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The Pacific Halibut Stock Assessment of 1997

by

Patrick J. Sullivan, Ana M. Parma, and William G. Clark

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ABSTRACT

Pacific halibut (*Hippoglossus stenolepis*) are assessed annually to formulate harvest guidelines and maintain the population at productive levels. Data are collected from scientific setline surveys, directed setline fisheries, sport fisheries, and fisheries targeting on other species where halibut is taken as bycatch. These data provide information on total harvest, size and age composition, catch per unit effort (CPUE), and bycatch mortality at length. A size-age-structured model, which accounts for changes in selectivity induced by trends in growth, is used for analysis. The model represents growth rates as varying gradually over time. Catchability and selectivity at size and age are also represented as time varying for the commercial component of the system, while catchability is held fixed and selectivity is assumed constant at either size or age for the survey component. The 1997 assessment indicates that total population biomass has remained steady or decreased recently in the central Gulf of Alaska while increasing in British Columbia and south. Recruitment estimates are uncertain due to a dramatic decrease in growth that reduces selectivity on younger age-classes. Harvest guidelines are provided under a constant harvest rate policy applied to model estimates while taking into account the effect of bycaught sublegal-sized halibut as pre-recruit mortality. Harvest rates in the range 0.20-0.25 should achieve close to maximum yields under different recruitment scenarios while maintaining a high probability that stock levels stay within the range of historical abundance.

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INTRODUCTION

Pacific halibut are managed by the International Pacific Halibut Commission (IPHC) under authority of the Pacific Halibut Convention of 1923 and through subsequent agreements between Canada and the United States. The IPHC uses annual assessments to monitor population trends and develop harvest guidelines for regulatory areas within its jurisdiction (Figure 1). Assessment approaches have varied over the years (Quinn et al. 1985, Sullivan and McCaughran 1995) and included cohort analysis

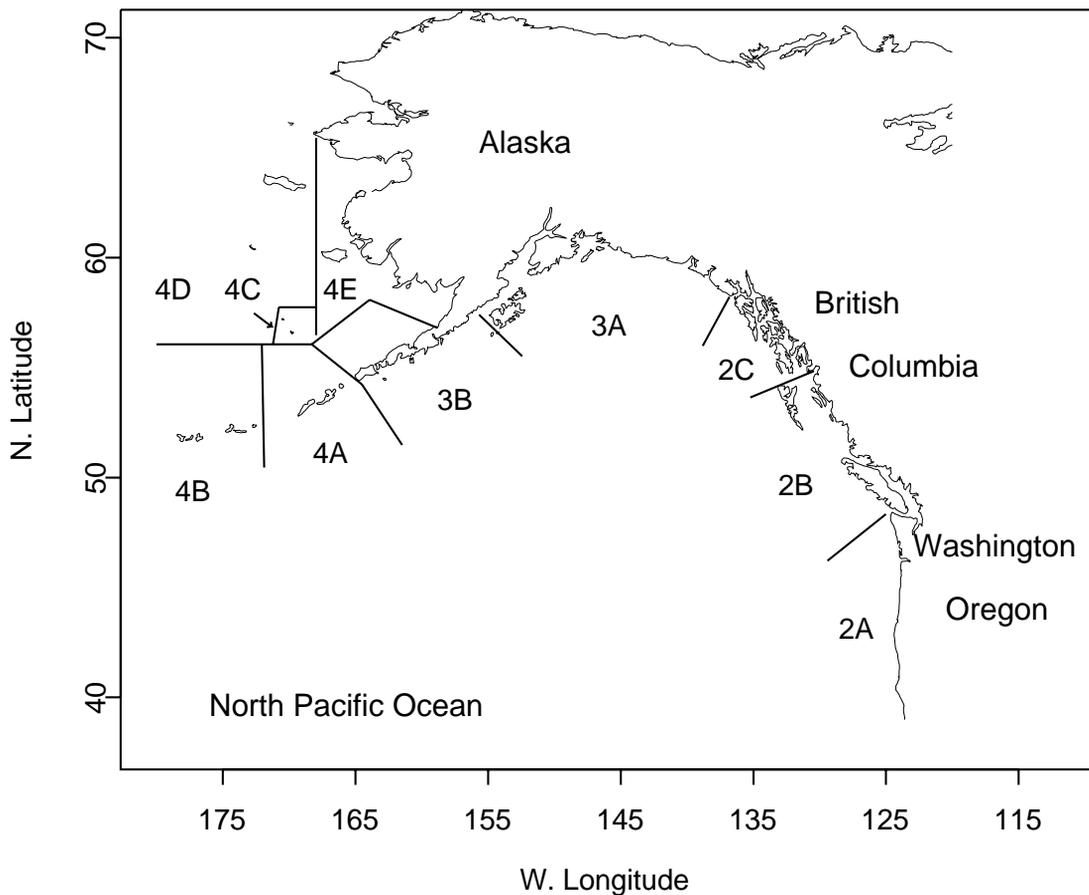


Figure 1. IPHC regulatory areas.

(Hoag and McNaughton 1978), annual surplus production methods (Quinn et al. 1984) and separable catch-at-age analysis (Deriso et al. 1985). Recent changes in the biology of Pacific halibut and its fishery have led to substantial revisions of the assessment procedures. Individual quota-share programs were implemented in British Columbia in 1991 and in Alaska in 1995, changing how the fishery was prosecuted and how data were collected (Sullivan and Rebert 1998). A dramatic decrease in individual growth rate occurred in the 1980s, resulting in delayed recruitment, smaller size at age, and decreased selectivity of younger halibut, all of which biased downward the estimates of abundance (Clark et al. 1999).

In order to deal with these changes, the model used to estimate stock biomass was restructured to account for trends in catchability, selectivity, and growth. The reduction in size at age was represented by modeling growth as a nonstationary stochastic process. Selectivity was made a function of both size (length) and age so that a decrease in size resulted in a decrease in selectivity. In addition, to provide a better and broader set of information for the size-age model, data on size at age and catch rates from IPHC scientific setline surveys and data on bycatch mortality occurring outside the directed fishery were developed for use as input to the assessment.

Changes in assessment methods as well as changes in halibut life history parameters led to a revision of the harvesting strategy. In this report we document the assessment procedures currently in use. Three major components of the assessment are discussed: (1) the data input to the assessment, (2) the model by which it is analyzed, and (3) the evaluation of alternative harvest rates. Procedures are illustrated using results from the assessments of Areas 2AB and 3A for 1997.

DATA USED IN THE ASSESSMENT

Scientific and commercial data collection at the IPHC has a long history. Starting with tagging experiments in 1925 to explore the effects of migration and progressing to the present where both fishery independent and fishery dependent data sources are employed, the IPHC endeavors to develop and maintain a comprehensive data repository that is useful to understanding and managing Pacific halibut. These data now include information from commercial logbooks, dealer tickets, market samples from off-loaded commercial harvests, landings from sport fisheries and other directed sources, bycatch mortality incurred outside the directed fishery, as well as data collected on systematic scientific setline surveys and field and laboratory experiments. The data are monitored for quality assurance and kept on record for use in the population assessment and for other research. What is provided here is an overview of these data as they are used for population assessment. More detailed information on halibut data and data collection practices can be found in other reports contained in the IPHC Scientific and Technical Report series.

Commercial Catch per Unit of Effort

Catch per unit of effort (CPUE) is commonly used in fisheries as an index of relative abundance over time and across areas. Besides abundance, trends in commercial CPUE statistics may also be affected by factors that change with a changing fishery. For example, the transition from J-hooks to circle-hooks in the

Pacific halibut fishery in 1983 more than doubled the catch efficiency of the gear as discussed below. Catchability, however, may also be affected in ways that are more difficult to correct. For this reason, commercial CPUE is no longer used as the sole index of trends in abundance, and is now used in conjunction with survey CPUE to monitor trends and estimate changes in catchability in the commercial fishery. Adjustments, corrections, and standardizations are still made to the commercial CPUE data to maintain as much consistency as possible in the statistic.

Halibut commercial CPUE (Table 1) is computed using data gathered from IPHC commercial logbook records. Only catch records for which there are corresponding effort records are used. Effort is measured as the total length of setline gear, in number of standard skates, used in taking the catch. CPUE for a given year and IPHC regulatory area is computed as the sum of all the catches divided by the sum of all the matching efforts:

$$CPUE = \frac{\sum \text{Catch}}{\sum \text{Effort}}.$$

Table 1. Commercial CPUE (pounds per skate net weight, circle-hook equivalent).

Year	2A	2B	2C	3A	3B	4	Total
1974	130.7	141.0	126.0	142.4	124.7	301.1	137.9
1975	130.6	148.7	117.4	145.3	149.3	210.7	139.7
1976	71.7	116.7	92.8	131.5	142.2	184.2	118.5
1977	182.2	135.3	99.4	134.6	161.3	176.2	133.1
1978	85.5	138.0	124.1	171.9	116.4	166.6	148.0
1979	110.0	105.8	176.6	189.0	80.8	146.1	154.6
1980	82.0	148.3	183.7	278.3	315.1	177.7	210.9
1981	67.7	154.3	313.7	327.7	387.2	249.9	254.6
1982	47.3	149.1	321.4	373.1	461.7	219.9	274.2
1983	NA	NA	NA	NA	NA	NA	NA
1984	69.0	146.6	280.8	500.3	475.2	235.6	288.0
1985	69.2	143.1	340.7	509.9	602.4	304.8	310.0
1986	60.9	118.2	294.0	517.9	514.8	276.5	287.7
1987	58.6	128.4	260.3	503.6	476.1	298.1	276.9
1988	171.4	131.6	281.3	502.8	654.2	296.4	309.4
1989	112.4	133.2	258.0	456.0	590.0	306.4	300.2
1990	168.4	173.9	269.1	352.9	483.6	336.2	302.0
1991	164.3	156.4	233.2	318.6	466.4	366.3	284.9
1992	113.9	186.6	230.5	397.1	440.2	312.4	304.4
1993	155.0	211.9	255.1	390.8	504.6	336.9	312.1
1994	92.4	212.5	187.5	330.2	355.9	247.1	255.5
1995	88.9	205.5	231.5	389.7	476.6	271.9	283.4
1996	154.9	221.0	221.0	442.3	461.6	339.9	311.3
1997	189.6	243.4	259.5	436.3	521.7	321.7	339.5

This is equivalent to an effort-weighted sum of each logbook's CPUE index:

$$CPUE = \frac{\sum_i \text{Effort}_i \text{CPUE}_i}{\sum_i \text{Effort}_i},$$

where i corresponds to each logbook record. In the assessment model total effort is used as an index of relative fishing mortality level. The estimated CPUE (pounds per skate) is divided into the recorded total catch (pounds) to provide the estimate of total effort for that regulatory area and year:

$$\text{Total Effort} = \frac{\text{Total Catch}}{\text{CPUE}}.$$

The data are checked for completeness and only those records that can be related to the standard and most common fishing gears are used. All useable effort data are standardized to units of effective skates, where an effective skate is given in units of circle-hook skates, adjusted to 18-foot spacing or the equivalent of 100 hooks per skate:

$$N_{\text{EffectiveSkates}} = N_{\text{Skates}} 1.52(1 - \exp(-0.06 H_{\text{Spacing}})) \frac{L_{\text{Skate}}}{100 H_{\text{Spacing}}} H_{\text{HookType}} + 0.5,$$

where H_{Spacing} is hook spacing in feet (e.g., 18 for the standard gear where the hooks are placed every 18 feet), L_{Skate} is skate length, and H_{HookType} is a multiplier that accounts for fishing power differences among hook types. The catchability of the hooks, and in turn the number of effective skates that a given number of hooks represents, increases with hook spacing. This increase is rapid when hooks are closer than 18 feet (the standard) and slow thereafter. The hook-spacing adjustment follows Hamley and Skud (1978), while the hook-type adjustment is based on work discussed in Quinn et al. (1985).

