is not modelled between areas.
Sector ( $f$ )	1	$\infty$	Number of fishing-related sectors, which includes fisheries and surveys. Sectors typically will operate at the region level or a finer scale, but there may be a case where a sector operates across regions (which is unlikely for the Pacific halibut operating model).

The state object is the key component of the population dynamics and must contain sufficient information to determine the population dynamics as well as any intermediate calculations, such as fishery catches. In this implementation, there are always six partitions, but some operating models may have a partition with only one element, effectively eliminating that dimension. For example, a single-sex, single-area model with no maturity partition would simply be a matrix of years and ages.

## 2.2 Notation

Notation of the variables in the operating model uses the concept of defining a quantity of the population (such as numbers or biomass), subscripted by various characteristics (intrinsic and extrinsic) and superscripted by specific concepts (such as spawning or exploitable). The subscripts reflect the intrinsic and extrinsic characteristics of the population by listing the intrinsic characteristics first (age and sex), followed by the extrinsic characteristics (time, region/area, and fishing sector). The possible subscripts are defined in Table 2.1 and are always subscripted in the order presented in those tables. For example, the numbers for age and sex in a year and region is  $N_{a,s|y,r}$ . When a subscript is not included, it is implied that the quantity is a summation over that index (or the index doesn't apply, as in the case of fishing sector) and ambiguity will be alleviated using the letter associated with the index when necessary (e.g.,  $N_{s=1|y=1}$  is the number of females in year 1 summed over all ages and regions).

Variables specific to a fishing sector ( $f$ ) include a subscript for that sector at the end. For example, the catch-at-age for females from sector  $f$  in year 1 and region 2 would be notated as  $C_{a,1|1,2,f}$ . Fishery sectors typically will operate at the region level or a finer scale, but the region subscript is retained for clarity and in case a sector does operate across regions (which is unlikely for the Pacific halibut operating model).

Finally, superscripts are used to notate specific concepts such as spawning biomass, which would be notated as  $B_{s=1|y=1}^{sp}$  to represent the spawning biomass for females in year 1 over all regions. Additionally, a superscript that is a number between 0 and 1 indicates the time in the year that the quantity is calculated. For example,  $B^{sp,0}$ ,  $B^{sp,0.5}$ ,  $B^{sp,1}$  would be the spawning biomass calculated at the beginning of the year, middle of the year, and end of the year, respectively. Possible superscripts and their definition are shown in Table 2.2.

Table 2.2: Superscripts for variable and their meaning.

Variable	Description
ma	Mature
sp	Spawning. Most often used with $B^{sp}$ to represent spawning biomass.
sr	Selected and retained referring to the fish that are landed by a fishery sector.
n	Numbers. Indicates that a quantity, such as catch, is in numbers ( $C^n$ ). Note that if a superscript is not used on catch, it is in weight.
'	Denotes update made to numbers-at-age after partial timestep is complete, which includes the effect of movement but not mortality.
''	Denotes update made to numbers-at-age at the end of the timestep.
Number 0-1	A number between zero and 1 (inclusive) indicates the time within the year. For example, 0 indicates beginning of the year, 0.5 indicates middle of the year, and 1 indicates end of the year.

## 2.3 Population dynamics

The population dynamics are modelled as an age-structured annual process accounting for changes in the numbers-at-age for each partition within the state (e.g., age and sex).

The sequence of processes from the start of the time-step (typically annual) is

1. *age increment*,

2. *recruitment (based on spawning biomass in a previous time-step),*
3. *movement,*
4. *mortality.* The sequence of mortality from all sources is theoretically described below, but does not need to be specifically modelled as such because the mortality calculations will appropriately account for the sequence, as described in Section 2.3.4 and a later Appendix.
  - (a) portion of natural mortality,
  - (b) fishing mortality for one or more sectors,
  - (c) portion of natural mortality,
  - (d) fishing mortality for one or more sectors,
  - (e) etc., until a full time-step of natural mortality has been applied
5. *spawning.*

The state object ( $N_{a,s|t,r}$ ) is updated at three different points in the annual process, and superscripts note the time point.

- $N$  : Beginning of the time-step after age increment and recruitment of age 0.
- $N'$  : After movement before mortality
- $N''$  : End of the time-step, after all natural and fishing mortality

At any point in time, the biomass may be desired and can be calculated from numbers-at-age ( $N_{a,s|t,r}$ ) and weight-at-age ( $W_{a,s|t,r}$ ).

$$(2.1) \quad B_{a,s|t,r} = N_{a,s|t,r} W_{a,s|t,r}$$

Various partitions of biomass may be desired. For example, spawning biomass is the weight of spawning fish, and exploitable biomass is the weight of fish available to a specific fishery. Biomass can also be calculated at specific points of time in the time-step. These various types of biomass will be defined in the sections below, and will be noted with a superscript. For example, spawning biomass is  $B_y^{sp}$ .

This section describes the technical specifications of the general population dynamics and how the historical population can be modelled given inputs such as catch and weight-at-age, as well as parameters that may be fixed or estimated from data. Conditioning the operating model is the process of determining the range of parameters and hypotheses that describe the observations, and is covered in Section 2.4. Projecting the population forward in time is discussed in Section 2.5, and involves defining random and fixed processes such as recruitment and changes in weight-at-age.

### 2.3.1 Age increment

The numbers-at-age at the beginning of the time-step with an annual time-step is obtained by incrementing the previous time-step's age class to one time-step older and calculating recruitment for age 0.

$$(2.2) \quad N_{a,s|t,r} = \begin{array}{ll} R_{s|t,r} & a = 0 \\ N_{a-1,s|t-1,r} & 1 \leq a < A \\ N_{a-1,s|t-1,r} + N_{A,s|t-1,r} & a = A \end{array}$$

### 2.3.2 Recruitment

Recruitment is a function of the spawning biomass calculated from the end of the previous time-step after all of the processes (movement and mortality) have occurred. See Section 2.3.6 for a description of spawning biomass.

$$(2.3) \quad R_{s|t,r} = p_{y,r} \times f(B_{s=1|t-1}^{sp,1}) \times e^{(\varepsilon_{y,r} - \frac{\sigma_{R}^2}{2})} \times e^{I_y \delta}$$

where  $p_{y,r}$  is the proportion recruiting to region  $r$  in time-step  $t$ ,  $f(B_{s=1|t-1}^{sp,1})$  is the equilibrium stock-recruit relationship using the end of the time-step spawning biomass (superscripts) for females from the previous time-step (subscripts),  $e^\varepsilon$  is the annual deviation in recruitment for time-step  $t$ , and  $e^{I_y \delta}$  is an overall adjustment for recruitment regime shift.

TODO: Additional details on recruitment need to be added, including the definition of lognormal deviations, environmental effects, and summing proportions across regions.

#### Density-dependent Recruitment

Density-dependence in the spawner-recruit relationship is modelled using a Beverton-Holt formulation.

$$(2.4) \quad f(B_{s=1|t-1}^{sp,1}) = \frac{aB_{s=1|t-1}^{sp,1}}{1 + bB_{s=1|t-1}^{sp,1}}$$

where the parameters  $a$  and  $b$  are determined from steepness ( $h$ ), unfished equilibrium recruitment ( $R_0$ ), and unfished equilibrium spawning biomass ( $B_0^{sp}$ ).

#### Recruitment Distribution

TODO:

### 2.3.3 Movement

In its most simple form, movement (also called migration) is the proportion of individuals that move from region  $j$  to region  $k$  (individuals can only move among regions and movement among areas within regions is not explicitly modelled). The probability that the individual stays in its current region is equal to one minus the sum of the probabilities of moving out of the current region. Movement is specific to the partitions age, sex, time-step, and region.

One of the most common ways to model movement is using a transition matrix. Let  $\Psi_{j \rightarrow k}$  be the instantaneous movement from region  $j$  to region  $k$  expressed as the proportion of the population in region  $j$  moving to region  $k$ . The diagonal of the transition matrix will be the proportion that stay in region  $j$  and the off-diagonal of  $\Psi_{j \rightarrow k}$  will represent the proportion that move out of region  $j$ . The row of the matrix corresponds to  $j$  (from) and  $k$  corresponds to the column of the matrix (to). Each row of the transition matrix should sum to 1. Each dimension of the transition matrix will be equal to the number of regions. For example, let there be  $n$  regions ( $R = 1..n$ ), then the transition matrix for each age, sex, and time-step will look like:

$$M = \begin{bmatrix} \Psi_{R=1} & \Psi_{1 \rightarrow 2} & \cdots & \Psi_{1 \rightarrow n} \\ \Psi_{2 \rightarrow 1} & \Psi_{R=2} & \cdots & \Psi_{2 \rightarrow n} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{n \rightarrow 1} & \Psi_{n \rightarrow 2} & \cdots & \Psi_{R=n} \end{bmatrix}$$

The numbers-at-age in region  $j$  after movement, for a given age, sex, and time-step, is determined from the following equation.

$$\begin{aligned} N'_{a,s,t,r=j} &= N_{|r=j} - N_{|r=j} \sum_{k \neq j} \Psi_{a,s|t,j \rightarrow k} + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\ &= N_{|r=j} \left( 1 - \sum_{k \neq j} \Psi_{a,s|t,j \rightarrow k} \right) + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\ &= N_{|r=j} \Psi_{a,s|t,j \rightarrow j} + \sum_{k \neq j} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \\ (2.5) \quad &= \sum_{k \in r} N_{|r=k} \Psi_{a,s|t,k \rightarrow j} \end{aligned}$$

### Movement parameters

There are two options for the construction of the transition matrix: 1) entered as simple proportions in an array by time-step, age, sex, and of course regions (with filling in duplicated partitions made easy), or 2) parameterize the proportions-at-age as a function of age and modify the parameters of the function for each time-step and sex. An example could be the use of a logistic function or a type-II functional form, outlined below.