Until 1983 the halibut fishery used a J-shaped hook. In that year a new circular-shaped hook was introduced, which proved to be much more effective in hooking halibut. The "circle hook" has a point that bends around and inward toward the shank, and in fact it resembles the hooks traditionally used by native fishers. In 1984 the Commission fished both hook types at the same stations in Alaska and British Columbia. Overall the circle hooks caught 2.2 times as much legal-sized halibut in weight and 2.4 times as much in number. In the assessment, the factor $0.45=1/2.2$ is used to convert historical J-hook commercial effort to the present circle-hook standard. In summary, the hook-type adjustment factors H_{HookType} are 1.00 for circle (currently the standard and denoted as "C" in the database), 0.45 for J-hook (denoted as "J" or "J"), and 0.73 for mixed circle and J-hooks (denoted as "M").

Gear with hook spacing less than four feet, which is commonly used for sablefish (*Anoplopoma fimbria*) fishing, is normally excluded in the effective-skate calculation. In Area 2A, however, gear with spacing less than four feet makes up a

significant portion of the data, and thus are included in the effective-skate calculations with the spacing indicator (H_{Spacing}) set at four feet. Analyses conducted in 1993 showed no change in trend with the inclusion of these data, but a significant increase in precision.

In the commercial setline halibut fishery, two types of fishing gear are commonly employed: fixed-hook gear, where the gangion is permanently tied onto the groundline, and snap-hook gear, where the pre-baited gangion is snapped onto the groundline as it is unreeled during setting. Fixed-hook gear is preferred in Alaska, while snap-hook gear is preferred in British Columbia. Various conversion factors have been used over the years to combine types for cross-area comparisons, but none has proved consistently satisfactory. Myhre and Quinn (1984) showed in side-by-side setline experiments that the efficiency of the two gears was the same when fished identically, but indicated that how the gear was employed (e.g. use of different bait sizes or haul speeds) could be a factor. Sullivan and Rebert (1998) showed fixed-hook gear to be about 8% more efficient than snap-hook gear for the British Columbia commercial fishery after accounting for year, area, season, and vessel-size class. Plots of commercial CPUE for the two gear types show some regional differences in both gear specific CPUE levels and gear use (Figure 2). Changes in the relative amount of

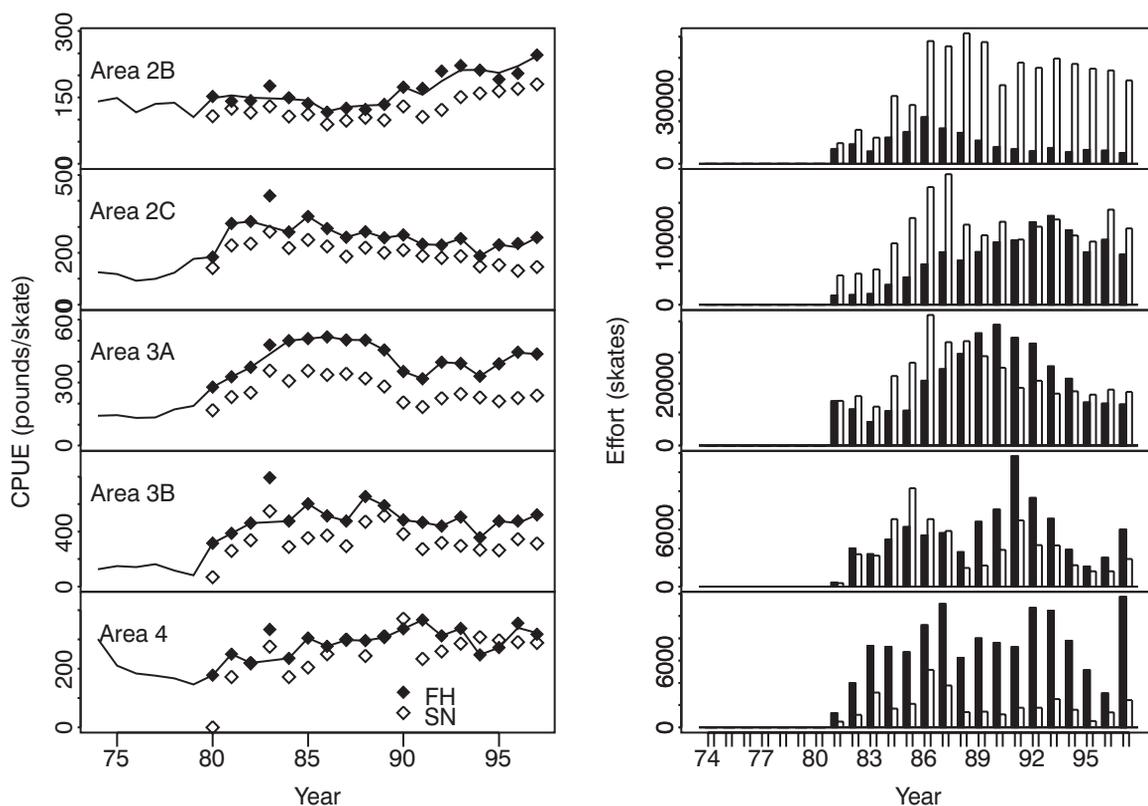


Figure 2. Estimated fixed-hook and snap-hook CPUE (shown on the left respectively as filled and unfilled diamonds) compared with estimates used historically (shown as a solid line) for each IPHC area. Total recorded effort in effective skates for fixed-hook and snap-hook (shown on the right respectively as filled and unfilled bars).

effort contributed by the two gears, however, suggest that a strictly effort weighted combination would be inappropriate. Fixed-hook CPUE data alone was used in the 1997 assessments for Alaskan IPHC regulatory areas, while a 50:50 combination of fixed-hook and snap-hook CPUE data was used in the assessment for British Columbia and the Pacific states.

Significant changes in catch efficiency are likely occurring as a result of the change to individual quota-share management in both British Columbia and Alaska. Sullivan and Rebert (1998) indicated in their analysis of the British Columbia individual quota system that some fishers moved to grounds with higher halibut densities, while others moved towards grounds which were closer to the home port. As indicated by Quinn et al. (1982), a redistribution of effort on the grounds could bias the CPUE estimates. Indeed, trends in the CPUE index over the period of transition to individual quotas changed depending on how the index was computed: an area-season weighting of CPUE showed less of an upward trend than the conventional effort weighting scheme. A correction for statistical area and seasonal differences is not implemented in the current assessment and is contingent on data availability at the appropriate scale for all areas and years.

Commercial Catch at Age and Size

Total commercial catch (Table 2) and sampled number at age and size in the catch are important measures of mortality and year-class strength for the assessment.

Table 2. Commercial catch (million pounds).

Year	2A	2B	2C	3A	3B	4	Total
1974	0.52	4.62	5.60	8.19	1.67	0.71	21.31
1975	0.46	7.13	6.24	10.60	2.56	0.63	27.62
1976	0.24	7.28	5.53	11.04	2.73	0.72	27.54
1977	0.21	5.43	3.19	8.64	3.19	1.22	21.88
1978	0.10	4.61	4.32	10.30	1.32	1.35	22.00
1979	0.05	4.86	4.53	11.34	0.39	1.37	22.54
1980	0.02	5.65	3.24	11.97	0.28	0.71	21.87
1981	0.20	5.65	4.01	14.22	0.45	1.19	25.72
1982	0.21	5.54	3.50	13.53	4.80	1.43	29.01
1983	0.26	5.44	6.40	14.11	7.75	4.42	38.38
1984	0.43	9.05	5.85	19.97	6.50	3.16	44.96
1985	0.49	10.39	9.21	20.85	10.89	4.28	56.11
1986	0.58	11.22	10.61	32.79	8.83	5.59	69.62
1987	0.59	12.25	10.68	31.32	7.76	6.88	69.48
1988	0.49	12.86	11.37	37.86	7.08	4.69	74.35
1989	0.47	10.43	9.53	33.73	7.84	4.93	66.93
1990	0.32	8.57	9.73	28.85	8.69	5.43	61.59
1991	0.36	7.17	8.69	22.86	11.93	5.99	57.00
1992	0.44	7.63	9.82	26.78	8.62	6.61	59.90
1993	0.52	10.63	11.29	22.74	7.86	6.25	59.28
1994	0.39	9.91	10.38	24.84	3.86	5.37	54.75
1995	0.31	9.62	7.76	18.34	3.12	4.74	43.89
1996	0.30	9.53	8.80	19.69	3.81	5.31	47.44
1997	0.40	12.20	9.89	24.68	9.10	8.79	65.05

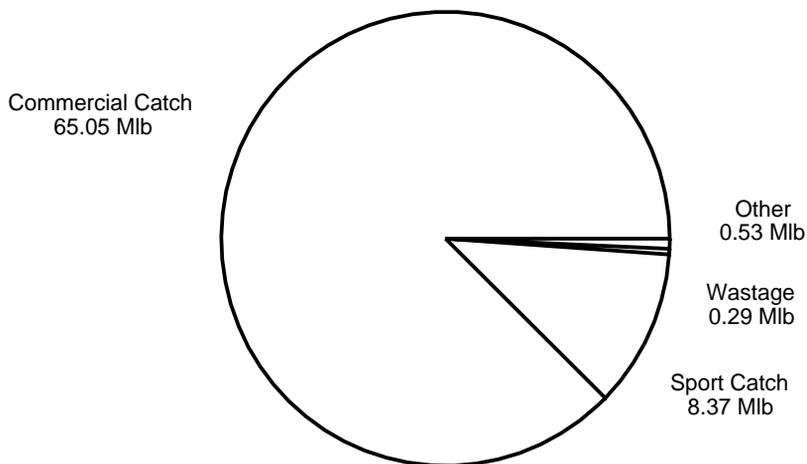


Figure 3. Distribution of legal-sized removals (net weight of halibut > 82 cm).

Total commercial catch historically was tallied from landing tickets submitted by fish buyers. In Alaska and British Columbia, the tallies now result from the electronic recording of landings through individual quota monitoring systems. Tickets were gathered, entered on computer by IPHC staff, and compared with electronic quota tallies obtained in 1991 and 1992 in the British Columbia fishery and in 1996 and 1997 in the Alaska fishery. This procedure helped verify the new process and highlighted critical components of the data (e.g. vessel identifier) that were going unrecorded.

IPHC port samplers measure fish and collect otoliths at major halibut ports to determine the length and age composition of the commercial landings. The sex of fish cannot be determined from market samples because the fish are eviscerated at sea. The sampling process has varied over the years (Quinn et al. 1983, Quinn et al. 1985) according to the needs and resources of the Commission. Since 1990 the aim has been to sample 2000 fish from the landings in each regulatory area, which provides estimates of proportion at age with a coefficient of variation of about 7% for fish in the most numerous age groups (Gilroy et al. 1995). Landings are sampled throughout the season at a constant rate calculated to result in approximately the desired sample size. For each sampled landing, the sample is a fixed proportion (e.g., 1%) of the weight of the landing. Fish are selected by one or another randomization method until the weight of the sample reaches the target amount.

Sport Harvest

Catch in the recreational fisheries is rather variable. In 1997, however, sport catch made up about 11% of the total removals coastwide (Figure 3), and about 18% of the removals in Area 3A. Sport catches are estimated by state and provincial agencies, which use creel census (Oregon and Washington) and postal surveys (Canada and Alaska). These estimates are provided to the IPHC where they are added to total harvest and are assumed to have the same age and size composition as that shown in

the commercial catch. State agencies in Alaska have been gathering age and size information in recent years so that estimates of the age composition of the sport catch can be obtained for future assessments.

Wastage and Other Removals

Wastage is the mortality of halibut due to discarding and loss of gear in the directed fishery. Only the legal-sized component of the wastage is included as removals in the assessment. Wastage of sublegal fish (fish below the 81cm commercial size limit) represents mortality of halibut prior to recruitment, and therefore is accounted for as reduced recruitment when we evaluate the harvesting strategy. Legal-sized wastage is estimated from gear loss rates recorded on fishing logs (i.e. number of skates lost to number of skates hauled). This rate, applied to total effort and CPUE, provides the wastage loss in pounds net weight. Wastage loss rates have dropped significantly in recent years with the adoption of individual quota systems. In 1997, wastage mortality of legal-sized halibut amounted to less than 1% of total removals, excluding bycatch (Figure 3).

A miscellaneous category of other removals exists that includes otherwise unrecorded subsistence and take-home halibut. These removals are small (on the order of 1% of total removals, excluding bycatch) and, in Alaska, were estimated from personal interview surveys on consumption of halibut in coastal communities.

Table 3. Total directed removals (million pounds net weight, excludes bycatch)

Year	2A	2B	2C	3A	3B	4	Total
1974	0.52	4.62	5.60	8.19	1.67	0.71	21.31
1975	0.46	7.13	6.24	10.60	2.56	0.63	27.62
1976	0.24	7.28	5.53	11.04	2.73	0.72	27.54
1977	0.22	5.45	3.26	8.84	3.19	1.22	22.18
1978	0.11	4.62	4.40	10.58	1.32	1.35	22.38
1979	0.06	4.88	4.70	11.70	0.39	1.37	23.11
1980	0.04	5.66	3.57	12.46	0.28	0.71	22.72
1981	0.22	5.67	4.33	14.97	0.45	1.20	26.84
1982	0.26	5.61	3.99	14.25	4.80	1.44	30.34
1983	0.32	5.54	6.95	15.06	7.75	4.42	40.05
1984	0.55	9.17	6.47	21.00	6.50	3.17	46.86
1985	0.68	11.02	10.11	22.99	11.09	4.44	60.33
1986	0.92	11.80	11.77	36.56	9.23	5.91	76.18
1987	1.04	12.95	11.83	34.89	8.10	7.17	75.97
1988	0.74	13.41	12.65	42.63	7.20	4.80	81.43
1989	0.80	11.11	11.28	38.19	8.03	5.08	74.50
1990	0.52	9.41	11.30	33.38	8.91	5.69	69.21
1991	0.52	7.88	11.41	29.23	12.41	6.52	67.96
1992	0.70	8.36	12.10	31.81	8.83	6.89	68.69
1993	0.77	11.68	13.40	28.67	7.98	6.56	69.07
1994	0.57	10.94	12.72	30.51	3.96	5.64	64.34
1995	0.55	10.62	9.57	23.06	3.19	4.90	51.89
1996	0.53	10.52	10.38	24.79	3.89	5.55	55.65
1997	0.76	13.19	11.76	30.26	9.21	9.05	74.23

Total Directed Removals

Mortality due to all directed fisheries (Table 3) is the sum of the removals due to each of the specific sources discussed above. For the purposes of the assessment these removals are all assumed to have the same age and length composition. Bycatch mortality occurring outside the directed fishery differs significantly in size composition. Consequently, it is tallied and incorporated into the assessment separately, as discussed below.

Bycatch

Halibut is caught as bycatch in fisheries targeting on other species but its retention is prohibited by law. The mortality rates of discarded bycaught halibut vary between 16%-100% depending on gear type (Williams et al. 1989 and Williams, personal communication¹) and method of release (Hoag 1975). Bycatch mortality makes up a significant portion of all removals (Table 4, Figure 4). In 1997, it was estimated that 5.2 million pounds of legal-sized halibut mortality took place outside of the directed halibut fisheries. Additionally, 7.8 million pounds of sublegal-sized

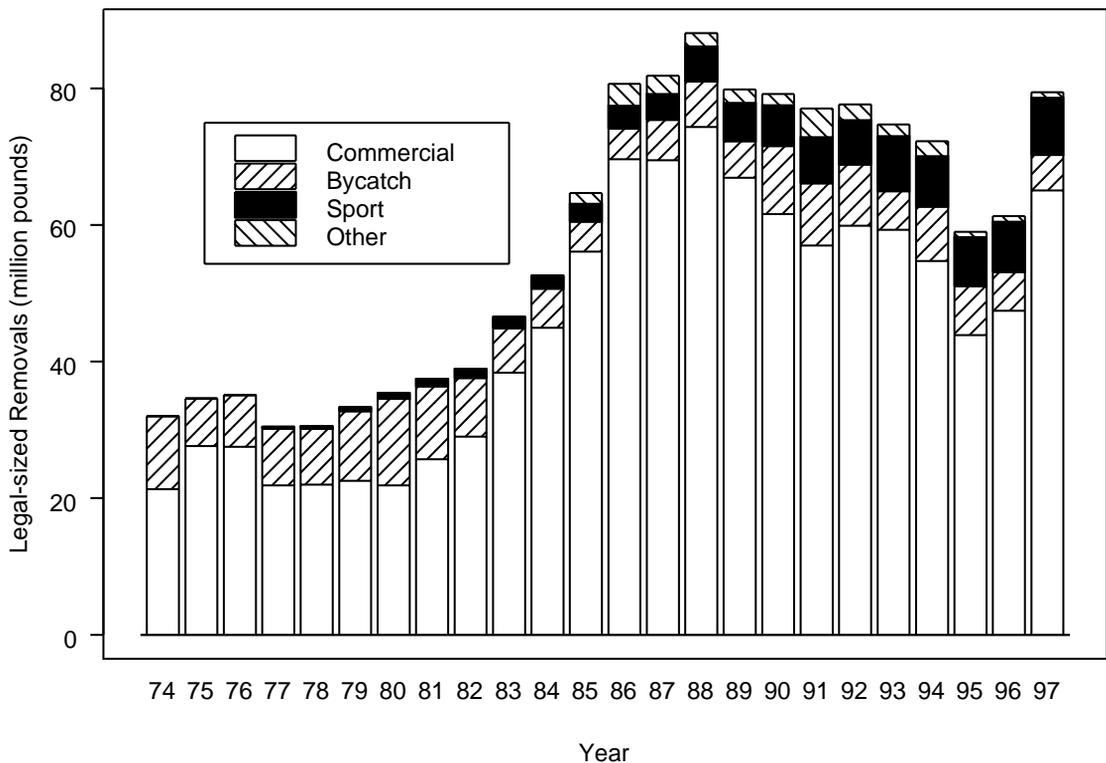


Figure 4. Annual removals of legal-sized (> 82 cm) Pacific halibut by harvest category in millions of pounds net weight.

¹ Williams, G. H. International Pacific Halibut Commission. P.O. Box 95009, Seattle, Washington, 98145-2009.

Table 4. Legal-sized bycatch mortality (million pounds, net weight).

Year	2A	2B	2C	3A	3B	4	Total
1974	0.25	0.90	0.37	4.48	2.82	1.89	10.71
1975	0.25	0.90	0.45	2.61	1.66	1.10	6.98
1976	0.25	0.94	0.50	2.74	1.94	1.18	7.56
1977	0.25	0.72	0.41	3.37	1.54	1.98	8.27
1978	0.25	0.55	0.21	2.44	1.31	3.40	8.16
1979	0.25	0.69	0.64	4.49	0.69	3.44	10.20
1980	0.25	0.51	0.42	4.93	0.87	5.71	12.69
1981	0.25	0.53	0.40	3.99	1.09	4.37	10.64
1982	0.25	0.30	0.20	3.20	1.68	2.95	8.58
1983	0.25	0.29	0.20	2.08	1.22	2.47	6.51
1984	0.25	0.52	0.21	1.51	0.92	2.29	5.70
1985	0.25	0.55	0.20	0.80	0.34	2.25	4.38
1986	0.25	0.56	0.20	0.67	0.20	2.61	4.50
1987	0.25	0.79	0.20	1.59	0.40	2.67	5.90
1988	0.25	0.77	0.20	2.13	0.04	3.27	6.66
1989	0.25	0.72	0.20	1.80	0.44	1.95	5.36
1990	0.25	1.03	0.68	2.63	1.21	4.16	9.97
1991	0.25	1.22	0.55	3.13	1.03	2.92	9.10
1992	0.28	1.02	0.57	2.64	1.12	3.34	8.97
1993	0.28	0.65	0.33	1.92	0.47	2.01	5.65
1994	0.28	0.57	0.40	2.35	0.85	3.48	7.93
1995	0.38	0.71	0.24	1.57	0.90	3.31	7.11
1996	0.38	0.14	0.24	1.16	0.77	2.97	5.66
1997	0.38	0.14	0.26	1.15	0.59	2.69	5.21

bycatch mortality was estimated to occur. As with the wastage, the legal-size component is input as mortality into the assessment, while the sublegal-component is viewed as reducing recruitment and dealt with by adjusting the harvest rate (Clark and Hare 1998). Unlike other removals, bycatch mortality is recorded only by length and is assumed to be observed without error in the assessment.