$$(2.6) \quad \text{Logistic : } \Psi_{a,s|t,k \rightarrow j} =$$

$$(2.7) \quad \text{Type II : } \Psi_{a,s|t,k \rightarrow j} =$$

### 2.3.4 Mortality

These operating models contain two types of mortality: natural mortality and fishing mortality. These are described below with definitions and mathematical equations. Total mortality, the summation of natural and fishing mortality, is noted as  $Z$  and is often modelled using a differential equation describing the instantaneous change with respect to a short period of time.

$$\frac{dN}{dt} = -Z \times N$$

Expanding to a single annual time step, the numbers in the next time-step, if mortality was the only process, are

$$(2.8) \quad N_{y+1} = N_y e^{-Z}$$

However, fishing mortality is often assumed to occur at a specific point in time.

$$(2.9) \quad N_{y+1} = N_y e^{-M} (1 - U_y)$$

where  $U_y$  is an annual exploitation rate. This formulation makes the calculations simpler, faster, and easier to interpret. These equations are general mortality equations, and the specific equations related to natural and fishing mortality for the operating models are described below. We follow the FAO definition (<http://www.fao.org/3/a0212e/a0212e12.htm>) and call the fishing mortality process in Equation 2.8 instantaneous and that in Equation 2.9 finite.

## Natural Mortality

Natural mortality represents mortality from all sources other than fishing (e.g., natural causes, predation, and emigration out of the area being modelled) and may reflect some processes that are not specifically accounted for in the model. Many fisheries models assume that natural mortality is constant over time, which will likely capture the general trend in abundance, but natural mortality likely varies from time-step to time-step. Therefore, the operating model allows for natural mortality that is age, sex, time-step, and region specific, but will likely assume a single value for natural mortality for each sex.

Given a single time step, the numbers-at-age in the next time step, without fishing and recruitment (i.e., no additions to the population), are

$$(2.10) \quad N''_{a,s|t,r} = N'_{a,s|t,r} e^{-M_{a,s|t,r}}$$

## Fishing Mortality

Fishing mortality can be modelled using the Baranov catch equation (an instantaneous formulation as with natural mortality), but it is simpler, faster, and more interpretable to model fishing mortality as a finite exploitation rate (also called Pope's approximation). This assumes that fishing occurs at a specific point in time, which will be an important assumption to consider when the fishery operates year round and at high mortality rates. For most applications, especially Pacific halibut with relatively low fishing mortality rates and a defined season, the exploitation rate is a useful approximation.

$$(2.11) \quad U_{|t,r,f} = \frac{C_{|t,r,f}}{B^{sr,p}_{|t,r,f}}$$

where  $C_{|t,r,f}$  is the catch in time-step  $t$  and region  $r$  for sector  $f$ , and  $B^{sr,p}_{|t,r,f}$  is the selected-and-retained biomass for that fishery. The time-point is the proportion of the time-step,  $p_{|r,f}$ , at which the fishery occurs, as shown in Equation (2.12) and is commonly defined as 0.5. The fishing sectors typically operate at a scale finer than region, but region is used in the equations for fishing mortality because a single Pacific halibut may be available to any of the sectors in a region throughout an annual time step. Therefore, sectors are tracked at the region level, but may represent fishing in a particular area within a region (through selectivity and fishery timing,  $p_f$ ). If a sector operates at a greater scale than region, certain quantities will be calculated by region and summed across appropriate regions. Further detail on the spatial scale on which fisheries operate will be added at a later date.

The selected-and-retained biomass is calculated from selectivity ( $S$ ), proportion retained ( $R$ ), and mean weight-at-age ( $W$ ).

$$(2.12) \quad B^{sr,p}_{|t,r,f} = \sum_{a=0}^A \sum_s N_{a,s|t,r} S_{a,s|t,r,f} R_{a,s|t,r,f} W_{a,s|t,r} e^{-p_{|r,f} M_{a,s|t,r}}$$

where  $p_{|r,f}$  is the proportion of natural mortality that occurs before the “pulse” fishery takes place. The proportion retained represents fish that are caught and subsequently retained or landed. Fish that are released may suffer discard mortality, which is discussed below.

Calculating exploitation rates at the region level recognizes that all fish in a region are available to a fishery sector, and the exploitation rate is relative to the exploitable biomass in the region for that sector. Therefore, regions should be defined appropriately in accordance with fishery sectors.

Catch at age and sex in numbers ( $C^n$ ) or weight ( $C$ ) for a sector in an area can be determined from the exploitation rate ( $U$ ), selectivity ( $S$ ), proportion that subsequently die ( $R$ ), and mean weight-at-age ( $W$ ).

$$(2.13) \quad C_{a,s|t,r,f}^n = U_{|t,r,f} N_{a,s|t,r} S_{a,s|t,r,f} R_{a,s|t,r,f} e^{-p_{r,f} M_{a,s|t,r}}$$

$$(2.14) \quad C_{a,s|t,r,f} = U_{|t,r,f} N_{a,s|t,r} S_{a,s|t,r,f} R_{a,s|t,r,f} W_{a,s|t,r} e^{-p_{r,f} M_{a,s|t,r}}$$

and the total predicted catch for a sector in a region is

$$(2.15) \quad C_{|t,r,f}^n = \sum_{a=0}^A \sum_s C_{a,s|t,r,f}^n$$

$$(2.16) \quad C_{|t,r,f} = \sum_{a=0}^A \sum_s C_{a,s|t,r,f}$$

Discarded fish-at-age (fish caught but not retained) that suffer mortality after release are an additional source of mortality not captured in the retained catch. Discard mortality is calculated as

$$(2.17) \quad D_{a,s|t,r,f}^n = U_{|t,r,f} N_{a,s|t,r} S_{a,s|t,r,f} (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f} e^{-p_{r,f} M_{a,s|t,r}}$$

$$(2.18) \quad D_{a,s|t,r,f} = U_{|t,r,f} N_{a,s|t,r} S_{a,s|t,r,f} (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f} W_{a,s|t,r} e^{-p_{r,f} M_{a,s|t,r}}$$

where  $d_{a,s|t,r,f}$  is the discard mortality rate (DMR) and  $(1 - R_{y,a,s,f}) d_{y,a,s,r,f}$  is the proportion of selected fish that are released and do not survive. The summation of catch ( $C_{|t,r,f}$ ) and discarded fish that die ( $D_{|t,r,f}$ ) is the total mortality for sector  $f$  ( $TM_{|t,r,f}$ ).

Natural and fishing mortality can be accounted for simultaneously, and the numbers-at-age in the next time-step, after all mortality, are

$$(2.19) \quad N''_{a,s|t,r} = N'_{a,s|t,r} e^{-M_{a,s|t,r}} \left[ 1 - \sum_f U_{|t,r,f} S_{a,s|t,r,f} [R_{a,s|t,r,f} - (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f}] \right]$$

The exploitation rate is defined to be between zero and one, but it is possible that the exploitation rate may exceed one if the calculated exploitable biomass is less than the fix input catch for either a single fishing fleet or the summation of multiple fleets. If the sum of exploitation rates for a specific partition exceeds 1, a negative population size for that partition will occur. Therefore, a maximum exploitation rate ( $U_{max}$ ) must be specified, which is realistically less than one. To determine if the overall exploitation rate is greater than  $U_{max}$ , the partition-specific exploitation rates (e.g., age, sex, region, and fleet) for a time-step are summed across fleets within a region, and the maximum rate within a region over the partitions are determined. This is called  $U_{y,r}^{maxObs}$ .

$$(2.20) \quad U_{|t,r}^{maxObs} = \max_{a,s} \left( \sum_f S_{a,s|t,r,f} R_{a,s|t,r,f} U_{|t,r,f} \right)$$

If  $U_{y,r}^{maxObs} > U_{max}$ , then

$$(2.21) \quad U_{|t,r,f} = \frac{U_{max}}{U_{|t,r}^{maxObs}} \frac{C_{|t,r,f}}{B^{sr}_{|t,r,f}}$$

which is simply an adjustment to the original exploitation rate ( $U_{|t,r,f}$ ) calculated in Equation 2.11. When this adjustment occurs, the predicted catch will be different than the input catch, and a penalty should be applied since catches are considered observed inputs (not data with error). This penalty will be discussed in a later section on conditioning.

### 2.3.5 Maturity

Maturity and spawning may be separated into two separate states with maturity being a part of the state object, and the numbers of mature fish specifically tracked as part of the state. However, this operating model does not partition maturity in the state, but instead simply determines the numbers of mature fish from mature proportions-at-age (called the maturity curve).

$$(2.22) \quad N_{a,s|t,r}^{ma} = N_{a,s|t,r}'' \Omega_{a,s|t,r}$$

#### Maturity functional forms and parameters

The maturity curve ( $\Omega$ ) may be an empirical vector of proportions input by the user from externally estimated data. Alternatively, the vector or proportions may be determined from a functional form, such as a logistic equation, with appropriate parameters defined.

### 2.3.6 Spawning biomass

The number of spawning individuals is the number that are mature at age times the proportion spawning at age. This allows for the accounting of individuals that are mature (able to produce gametes) but are not actively spawning in that time-step (e.g., skip spawning), and those that are mature and actively spawning in that time-step.

$$(2.23) \quad N_{a,s|t,r}^{sp} = N_{a,s|t,r}^{ma} \Phi_{a,s|t,r}$$

The spawning biomass ( $B^{sp}$ ) is calculated as follows.

$$(2.24) \quad B_{|t,r}^{sp} = \sum_{a=0}^A \sum_s N_{a,s|t,r}^{sp} W_{a,s|t,r}^1$$

where  $W_{a,s|t,r}^1$  is the weight-at-age for that age, sex, time-step, and region at the end of the time-step.

#### Spawning proportion parameters

The proportions spawning ( $\Phi$ ) is a vector of proportions-at-age input by the user from externally estimated data. Typically, this vector contains a value of 1 for all ages because there is currently a paucity of information for many fish stocks.