IPHC Setline Survey Data

The IPHC conducts surveys to collect fishery independent indices of relative abundance and stock structure, as well as specimen data. Surveys are conducted during the summer (mostly between June and August) when halibut are distributed on their feeding grounds. In the past, these surveys were only conducted periodically and in just a few areas as the cost in staffing vessels and processing data was considered high relative to the information obtained, given what was available from logbook records and the market sample program. Recent changes in the stock, especially the reduction in size at age that resulted in reduced availability of younger age classes to the fishery, and recent changes in management, specifically the implementation of individual quota systems in British Columbia and Alaska, have changed the information available in the fishery-dependent data. IPHC systematic setline surveys

were reestablished in Areas 2B and 3A in 1993 after a seven-year hiatus. Areas 2A, 2C and 3B were later added and in 1997 the survey was expanded to include the entire coastline from Oregon north through the eastern Bering Sea region. The data gathered include CPUE in numbers per skate (Table 5), proportion at age and length for males and females, distribution of length at age, sex ratio, and sexual maturity.

Analysis of results from the 1984 surveys, in which circle-hooks and J-hooks were both fished at the same stations, showed that circle-hooks caught almost 4 times as many small (60-70 cm) fish as did J-hooks, but only about 2 times as many large (>120 cm) fish, with intermediate values for fish of intermediate sizes. Details are given in Appendix 1. In the assessment, the different catchability and selectivity of circle-hooks and J-hooks are incorporated by fitting the predicted relative ratios for different size categories to the results observed in the 1984 surveys.

ASSUMPTIONS ABOUT STOCK STRUCTURE

The assessments are conducted separately for each IPHC regulatory area (Figure 1), assuming that the populations on which fishing takes place remain closed

Table 5. IPHC setline survey CPUE in average number of halibut per effective skate and associated coefficient of variation (CV).

Year	Area 2AB		Area 2C		Area 3A	
	CPUE	CV	CPUE	CV	CPUE	CV
1974						
1975						
1976	2.30	0.12				
1977	1.56	0.13			5.56	0.06
1978	1.84	0.12			3.66	0.06
1979					5.25	0.07
1980	2.99	0.10			6.86	0.06
1981	1.86	0.11			10.21	0.06
1982	2.31	0.12	11.28	0.09	11.53	0.06
1983	3.11	0.14	10.89	0.09	9.29	0.04
1984	4.74	0.08	13.21	0.11	13.33	0.04
1985	4.30	0.11	11.53	0.10	15.74	0.04
1986	2.70	0.10	9.44	0.09	9.58	0.06
1987						
1988						
1989						
1990						
1991						
1992						
1993	6.63	0.10			17.94	0.06
1994					19.01	0.05
1995	8.95	0.08			23.10	0.06
1996	9.20	0.09	16.89	0.09	18.54	0.07
1997	9.07	0.08	21.77	0.09	26.46	0.06

to the effects of migration. Such an assumption is believed to be valid even though halibut migrate north and west to spawn in the winter (Skud 1977) because the fishery takes place primarily during the spring and summer when halibut are present on their summer feeding grounds. Adult halibut are known to have a high fidelity to these grounds (Skud 1977) and attempts to account for the effects of whatever migration does take place (Quinn et al. 1985) have not resulted in significant differences. An exception may be migration from Area 3B into the eastern Gulf of Alaska which, according to some of the available estimates from tagging (Deriso and Quinn 1983, Hilborn et al. 1995), may be significant at least for the youngest ages.

While the stocks are considered separate for assessment purposes, harvest guidelines are derived by assuming that the stocks in the Gulf of Alaska (Areas 2 and 3) constitute a single reproductive unit. This is supported by the long duration of the pelagic egg and larval phases, during which extensive intermingling of fish spawned in different grounds should occur while larvae drift north and west carried by the prevailing ocean currents (Skud 1977). The Bering Sea stock is considered a separate reproductive unit, therefore not included in estimates of trends in stock and recruitment used to evaluate harvest rates. Results from analysis of microsatellite DNA (Bentzen et al. Unpub.²), although limited, are consistent with a single panmictic stock in the NE Pacific, but a separation between this and the populations in Russian waters. Unfortunately, samples from the eastern Bering Sea were not available for this study. It should be noted that even if these were indeed separate stocks, the two areas are interrelated through transport of larvae spawned in the Gulf of Alaska into the Bering Sea (St-Pierre 1989), and migration of juveniles from the Bering Sea to the Gulf (Skud 1977). Halibut recruitment in the Gulf is therefore affected by bycatch of juveniles in the Bering Sea (Clark and Hare 1998), and estimates of recruitment used for stock-recruitment analyses are adjusted to account for this pre-recruit mortality.

SIZE- AND AGE-STRUCTURED MODEL

Pacific halibut have undergone a rapid reduction in individual growth rates in recent years, with average weight-at-age for the dominant age groups in the catch about half of what it was 20 years ago (Figure 5). This has a number of consequences for halibut stock assessment and management. Stock assessments conducted in the late 1980s and early 1990s used a catch-age model (CAGEAN – Deriso et al. 1985) which assumes that fishing mortality can be partitioned into a constant age-specific selectivity component, and a time-dependent full-selection fishing mortality component. This assumption can work well even though fishing gear may be size-selective when fish maintain roughly a constant size-at-age, and when other factors such as type of gear used and targeting practices remain stable. Given recent changes observed in halibut growth, however, the assumption was considered to be problematic and to severely bias the assessments. Due to the constant-selectivity assumption, a low representation of the younger age classes in the landings resulted in drastically declining recruitment estimates in the early 1990s. Initial estimates were later adjusted

² Bentzen, P., J. Britt, and J. Kwon. Unpub. Genetic variation in Pacific halibut (*Hippoglossus stenolepis*) detected with novel microsatellite markers. [In] Int. Pac. Halibut Comm. Rpt. of Assessment and Research Activities 1998: 229-242.

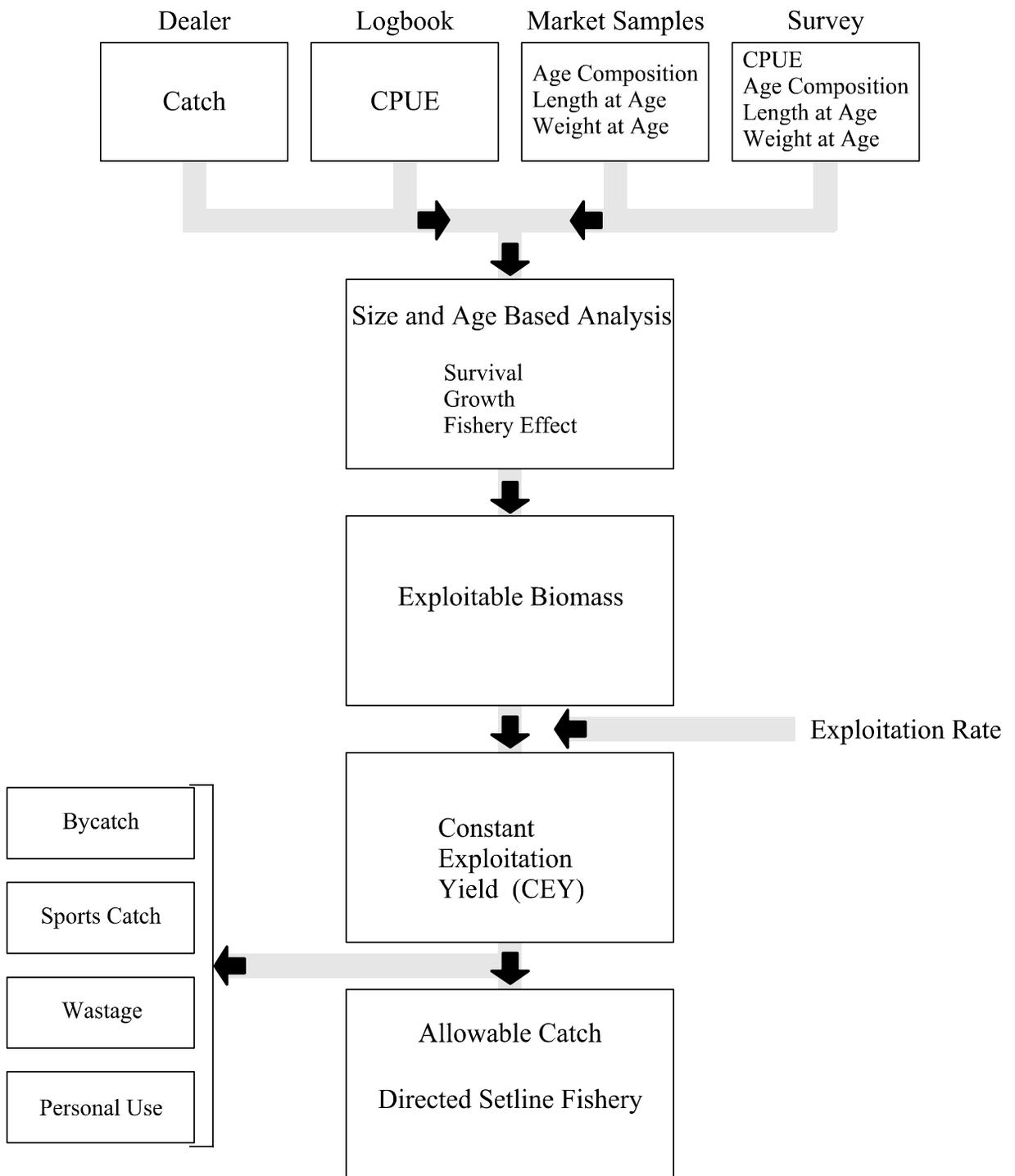


Figure 5. Overview of Pacific halibut stock assessment procedure.

upwards in successive assessments as fish grew and became vulnerable to the setline gear. As a result, stock assessments showed a strong retrospective pattern, in which estimates of exploitable biomass for past years were consistently adjusted upwards in every successive assessment and, while stock levels appeared to be declining rather steeply, quotas remained stable. To address these problems, an alternative assessment model was developed which accounts for possible changes in selectivity with age that result from changes in size-at-age.

The model, which was first implemented in 1996, is described here and the criteria used for parameter estimation are specified. An outline of the model relative to the dynamics of the age classes represented in the exploited stock is presented first. This component is similar to previous age-structured models used on halibut and so its development should be familiar. This is followed by a reformulation of selectivity at age as a dynamic function of the size distribution of each age class in the population coupled with a size-based selectivity function. The effect of the minimum size limit on the catch age composition is modeled explicitly; other parameters controlling the size-based selectivity are allowed to change gradually over time for the commercial component of the system, while they are held constant at either size or age for the survey component. Finally, a model of how the size-distribution at age changes with time and through the effect of size-selective mortality is developed. Size-selective mortality couples growth and fishing mortality into a size-age dynamic model for each cohort.