### 2.3.7 Size-at-age

Growth is not modelled specifically (e.g, length-at-age), but weight-at-age is used to calculate biomass-at-age from numbers-at-age. Mean weight-at-age, sex, and region for a particular time-step ( $W_{a,s|t,r}$ , which will simply be referred to as weight-at-age regardless of the various partitions), is input by the user. Projecting variability in weight-at-age is discussed in Section 2.5.

### 2.3.8 Initial population

The initial population is the partitioned population numbers at the start of the first modelled time-step, and is based on unfished equilibrium recruitment ( $R_0$ ) with three potential adjustments.

1. An overall adjustment ( $e^{\delta|I}$ ) that changes  $R_0$  (i.e., the overall scale of recruitment) and could mimic a different regime that influenced the initial population.
2. Cohort ( $a$ ) specific adjustments to account for recruitment variability ( $\varepsilon_{I,a}$ ).
3. Adjustments by age to account for an average level of fishing that occurred before the initial time-step ( $1 - \sum_f S_{a,s|I,r,f} U_{|I,r,f}$ ).

Calculating the initial equilibrium population size is not a simple calculation when region is in the partition and initial recruitment deviations are used. It is easiest to build up the population sequentially by each cohort that makes up the initial population using the sequence of processes described in Section 2.3: recruitment, movement, and mortality. This will be time-consuming, however, but only has to be done while an operating model is being conditioned.

The numbers at age 0 for the cohort that is age  $a$  in the initial time-step ( $I$ ) for each sex and region is

$$(2.25) \quad N_{0,s|I-a,r} = p_s p_{a|I,r} e^{\delta|I} R_0 \left( 1 - \sum_f S_{0,s|I,r,f} U_{|I,r,f} \right) e^{\left( \varepsilon_{a|I} - \frac{\sigma_R^2}{2} \right)}$$

where  $p_s$  is the proportion of sex  $s$  at birth ( $\sum_s p_s = 1$ ) and  $p_{a|I,r}$  is the proportion of cohort  $a$  recruiting to region  $r$  in the initial time-step ( $\sum_r p_{a|I,r} = 1$ ). It is assumed that there is a single selectivity curve for the initial time period that applies equally to all cohorts and a constant exploitation rate for all cohorts over their life-span before the initial time-step.

Equation 2.25 calculates the number for each cohort when they were age 0 prior to the initial time-step. To calculate the numbers-at-age in the initial time-step, the annual process for each cohort is iterated up to the age that each cohort would be in the initial time-step. For example, to calculate the cohort that is age 3 in the initial population, Equation 2.25 would first determine the number of that cohort that were born into the population three time-steps prior. Then, the population dynamics would apply to that cohort for three iterations (0 to 1, 1 to 2, and 2 to 3) to determine the numbers in that cohort at age 3 in the initial time-step. Therefore, the annual process for the cohort that is age  $a$  in the initial time-step is iterated from  $i = a \dots 1$  in the following equations. It begins by incrementing the annual process.

$$(2.26) \quad N_{a-i+1,s|I-i+1,r} = N_{a-i,s|I-i,r}$$

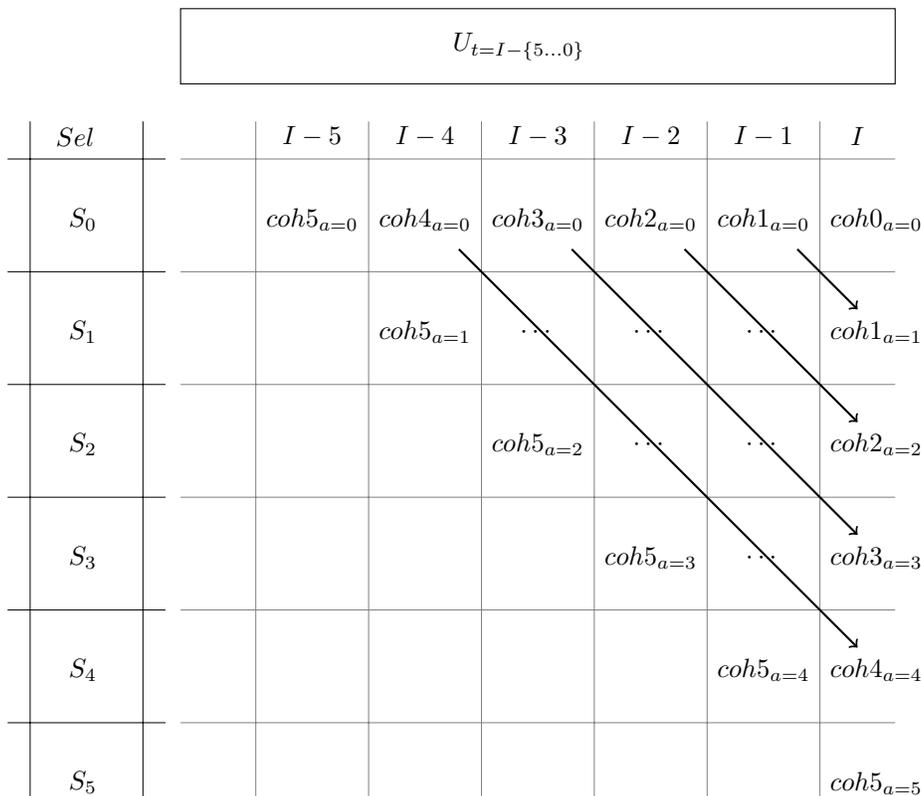
Then, movement from region  $j$  to  $k$  is applied. The subscripts for age, sex, and time-step are dropped for clarity in the derivation below, but are noted in the final equation. Also note that the movement-at-age,  $Psi$ , does not change in time-steps prior to the initial time-step.