Abundance Dynamics

The population abundance N of a cohort at age $a+1$ in year $t+1$ is related to the cohort's abundance at the previous age a and year t by:

$$N_{a+1,t+1} = N_{a,t} e^{-M} \left(1 - {}_bH_{a,t}\right) \left(1 - {}_cS_{a,t} \left(1 - e^{-F_t}\right)\right) \quad \text{for } a = 6, \dots, 18 \quad (1)$$

where M is the instantaneous rate of natural mortality, ${}_cS_{a,t}$ is the commercial selectivity of fish of age a at time t , F_t is the instantaneous commercial fishing mortality at time t for fully-selected fish, and ${}_bH_{a,t}$ is the annual rate of bycatch mortality at age a and time t , which results from fisheries targeting on other species. Age classes from age 6 to 20 are considered, where age 20 is actually a “plus” age-group which accumulates all fish of age 20 and older. The notation used in this and subsequent equations is summarized in Appendix 2.

The representation of fishing mortality differs from the familiar Baranov equation in that fishing is assumed to take place in a short period in the middle of the year, and selectivity at age is modeled as the *fraction* of each age class that is recruited to the exploitable stock and suffers an instantaneous fishing mortality equal to F_t . In other words, we equate selectivity with availability (Ricker 1975) and assume that all available fish are fully vulnerable. In the more familiar formulation, selectivity is equated with vulnerability, which affects the instantaneous rate of fishing mortality of different sizes of ages, and differences in availability of different stock components are ignored. Either formulation should be adequate in practice to explain differences in age composition between the population and the catches. The formulation above is computationally straightforward for use in determining effects on survivorship and size-at-age, and it is consistent with the definition of exploitable biomass used to

compute recommended catch levels. Note that the selectivity component ${}_cS_{a,t}$ is a function of both age and time, unlike standard separable age-structured models which assume that selectivity is a function of age alone and is constant over time.

Assuming in addition that bycatch mortality takes place prior to the fishing season, the catch associated with the directed commercial fishery ${}_cC$ follows:

$${}_cC_{a,t} = N_{a,t} e^{\frac{-M}{2}} (1 - {}_bH_{a,t}) {}_cS_{a,t} (1 - e^{-F_t}) \quad (2)$$

Note that equations (1) and (2) can be combined to give

$$N_{a+1,t+1} = \left(N_{a+\frac{1}{2},t+\frac{1}{2}} - {}_cC_{a,t} \right) e^{\frac{-M}{2}},$$

$$\text{where } N_{a+\frac{1}{2},t+\frac{1}{2}} = N_{a,t} (1 - {}_bH_{a,t}) e^{\frac{-M}{2}}$$

Age composition of the survey catches is given by

$${}_sP_{a,t} = \frac{{}_sS_{a,t} N_{a,t}}{\sum_i {}_sS_{i,t} N_{i,t}}, \quad (3)$$

where ${}_sS_{a,t}$ is survey selectivity at age and time, parameterized as described below. Predicted values of ${}_cC_{a,t}$ and ${}_sP_{a,t}$ are fitted to the observed catches for parameter estimation.

Catchability

Two abundance indices are used in the estimation: commercial CPUE (or effort) and survey CPUE. Strict proportionality between biomass and commercial CPUE was not assumed, as the catchability of the commercial fleet was allowed to vary according to a random walk model so that:

$$\ln({}_cQ_{t+1}) = \ln({}_cQ_t) + {}_q\varepsilon_t, \quad (4)$$

where ${}_q\varepsilon_t \sim N(0, {}_q\sigma^2)$. The parameter ${}_q\sigma^2$ is used to control the amount of year-to-year variation allowed in ${}_cQ_t$. Random walk models of this type were first used in fisheries models by Gudmundsson (1994) and Fournier et al. (1998). Similar formulations are used for other model parameters as well, whenever time-series trends are considered likely. The effective commercial effort can be predicted by assuming that mortality F_t for fully vulnerable fish is related to fishing effort according to:

$${}_c E_t = \frac{F_t}{{}_c Q_t}. \quad (5)$$

The survey catch in numbers per unit effort is predicted as

$${}_s CPUE_t = {}_s Q e^{\frac{-M}{2} \sum_a {}_s S_{a,t} N_{a,t}}, \quad (6)$$

where ${}_s Q$ is the catchability coefficient for the surveys, which is assumed to be constant except for an adjustment factor incorporated to account for a change in hook type as explained below. Note that equations (3) and (6) imply that the survey is assumed to take place prior to the commercial fishery when, in reality, a variable fraction of the commercial catch is taken before or during the survey. Alternative, more realistic formulations could be explored, but it is not clear that the problem is worth the increase in model complexity, relative to other sources of inter-annual survey variability.

Selectivity

Selectivity, the relative catchability of fish of different ages and sizes, is usually modeled as a function solely of age. In the so-called separable models (e.g. CAGEAN), age-specific selectivity is assumed to be time-invariant. Such an assumption results in a considerable reduction in the number of parameters that need to be estimated in catch-age analysis. The assumption is valid when capture is an age-dependent process as, for example, when organisms recruit to the fishery at a certain life stage, and when the size-at-age is relatively stable with time. The distribution of size at age of Pacific halibut has changed over time, with fish in the catch being about 20% smaller (in length) at age now than they were in the early 1980s. By not accounting for this change and by assuming that selectivity is constant at age, erroneous time trends can be introduced into the estimation procedure. To address this issue we modeled the change in commercial selectivity at age by tracking how size at age changes in the population while assuming that selectivity at size can only change very slowly through time. In this manner, the effect of the minimum size limit on the age composition of the catches could be explicitly incorporated, while at the same time we allowed for trends in size-selectivity that may occur particularly when size-at-age changes. Following Deriso and Parma (1989), the expected selectivity at age a and time t was computed by integrating the selectivity at size ${}_c s_t(X)$ over the distribution of size at age:

$${}_c S_{a,t} = \int_{-\infty}^{\infty} {}_c s_t(X) \phi(X | \mu_{a,t}, \sigma_{a,t}^2) dX \exp\{\epsilon_{sel, a,t}\} \quad (7)$$

where size is represented by log-length, X , and the function $\phi(X | \mu_{a,t}, \sigma_{a,t}^2)$ represents the probability that a fish of age a at time t is of log-length X , assumed to be Gaussian

with mean $\mu_{a,t}$ and variance $\sigma_{a,t}^2$. Small random deviations are allowed in selectivity for ages 6-10 by assuming that $\text{sel } \varepsilon_{a,t} \sim N(0, \text{sel } \sigma_t^2)$, giving greater flexibility to the fit for these less vulnerable age classes. Selectivities of age classes older than 10 are as predicted from their size-distribution coupled with the size-based selectivity (i.e. $\text{sel } \varepsilon_{a,t} = 0$ for $a > 10$). Selectivity at size for the commercial fishery at time t is represented in terms of the legal size (81 cm) and two parameters (X_t^{full} and n_t):

$$c_s s_t(X) = \begin{cases} 0, & \text{for } X < \ln(81) \\ \exp\left\{\frac{-(X - X_t^{\text{full}})^2}{2v_t}\right\} & \text{for } \ln(81) \leq X \leq X_t^{\text{full}} \\ 1, & \text{for } X > X_t^{\text{full}} \end{cases} \quad (8)$$

Selectivity is zero for fish smaller than the legal size, increases according to a half Gaussian curve scaled to reach a maximum of one at $X = X_t^{\text{full}}$, the size (log-length) at full selectivity, and equals one beyond X_t^{full} . Equations (1), (7), and (8) imply that all discarded sublegal fish are assumed to survive. The parameters X_t^{full} and n_t are allowed to change over time according to a random walk model with constraints on the variances of the year-to-year deviations:

$$\begin{aligned} X_{t+1}^{\text{full}} &= X_t^{\text{full}} + X_{\text{full}} \varepsilon_t & \text{where } X_{\text{full}} \varepsilon_t &\sim N(0, X_{\text{full}} \sigma_t^2) \\ \ln(v_{t+1}) &= \ln(v_t) + v \varepsilon_t & \text{where } v \varepsilon_t &\sim N(0, v \sigma_t^2) \end{aligned} \quad (9)$$

The formulation is similar to that used for $\ln(Q_t)$ except that the variances for the normal deviations are year-specific. This was done so as to allow selectivity to change more when growth rates are changing rapidly; very little change was allowed during periods of relatively stable size at age.

The size-selectivity of the longline survey is assumed to have the same functional form as the commercial selectivity except for the discontinuity at the legal size limit, which does not apply. In 1984, the hook type used in the surveys was changed from J-hook to the more efficient circle-hook used by the commercial fleet since the early 1980s. In order to estimate the relative efficiency of the two hook types, parallel sets were fished on each survey station in 1984, one with each hook type. The ratio of catches by 10-cm size category showed that the circle-hook selected fish of smaller sizes than the J-hook. Details of this analysis are presented in Appendix 1. The circle-hook was estimated to be twice as effective as the J-hook at catching large, fully selected fish. In order to account for these experimental results, the ratio of the catchabilities of the hooks for fully-selected sizes is assumed known and equal to two, and selectivity parameters are estimated separately for the two time periods with an overlap in 1984. The size-specific ratios of catches obtained with the two hook types in 1984 are predicted from the ratios of expected catches, computed by summing across age groups. Predicted catch ratios, ratio_t , are fitted to the observed ratios, $\text{ratio}_t^{\text{obs}}$, as explained below.

Two model formulations are considered with respect to survey selectivity: (a) selectivity at length is constant over time except for the change in 1984 associated with the change in hook type, and (b) selectivity at length changes over time so that the coupling of changing size at age and changing survey size-selectivity results in constant age-specific selectivity (except, again, for the change in hook type). In the first formulation, surveys conducted using the same hook are assumed to index population abundance by size-category, and so selectivity parameters X_t^{full} and n_t are constant for each hook type. In the latter, selectivity parameters X_t^{full} and n_t are allowed to change according to a random walk, but the variability in the derived selectivities at age is severely penalized so that selectivities are effectively constant at age. The first assumption would be more appropriate if survey selectivity reflected mostly the properties of the fishing gear as it interacts with fish of different sizes. The second would be preferred if the availability of fish of different age classes on the surveyed grounds were the dominant factor in determining survey selectivity.

The change in hook type experienced by the commercial fleet must have also affected the commercial selectivity. But because commercial selectivity is expected to change also in response to changes in targeting practices, a model that allowed for varying selectivities over time as explained above was preferred to just a break for the change in hook type.

Growth Dynamics

The selectivity and size distribution in the catch of fish of a given age-class depend on their size distribution in the population, which is not directly observed. Thus, the growth dynamics must be modeled as well. In the absence of size-selective mortality, the median length-at-age $m_{a,t} = \exp(\mu_{a,t})$ is assumed to propagate according to

$$m_{a+1,t+1} = \alpha_t + \beta m_{a,t} \quad \text{for } a=6,7,\dots \quad (10)$$

with time-varying initial size $m_{6,t}$ and intercept α_t . When the growth coefficient β is less than one, this representation corresponds to a von Bertalanffy model (applied to median length at age) with a time trend in the parameter corresponding to the asymptotic length $L_\infty = \alpha_t / (1 - \beta)$, and a time trend in size at age 6 (the age of recruitment). When $\beta = 1$, growth is linear with time-varying slope and initial size. The time-series trend in the mean log-length at recruitment $m_{6,t}$ is modeled as a random walk

$$\mu_{6,t+1} = \mu_{6,t} + \mu \varepsilon_t, \quad (11)$$

where $\mu \varepsilon_t \sim N(0, \mu \sigma^2)$. The growth intercept α_t is modeled as a cubic polynomial function of t

$$\alpha_t = p_0 + p_1 t + p_2 t^2 + p_3 t^3. \quad (12)$$

Thus, changes in modeled growth rate result from changes in the size at recruitment of each cohort, and an additive year effect on the annual growth increment applied across all cohorts. The slope term could easily have been generalized to cover higher order year- or age-specific effects, but because halibut growth is almost linear

(β is close to 1), the present form adequately predicted observed changes in size at age. Likewise, higher-order polynomials and random walk models for α_t resulted in very similar fits to the data. Autoregressive models, however, may be preferable for exploring possible factors underlying the observed trends in annual growth rates.

The variance of log-length at age $\sigma_{a,t}^2$ is linked to the mean $\mu_{a,t}$ by

$$\sigma_{a,t}^2 = [c + d\mu_{a,t}]^2. \quad (13)$$

If d is set to zero, $\sigma_{a,t}^2$ is constant and equal to c^2 , so that the coefficient of variation of length-at-age is constant and equal to $CV[L] = \sqrt{\exp(c^2) - 1} \cong c$. The variance relationship is assumed to hold even when $\mu_{a,t}$ changes due to size-selective mortality.

The effect of size-selective mortality on the size distribution at age is incorporated by adjusting the mean log-length at age, from $\mu_{a,t}$ (the mean prior to the fishing season) to $\mu_{a,t}^+$ (the mean at a time immediately following fishing). Realistically, the nature of the distribution should also be affected, but we assume that a Gaussian function is still an adequate approximation of the distribution of log-length after the fishery. We let the variance follow again as the square of a linear function of the mean as stated above. If the means change as a result of changes either in the environment or due to size selection, the variances will change as well in a corresponding manner.

Because larger fish are selectively removed by the fishery, $\mu_{a,t}^+$ is smaller than $\mu_{a,t}$. The mean log-length of fish that survive fishing is given by:

$$\mu_{a,t}^+ = \frac{\int_{-\infty}^{\infty} X (1 - {}_c s_t(X)) {}_c H_t \varphi(X | \mu_{a,t}, \sigma_{a,t}^2) dX}{\int_{-\infty}^{\infty} (1 - {}_c s_t(X)) {}_c H_t \varphi(X | \mu_{a,t}, \sigma_{a,t}^2) dX}, \quad (14)$$

where $(1 - {}_c s_t(X)) {}_c H_t$ corresponds to survivorship from fishing, with ${}_c H_t = (1 - \exp(-F_t))$ representing the harvest fraction of fish larger than the size at full selectivity, and $\varphi(X | \mu_{a,t}, \sigma_{a,t}^2)$ the probability density function of log-length X prior to fishing, as specified by its mean $\mu_{a,t}$ and variance $\sigma_{a,t}^2$. The denominator of the equation above corresponds to the fraction of fish of age a that survive after the fishing season.

The median length at age $a+1$, prior to the next fishing season, is predicted based on $m_{a,t}^+ = \exp(\mu_{a,t}^+)$ as:

$$m_{a+1,t+1} = \alpha_t + \beta m_{a,t}^+. \quad (15)$$

The corresponding mean of log-length prior to the next fishing season is $\mu_{a+1,t+1} = \ln(m_{a+1,t+1})$, which is used to calculate $\sigma_{a+1,t+1}^2$ as in equation (13). The two parameters that specify the probability density function of X prior to the fishing season at time $t+1$ are thus obtained and a new recursive cycle can be applied.

Given that the probability density function of the log-length-at-age for a cohort is represented by $\varphi(X|\mu_{a,t}, \sigma_{a,t}^2)$, the mean and variance of the log-length-at-age in the catch can be predicted as the first and adjusted second moments normalized by the average selectivity at age:

$$\begin{aligned} {}_c\mu_{a,t} &= \frac{\int_{\ln(81)}^{\infty} X {}_c s_t(X) \varphi(X|\mu_{a,t}, \sigma_{a,t}^2) dX}{\int_{\ln(81)}^{\infty} {}_c s_t(X) \varphi(X|\mu_{a,t}, \sigma_{a,t}^2) dX} \\ {}_c\sigma_{a,t}^2 &= \frac{\int_{\ln(81)}^{\infty} X^2 {}_c s_t(X) \varphi(X|\mu_{a,t}, \sigma_{a,t}^2) dX}{\int_{\ln(81)}^{\infty} {}_c s_t(X) \varphi(X|\mu_{a,t}, \sigma_{a,t}^2) dX} - {}_c\mu_{a,t}^2 \end{aligned} \quad (16)$$

Note that the integration is done across all sizes above the legal size limit ($\ln(81)$). Similar equations are used to predict the mean log-length at age ${}_s\mu_{a,t}$ and the variance of log-length at age ${}_s\sigma_{a,t}^2$ for survey catches; but because survey selectivity is not restricted by the legal size limit, the lower limit of integration is set to $-\infty$. The specific assumptions made about the probability density function of X and the shape of $s_t(X)$ lead to a numerically efficient algorithm. Selectivities at age $S_{a,t}$ and the moments of the distribution of X , in the catch and among the survivors, can be expressed simply as functions of standard Gaussian cumulative distributions as explained in Appendix 5.

Bycatch of Legal-Sized Halibut

Accounting for bycatch of legal-sized halibut is complicated because the age composition of the bycatch is unknown and only size compositions are available. Furthermore, modeling bycatch mortality by size and age is difficult due to changes in targeting practices and gear used in the different fisheries involved. As a consequence, no attempt was made to model the bycatch process as it is done with the directed catch. Instead, bycatch at length is apportioned into age classes internally in the model using the predicted size-age compositions, and it is then treated as known removals. To do this, the fraction of individuals in each 10-cm size category l is computed for each age a using the modeled size distributions at age:

$$f_{l|a,t} = \int_{X \in l} \varphi(X|\mu_{a,t}, \sigma_{a,t}^2) dX .$$

Then the age proportions for each size category are given by

$$f_{al,t} = \frac{N_{a,t} f_{l|a,t}}{\sum_i N_{i,t} f_{l|i,t}}.$$

By assuming that bycatch data are free of error, and that bycatch mortality occurs just prior to commercial fishing, the finite rate of bycatch mortality by age can be computed as:

$${}^b H_{a,t} = \frac{\sum_l {}^b C_{l,t}^{\text{obs}} f_{al,t}}{N_{a,t} e^{-\frac{M}{2}}}.$$

Objective Function

Model predictions are fitted to four types of observations: catch at age, effort or CPUE, size (length) at age, and circle-J-hook catchability ratios. These data come from the commercial fishery and longline survey samples. Bycatch mortality, on the other hand, is treated as known and subtracted out from each cohort based on the predicted age composition at size as discussed above. Thus, no fitting criterion for the bycatch component is discussed here.

Parameters in Table 6 are estimated for each IPHC regulatory area by minimizing differences between observations and model predictions. The objective functions as specified below include a likelihood component representing the statistical goodness-of-fit and a component that penalizes variability in parameter trends through an *a priori* weighting. Log-normal errors are assumed throughout and variances of the observations are assumed to be proportional to the square of their respective standard errors (se) divided by a weighting factor λ_k specific to the type of data as explained below. The full weighted residual sum of squares RSS is computed by summing the following components:

Catch-at-age equations:

$$RSS_{c:c} = \lambda_{c:c} \sum_a \sum_t \left[\frac{\ln({}_c C_{a,t}^{\text{obs}}) - \ln({}_c C_{a,t})}{\text{se}(\ln({}_c C_{a,t}^{\text{obs}}))} \right]^2$$

$$RSS_{s:c} = \lambda_{s:c} \sum_a \sum_t \left[\frac{\ln({}_s P_{a,t}^{\text{obs}}) - \ln({}_s P_{a,t})}{\text{se}(\ln({}_s P_{a,t}^{\text{obs}}))} \right]^2$$

Table 6. Estimated model parameters

Parameters		Number	Equation
$N_{6,t}$	Recruitment	T	(1)
$N_{a,1}$	Initial abundance	$A-1$	(1)
F_t	Fishing mortality	T	(1)
${}_c Q_1$ and ${}_q \varepsilon_t$	Commercial catchability	T	(4)
${}_s Q$	Survey catchability	1	(6)
$X_1^{\text{full}}, \nu_1,$ ${}_x \text{full} \varepsilon_t$ and ${}_v \varepsilon_t$	Size-selectivity	$2 \times T + 2 \times \text{number of surveys}$ (or 2 if surveys have constant size-selectivity)	(8)-(9)
${}_{\text{sel}} \varepsilon_{a,t}$	Deviations in selectivity at age	$(10-5) \times T$	(7)
$\mu_{6,1}$ and ${}_{\mu} \varepsilon_t$	Mean log-length at age 6	T	(11)
α_0 and β_0	Length at age in year 1	2	(17)
p_0, \dots, p_3	Polynomial for α_t , the intercept of growth equation	4	(12)
β	Slope of growth equation	1	(10)
c	Coefficient of variation of size at age	1	(13)

Effort/CPUE equations:

$$RSS_{c:e} = \lambda_{c:e} \sum_t \left[\ln(E^{\text{obs}}_t) - \ln(E_t) \right]^2$$

$$RSS_{s:e} = \lambda_{s:e} \sum_t \left[\frac{\ln({}_s CPUE^{\text{obs}}_t) - \ln({}_s CPUE_t)}{\text{se}(\ln({}_s CPUE^{\text{obs}}_t))} \right]^2$$

Length equations:

$$RSS_{c:\mu} = \lambda_{c:\mu} \sum_a \sum_t \left[\frac{\ln({}_c \mu^{\text{obs}}_{a,t}) - \ln({}_c \mu_{a,t})}{\text{se}(\ln({}_c \mu^{\text{obs}}_{a,t}))} \right]^2$$

$$RSS_{c:\sigma} = \lambda_{c:\sigma} \sum_a \sum_t \left[\frac{\ln({}_c \sigma^2_{a,t}^{\text{obs}}) - \ln({}_c \sigma^2_{a,t})}{\text{se}(\ln({}_c \sigma^2_{a,t}^{\text{obs}}))} \right]^2$$

$$RSS_{s:\mu} = \lambda_{s:\mu} \sum_a \sum_t \left[\frac{\ln({}_s \mu^{\text{obs}}_{a,t}) - \ln({}_s \mu_{a,t})}{\text{se}(\ln({}_s \mu^{\text{obs}}_{a,t}))} \right]^2$$

$$RSS_{s:\sigma} = \lambda_{s:\sigma} \sum_a \sum_t \left[\frac{\ln({}_s \sigma^2_{a,t}^{\text{obs}}) - \ln({}_s \sigma^2_{a,t})}{\text{se}(\ln({}_s \sigma^2_{a,t}^{\text{obs}}))} \right]^2$$