$$\begin{aligned}
 N'_{a-i+1,s|I-i+1,j} &= N_{|j} - N_{|j} \sum_{k \neq j} \Psi_{|j \rightarrow k} + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\
 &= N_{|j} \left( 1 - \sum_{k \neq j} \Psi_{|j \rightarrow k} \right) + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\
 &= N_{|j} \Psi_{|j \rightarrow j} + \sum_{k \neq j} N_{|k} \Psi_{|k \rightarrow j} \\
 (2.27) \qquad &= \sum_{k \in r} N_{a-i+1,s|I-i+1,k} \Psi_{a-i+1,s|I,k \rightarrow j}
 \end{aligned}$$

Finally, natural and fishing mortality is applied. Subscripts for age, sex, and time-step are dropped for clarity, but are the same as in the left side of the equation unless indicated (e.g.,  $M$ ,  $U$ ,  $S$ ,  $R$ , and  $d$ ).

$$(2.28) \quad N''_{a-i+1,s|I-i+1,r} = N'_{|r} e^{(-M_{|I,r})} \left[ 1 - \sum_f S_{|I,r,f} U_{|I,r,f} [R_{|I,r,f} - (1 - R_{|I,r,f}) d_{|I,r,f}] \right]$$

Graphically, the process will look like:



### Summary of initial population numbers

To summarize how the initial numbers in the partitions are completed, psuedo code is provided below.

1. Determine the inital number of age zero ( $i = 0$ ) fish for the cohort (“*coh*” in the schematic representation above) of age  $a$  from Equation 2.25, where  $a = 0, 1, 2, \dots, nA$ . The  $n$  is a multiplier on the plus group age  $A$  to simulate beyond the plus group to ensure the dynamics of the plus group are correct.
2. Loop over  $a = 1, 2, \dots, nA$ . For each  $a$ ,
  - (a) loop over  $i = a, a - 1, \dots, 2, 1$  applying Equations 2.26, 2.27, and 2.28 to build up the initial numbers for each cohort at age  $a$ .
3. sum numbers over  $a = A, A + 1, \dots, nA$  to create the plus group ( $A$ ).

A value of 3 is typically used for  $n$ , but it depends on the plus group age and the time

### Calculation of other initial population values

Initial spawning biomass is calculated after the initial numbers-at-age are completed.

$$(2.29) \quad B_{|I,r}^{sp} = \sum_{a=0}^A \sum_s N_{a,s|I,r}'' \Omega_{a,s|I,r} \Phi_{a,s|I,r} W_{y,a,s,r}^1$$

### Population Dynamics

The sequence

1. *age increment*,
2. *recruitment* (based on spawning biomass in the previous time-step),
3. *movement*,
4. *mortality*, and
5. *spawning*.

The notation

$N$  : Beginning of the time-step after age increment and recruitment.

$N'$  : After movement before mortality

$N''$  : End of the time-step, after natural and fishing mortality

The equations

$$N_{y,a,s,r,m} = \begin{cases} R_{s|t,r} & a = 0 \\ N_{a-1,s|t-1,r} & 1 \leq a < A \\ N_{a-1,s|t-1,r} + N_{a,s|t-1,r} & a = A \end{cases}$$

$$N'_{a,s|t,r} = \sum_{k \in r} N_{a,s|t,k} \Psi_{a,s|t,k \rightarrow r}$$

$$N''_{a,s|t,r} = N'_{a,s|t,r} e^{-M_{a,s|t,r}} \left[ 1 - \sum_f U_{|t,r,f} S_{a,s|t,r,f} [R_{a,s|t,r,f} - (1 - R_{a,s|t,r,f}) d_{a,s|t,r,f}] \right]$$

## **2.4 Conditioning the Operating Models**

### **2.4.1 Likelihoods**

## **2.5 Projecting the Operating Models**

### **2.5.1 Recruitment**

See IPhC-2018-MSAB012-07. Discuss regimes, PDO, and recruitment variability.

### **2.5.2 Fishing mortality**

Based on the management procedure. Mimic catch sharing plans by IPhC Regulatory Area to determine allocation across sectors.

### **2.5.3 Movement**

Annual variability in movement yet to be determined.

### **2.5.4 Weight-at-age**

Projected values of weight-at-age are modelled using a random walk to introduce inter-annual variability. Will need to do this by region as well and have regions in some sort of synchrony.

## Chapter 3

# Specifications for the MSE framework

### 3.1 Population structure

The current investigation of management procedures at IPHC is focused on the coastwide scale and the distribution of mortality limits.