Circle-J-hook-conversion equations:

$$RSS_{c-j} = \lambda_{c-j} \sum_l \left[\frac{(\text{ratio}_{l}^{\text{obs}} - \text{ratio}_l)}{\text{se}(\text{ratio}_{l}^{\text{obs}})} \right]^2$$

Thus the total sum of squares is given as

$$RSS = RSS_{c:c} + RSS_{s:c} + RSS_{c:e} + RSS_{s:e} + RSS_{c:\mu} + RSS_{s:\mu} + RSS_{c:\sigma} + RSS_{s:\sigma} + RSS_{c-j}.$$

The negative log-likelihood $-\ln L$ of the observations, up to an additive constant, is

$$-\ln L = 0.5 n_{\text{obs}} \ln(RSS),$$

where n_{obs} is the total number of observations. Note that this formulation corresponds to the concentrated likelihood, where the residual variance for all the standardized normal observations (i.e., the weighted residuals) is estimated as

$$\hat{\sigma}^2 = \frac{RSS}{n_{\text{obs}}}.$$

Parameter estimates are obtained by minimizing the overall objective function

$$f = -\ln L + \text{penalties},$$

where the term “penalties” corresponds to prior assumptions made about some of the stochastic processes involved, namely time-series trends in catchability (equation (4))

$$PSS_q = 0.5 \sum_t \frac{q \mathcal{E}_t^2}{q \sigma^2},$$

time-series trends in mean log-length at age 6 (equation (11))

$$PSS_{\mu} = 0.5 \sum_t \frac{\mu \mathcal{E}_t^2}{\mu \sigma^2},$$

time-series trends in the parameters of the size-selectivity function $s_t(X)$ for the commercial fishery and the survey when appropriate (equation (9))

$$PSS_{x_{\text{full}}} = 0.5 \sum_t \frac{x_{\text{full}} \mathcal{E}_t^2}{x_{\text{full}} \sigma_t^2} \text{ and } PSS_v = 0.5 \sum_t \frac{v \mathcal{E}_t^2}{v \sigma_t^2},$$

and random deviations in selectivity at age affecting the youngest age classes (equation (7))

$$PSS_{sel} = 0.5 \sum_{a,t} \frac{{}_{sel}\mathcal{E}_{a,t}^2}{{}_{sel}\sigma^2} \quad \text{for } a=6, \dots, 10.$$

The term “penalties” in the objective function is thus

$$\text{penalties} = PSS_q + PSS_\mu + PSS_{x_{full}} + PSS_v + PSS_{sel}.$$

The model was implemented using AD Model Builder (Otter Research Ltd. 1994), which uses automatic differentiation to compute the analytical derivatives for input to a quasi-Newton algorithm in order to minimize the objective function. The minimization is conducted in steps or phases of increasing complexity as specified by the user. Scale parameters such as average recruitment, average fishing mortality and catchabilities are estimated first, before trends in abundance and mortality are allowed. Process error parameters (e.g. time-series trends in catchability and selectivity) are estimated in later phases. While this is no guarantee that a global minimum will be attained, experience indicates that a careful choice of phases can substantially improve the performance of the minimization. Monte Carlo trials done using initial parameter values generated at random has shown good convergence in assessments for which there are good survey data.

Weighting Criteria

Relative weights are used to control the emphasis that different *RSS* components receive in the estimation. Weights should correspond to the level of information present in the data. Two methods are used for setting those weights.

First, a relative weighting of observations of the same type (e.g. within catch, or length category) is incorporated on an observation-by-observation basis. We used empirically computed coefficients of variation of the statistics whenever possible. Because errors are assumed to be log-normally distributed, the coefficients of variation of the observations approximate the standard deviations of the corresponding log-transformed variables. Catch-at-age observations are weighted based on the coefficient of variation of the age proportions in the market sample data. Weights on the log-length mean and variance observations in the commercial catch, for years prior to 1991 when fish lengths were estimated from otolith sizes, are calculated using the coefficient of variation of the estimated moments as determined by bootstrap methods. Weights on log-length observations taken since that time, when the fish were actually measured, were determined analytically from standard equations based on simple random sampling methods. Details are provided in Appendix 5. Survey catch observations are weighted using coefficients of variation determined for all years from standard equations based on simple random sampling. Coefficients of variation for age proportions and CPUE in the surveys are also estimated assuming simple random sampling.

Second, a differential weighting of data of different types is effected through the λ s, as in previous model formulations (Table 7). The λ s are equivalently expressed

Table 7. Lambda and associated SDWeight ($1/\sqrt{\lambda}$), with rationale for information content. SDWeight represents lambda's information weighting in units of standard deviation.

Component	Lambda	SDWeight	Rationale
Commercial catch:			
Catch at age	0.25	2	Twice sample CV
Effort	50.00	$\sqrt{2} / 10$	Approximately twice the mean variance of catch
Mean log-length at age	0.04	5	Less emphasis than catch on fit
Variance of log-length at age	0.50	$\sqrt{2}$	Twice sample variance
Survey:			
CPUE	0.25	2	Twice sample CV
Age composition	0.25	2	Twice sample CV
Mean log-length at age	0.04	5	Less emphasis than catch on fit
Variance of log-length at age	0.50	$\sqrt{2}$	Twice sample variance
circle/J hook ratio	5.00	$1/\sqrt{5}$	One fifth sample variance

by the square-root of the inverse of λ , which we call the standard deviation weight or SDWeight. The SDWeight is often more intuitive as it presents the weightings in units of standard deviation that may be compared with relative weightings derived via the sample-size calculations discussed above. Sample-based measures of uncertainty do not normally capture all the variability present in the process, so λ s lower than one are used in most cases to increase the variance assigned to the different components and downweight their influence in an *ad hoc* manner. The λ associated with effort is greater than one because its variance cannot be estimated empirically and the residual sum of squares alone is not fully indicative of the effort variation relative to the catch component. The λ s associated with the growth components (mean and variance of log-length at age) are lower as their sampling precision is high due to large sample sizes, but their presence in the model is to guide the selectivity curves and not dominate the fit. A greater emphasis is also placed on the circle-hook/J-hook observed catch ratios by setting λ_{c_j} to five, as the information in this unique experiment outweighs its relatively small number of available sample points. All weights affecting the *RSS* are relative, as an overall residual variance is estimated.

In addition to the relative weights affecting the observations' *RSS*, variances for the four random walk components are set *a priori*. As described earlier these represent trends in commercial catchability, initial size at age, and the two selectivity parameters. The degree of variability from year to year is controlled by the assumed distribution on the random variables \mathcal{E} . Here the variables are assumed to be Gaussian distributed with zero mean and variance σ^2 . In contrast to the variances of the

observations, variances of these processes are assumed known, which effectively creates a prior for the amount of random deviation. The random walk for log-catchability is assumed to have a variance equal to 0.03^2 , which allows a substantial change in catchability over the period covered by the assessment without discarding completely the information about fishing mortality contained in the effort data. While this assumption is considered adequate for periods of relative stability in the fishery, more abrupt changes may have resulted from the implementation of individual quotas in Canada and Alaska which entail a major change in the conduct of the fishery. An alternative formulation, which includes a break in the time series in the first year of IQ management, resulted in somewhat lower (ca. 7%) estimates of biomass. Starting in 1998, the assessments used this latter formulation as the default. The variance of the changes in log-length at recruitment was set at 0.1^2 , which is not restrictive relative to the degree of change observed in the data; allowing larger variances did not affect the estimates. Finally, the variances of the random walks for the two selectivity parameters ranged between 0.01 during periods of stable growth rates to 0.03 when growth rates were changing fast.

Number of Observations

The availability of survey information varies depending on the regulatory area: Areas 2B and 3A were surveyed more often from 1974 through 1986, and since 1993, while other areas were surveyed more sporadically; no setline surveys were conducted from 1987 to 1992. Observations for the commercial fishery are available for all years and age groups modeled, except for effort data for year 1983, when the commercial fleet was in the process of switching from using J hooks to using the more efficient circle hooks. If there are A age groups and T years, there typically will be $A \times T$ observations on commercial catch, $A \times T$ observations on log-length at age, $A \times T$ observations on variance of log-length at age, and $T-1$ observations on fishing effort. For the surveys, there will be a maximum of $A \times (T-7)$ observations on the proportions-at-age, and the mean and variance of log-length at age, and $T-6$ CPUE observations. Aging of survey samples collected in the current year is not completed at the time of the assessment, only survey CPUE is available. Under this scenario there are $(6 \times A \times T) - (A \times 7 \times 3) + 2 \times T - 7$ observations. This amounts to 1886 observations for data covering 1974 through 1997.

Fundamental Model Parameters

In order to define a set of estimable parameters and to make sure that the estimates have reasonable values, certain parameters are fixed while others are estimated under a specified set of constraints. The natural mortality parameter is one such parameter which is typically fixed. It is set here to $M = 0.2$ and assumed to be constant over all time periods and age-classes modeled.

The initial conditions for median size at age in the population at the start, $m_{a,1}, a \in \{6, \dots, 20+\}$, are constrained to follow a von Bertalanffy model:

$$m_{a+1,1} = \alpha_0 + \beta_0 m_{a,1} \quad \text{for } a = 6, 7, \dots \quad (17)$$

where $m_{6,1} = \exp(\mu_{6,1})$, and α_0 and β_0 are estimated parameters. The growth coefficients β and β_0 are constrained to be between 0.5 and 1.0, the log-length at full selectivity X_t^{full} in year $t=1$ is constrained to be less than $\ln(130)$, and a quadratic penalty is added to the objective function so as to force the predicted commercial and survey selectivities for the 20+ age group to be equal to one. This penalty has been eliminated in more recent assessments (starting in 1998) because it was causing the model to overestimate size at age in the older age groups. The variance of log-length at age is assumed to be constant by setting $d = 0$ in equation (13). Estimated parameters are shown in Table 6, although the actual minimization is conducted over a different parameter space. Re-parameterizations are used to reduce the correlation among estimated parameters, and transformations are used in some cases to constrain parameter values; the latter is done automatically by AD Model Builder when bounded parameters are specified.