To simulate the distribution of mortality and determine how management procedures meet area-specific objectives, the operating model will have to include multiple regions with migration between them. Biological Regions (Figure 3.1) have been defined based on current knowledge of movement as well as biological understanding. A Biological Region is larger than an IPHC Regulatory Area (Figure 3.1), but fisheries operate at the level of the Regulatory Area or finer. Movement will not be specifically modelled between areas within a region, but movement will always be modelled between regions. Even though the computer program for the operating model allows flexibility to define any arrangement of regions and areas, with movement modelled between them, it would be moot to model movement within a Biological Region on an annual time-step because it is assumed that a fish may be anywhere within the region within a year. The modelling of fisheries in separate areas is described below.

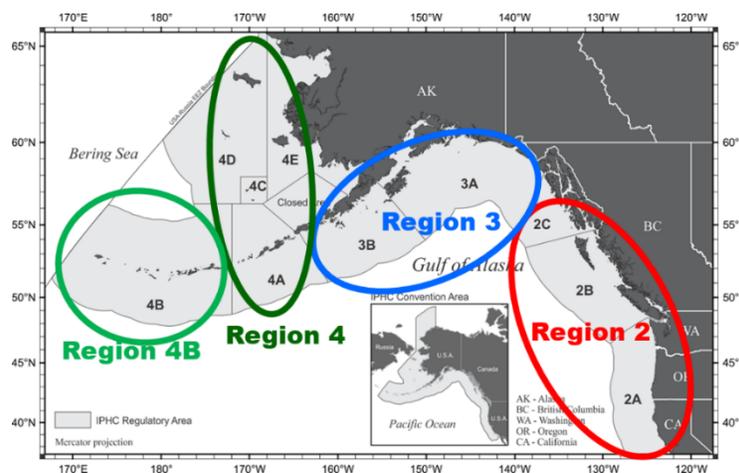


Figure 3.1: IPHC Regulatory Areas (grey shaded areas) and biological regions (colored circles).

### 3.1.1 Biological parameters

## 3.2 Structure of the fisheries

The annual mortality limits determined for various fishery sectors occur at the level of IPHC Regulatory Areas (Figure 3.1). However, some fisheries for Pacific halibut may operate at a finer scale than an IPHC Regulatory Area. The best definition of fishery areas will be determined from the objectives defined for the MSE, as well as the availability of knowledge to parameterize the fisheries for simulation, which is likely at the IPHC Regulatory Area level. This can be done when modelling the population at larger Biological Regions by defining separate sectors within a region, and using separate selectivity curves and exploitation rates to account for the availability of Pacific halibut to a particular sector (see Section 3.2.1). This assumes that each fishery within a Biological Region operates on the same pool of fish, but each fishery encounters those fish differently.

The various fishery sectors, or sources of fishing-related mortality, is likely to be those defined in Table 3.1. Notice that surveys are included with fishery sectors because they are a specific type of fishing-related mortality. Fishery sectors are often called fleets in fisheries models, but we chose to use sectors to describe the sub-divisions within specific areas. A sector will not span multiple areas, thus an area and a sector will be synonymous.

Table 3.1: Potential fishery sectors to include in the model. Each sector will occur in each region, and may be split further or possibly aggregated.

Sector	Regions	Regulatory Areas	Description
Commercial	All	All	Commercial longline fisheries targeting Pacific halibut.
Directed Fishery Discard Mortality	All	All	Commercial discard mortality that is not accounted for in the landings of Pacific halibut from longline fisheries. This could be undersized fish or mortality due to lost gear. This may be modelled separately but is likely to be modelled using retention in the commercial fisheries.
Recreational			A recreational fleet for each Regulatory Area will be modelled unless recreational fisheries with a Biological Region are similar and can be combined for convenience.
Subsistence			
Non-directed Fishery Discard Mortality	All	All	A fleet within each region to account for non-directed fishery mortality (i.e., bycatch).
Survey	All	All	The survey will be modelled within each Biological Region to account for the small amount of mortality.

### 3.2.1 Fishery parameters

#### Selectivity and availability

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## Appendix A

# Parameters and notation

Table A.1: Dimension and partition notation in the operating models (used as subscripts).

Parameter	Description
$y$	Year.
$a$	Age. A capital $A$ indicates the maximum age.
$s$	Sex, which includes female, male, and unsexed, in that order, labeled 1, 2, and 3.
$r$	Region with movement occurring between regions.
$r_l$	Area within region. Movement is not modelled between areas.
$im$	Immature state of the maturity partition.
$ma$	Mature state of the maturity partition.
$sp$	Spawning state calculated from the maturity partition. Note that this is not a specific partition in the state object.
$f$	Fishery sector.
$I$	Initial, meaning the starting time-step.

Table A.2: Parameters in the operating models.

Parameter	Description
<i>Population dynamics parameters</i>	
$N_{y,a,s,r,m}$	Numbers for year, age, sex, region, and maturity
$B_0$	Unfished equilibrium biomass
$R_0$	Unfished equilibrium recruitment
$B_{y,a,s,r,m}$	Biomass at the beginning of year $y$ (and possibly other partitions as noted)
$B_{y,a,s,r,m}^M$	Mature biomass at the beginning of year $y$
$B_{y,a,s,r,m}^S$	Spawning biomass at the beginning of year $y$
$B_{y,r,f}^R$	Biomass selected and retained by sector $f$ in year $y$
$M_{y,a,s,r}$	Natural mortality
$W_{y,a,s,r}$	Mean weight-at-age in year $y$ (and possibly other partitions)
$\Omega$	Proportion of mature individual at age, or the proportion transitioning from immature to mature at age, depending on partitioning maturity in the state object.
$\Phi$	Proportion of spawning individuals at age
$\Psi_{j \rightarrow k}$	Movement rate from area $j$ to area $k$
$\rho_1$	Parameter for Type II functional response of movement from area 1 to area 2
$\rho_2$	Parameter for Type II functional response of movement from area 1 to area 2
$p_s$	Proportion of females at birth.
$p_{y,r}$	Proportion of recruitment in region $r$ in year $y$ . Sums to one over regions.
$p_f$	Proportion of natural mortality that occurs before exploitation from sector $f$ occurs.

Table A.2 continued.

Parameter	Description
<i>Fishing mortality related parameters</i>	
$C_{y,a,s,r,f}$	Catch (in weight) for year, age, sex, region, and sector. Catch summed over age and sex ( $C_{y,r,f}$ ) is typically an input to the model.
$C_{y,a,s,r,f}^N$	Catch (in numbers) for year, region, and sector.
$U_{y,r,f}$	Exploitation rate for sector $f$
$S_{a,f}$	Selectivity-at-age for sector $f$
$R_{a,f}$	Proportion retained-at-age for sector $f$
<i>Survey parameters</i>	
$Z_j$	Survey index for year $j$
$q$	Survey catchability
$\tau_j$	Error in year $j$ for the survey series
$\sigma_{\tau_j}^2$	Total variability of the survey in year $j$
<i>CPUE parameters</i>	
$U_{u,i}$	CPUE for year $i$
$\alpha$	Multiplier in relationship between CPUE and abundance for the CPUE series
$\beta$	Nonlinearity parameter in relationship between CPUE and abundance
$\nu_i$	Error in year $i$ for CPUE series
$\sigma_{\nu_i}^2$	Total variability of the CPUE series in year $i$

## Appendix B

# Fishing mortality using exploitation rates

Fishing mortality is parameterized with an exploitation rate and is assumed to occur at a specific point in time after a proportion of the mortality ( $p_{|r,f}$ ) has occurred (Section 2.3.4). The derivation below shows that the equations in Chapter 2.3 are appropriate regardless of the proportion assigned to any fleet, which may differ between fleets. Let's assume there are two fleets with the mortality from fleet 1 occurring after three fifths of the natural mortality ( $p_{|1} = 0.6$ ) and the mortality from fleet 2 occurring after one-quarter of the natural mortality ( $p_{|2} = 0.25$ ). Simplifying the dynamics by ignoring region and sex, and assuming there is no discard mortality (including only the exploitation rate and selectivity) the catch (in numbers) for each fleet is

$$(B.1) \quad C_{a|t,f=1}^N = N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a}$$

$$(B.2) \quad C_{a|t,f=2}^N = N_{a|t} U_{|t,f=2} S_{a|t,f=2} e^{-0.25M_a}$$

The numbers-at-age in the next year, accounting for fishing mortality by removing the catch at the appropriate time, is

$$(B.3) \quad N_{a|t+1} = \left[ \left[ N_{t,a} e^{-0.25M_a} - C_{a|f=2}^N \right] e^{-(0.6-0.25)M_a} - C_{a|f=1}^N \right] e^{-(1-0.6)M_a}$$

Converting the catch ( $C^N$ ) to exploitation rates using equation B.1 and simplifying produces equation 2.19.

$$\begin{aligned}
\text{(B.4)} \quad N_{a|t+1} &= \left[ [N_{t,a} e^{-0.25M_a} - N_{a|t} U_{|t,f=2} S_{a|t,f=2} e^{-0.25M_a}] e^{-(0.6-0.25)M_a} - \right. \\
&\quad \left. N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a} \right] e^{-(1-0.6)M_a} \\
\text{(B.5)} &= [N_{t,a} e^{-0.25M_a} e^{-0.35M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) - N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a}] e^{-0.4M_a} \\
\text{(B.6)} &= [N_{t,a} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2}) - N_{a|t} U_{|t,f=1} S_{a|t,f=1} e^{-0.6M_a}] e^{-0.4M_a} \\
\text{(B.7)} &= [N_{t,a} e^{-0.6M_a} (1 - U_{|t,f=2} S_{a|t,f=2} - U_{|t,f=1} S_{a|t,f=1})] e^{-0.4M_a} \\
\text{(B.8)} &= N_{t,a} e^{-M_a} (1 - \sum_f U_{|t,f} S_{a|t,f})
\end{aligned}$$

This can be generalized to any set of  $p_{|f}$  as long as the proportions are sorted from smallest to largest in the derivation.