Derived Parameters

Derived parameters of management interest are total biomass

$$TB_t = \sum_{a=8}^{20+} w_{a,t} N_{a,t} ,$$

and exploitable biomass

$$B_t = \sum_{a=8}^{20+} {}_c S_{a,t} w_{a,t} N_{a,t} ,$$

where $w_{a,t}$ are smoothed weights at age in the commercial catch. Weight at age in the commercial catch is used in the calculation of exploitable biomass, rather than weight at age in the population. The exploitable biomass at the beginning of the year $T+1$ is predicted as

$$B_{T+1} = \sum_{a=7}^{19} {}_c S_{a+1,T} w_{a+1,T} N_{a,T} e^{-M} (1 - {}_b H_{a,T}) (1 - {}_c S_{a,T} {}_c H_T) + {}_c S_{20,T} w_{20,T} N_{20,T} e^{-M} (1 - {}_b H_{20,T}) (1 - {}_c S_{20,T} {}_c H_T) .$$

In all cases, aggregate estimates only include ages $a = 8$ in order to reduce the influence of younger, poorly estimated age classes. Recruitment is represented by the abundance of eight-year-old halibut.

Uncertainty of Parameter Estimates

AD Model Builder (Otter Research Ltd. 1994) provides standard deviations of estimated and derived model parameters as specified by the user. The covariance matrix of the parameter estimates is estimated by inverting the Hessian matrix and

using the delta method in the case of derived parameters, such as predicted exploitable biomass. IPHC staff is currently evaluating the use of Markov Chain Monte Carlo methods, which have been recently implemented in AD Model Builder, to express uncertainty and conduct simulations for policy evaluation.

RESULTS FROM THE 1997 STOCK ASSESSMENT

The procedure outlined above was first used in the assessment of Pacific halibut stocks in 1996. Its application led to a substantial increase in the estimates of exploitable biomass, in contrast to estimates derived under the separable age-structured analysis based on CAGEAN (Deriso et al. 1985). The increase in the estimates can be broken down into three major components. (1) Because selectivity is modeled as a function of fish size, the observed reduction in size at age has resulted in reduced catchability of younger age groups by setline gear through fish behavior and the effect of the legal size limit. The poor representation of younger halibut in the catch was attributed to poor recruitment in earlier assessments rather than to lower catchability due to smaller size. The new model estimates a lower catchability, and so the estimated abundance of both younger and older age groups has increased accordingly. (2) Bycatch mortality of legal-sized halibut is now included in the assessment along with other removals (i.e. commercial and sport catches, wastage, and personal use). The estimated biomass must increase to account for the increased removals. The magnitude of the increase depends on the amount of legal-sized bycatch mortality relative to total stock biomass in each area. (3) Trends in survey CPUE, now used in the assessment, either support trends seen in the commercial CPUE or they indicate greater increases in abundance since the mid 1980s.

In order to put the data and model discussed thus far into a more concrete context, estimates from the 1997 assessment for Area 2AB and Area 3A (Figure 1) are presented. The Area 2AB assessment represents the stock off IPHC Area 2A (Washington, Oregon, and northern California) and Area 2B (British Columbia). The Area 3A assessment represents the stock in the central Gulf of Alaska. (Area 3A input data are provided in Appendix 3). These two assessments span the range of results for areas with the longest history of surveys and highest exploitation rates. At the other extreme are Areas 3B and 4, where historical series of survey data are lacking, commercial CPUE is of limited value as the commercial fleet does not cover fully the grounds, and harvest rates have been low. As a consequence, catch-at-age analyses (both old and new methods) have performed poorly in these areas, substantially underestimating abundance as the recent coast-wide surveys indicate. Starting in 1997, the assessments of Areas 3B and 4 have been based on direct estimates of relative abundance provided by the surveys, scaled using the analytical estimate of exploitable biomass in Area 3A, or the sum of Areas 2 and 3A.

The assessment of Areas 2AB and 3A are also good examples because they provide some interesting contrasts in growth trends and recruitment patterns. The reduction in growth rate was most dramatic in the north (Area 3A), in contrast to the south (Area 2AB) where the decline was subtler (Figure 6). Individual quota management systems were also implemented at different times in the two areas, a vessel quota system in British Columbia in 1991, and a fisher quota system in Alaska in 1995.

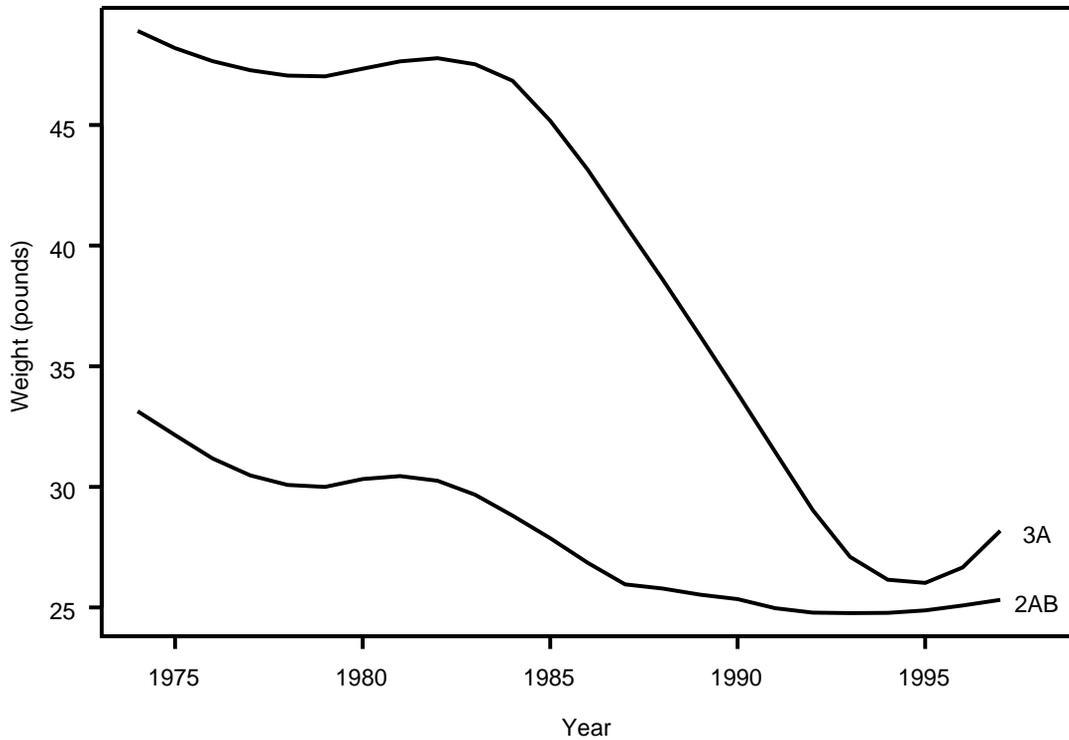


Figure 6. Pacific halibut smoothed weight at age 12 in the commercial catch for IPHC Regulatory Areas 2AB and 3A.

How survey selectivity operates is particularly relevant to the differences in each area's assessments. Surveys are designed to provide a consistent mechanism for taking observations over time, so that changes in survey CPUE reflect changes in population density rather than changes due to gear configuration or targeting. However, fish behavior at different sizes and life stages also influences their likelihood of being captured. For example, if the chance of a halibut getting caught were simply a function of size, with larger hooks catching larger fish, then with dropping size at age one would expect constant selectivity at size and dropping selectivity at age. On the other hand, if the chance of a halibut getting caught were more a function of age, with halibut suddenly appearing on the grounds at age eight, say, then with dropping size at age one might still expect constant selectivity at age despite the smaller size of the fish.

In a commercial fishery with a minimum size limit, a decrease in selectivity at age with a decrease in size is expected. In the survey (which captures and measures all fish) we expect selectivity to primarily reflect differences in vulnerability of fish of different sizes, which presumably should stay constant over time. Yet we see consistent differences in estimated selectivity between areas (Figure 7), despite the fact that the same gear is used throughout all surveyed areas. These differences indicate that other factors besides size affect selectivity, for example age-specific availability of halibut on the fishing grounds.

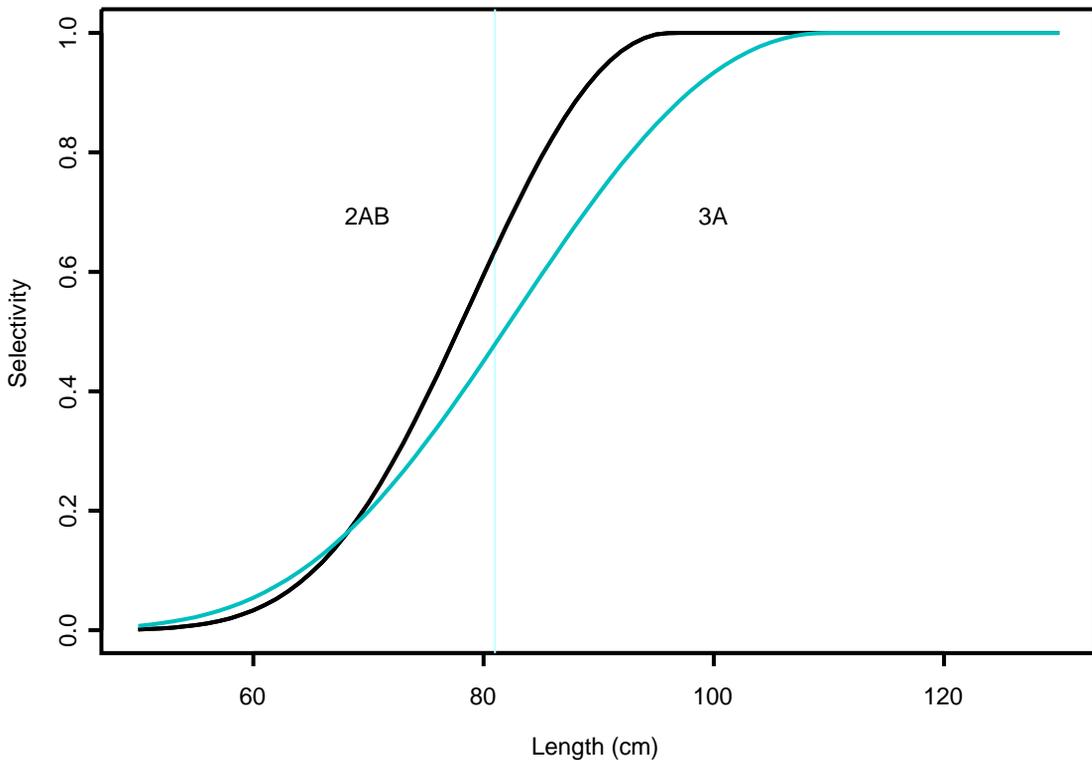


Figure 7. Comparison of survey selectivity at length between Areas 2AB and 3A from assessments assuming constant survey selectivity at length.

Because this uncertainty cannot be resolved at present, two assessments were conducted for each of the regulatory areas for which long-term surveys are available. One of them assumes that survey selectivity at *age* remains constant while size at age decreases; the other assumes that survey selectivity at *length* remains constant (Figure 8, Tables 8-11). Differences in the assessments are greatest for the most recent four to five years, and are most significant for the Area 3A assessment, where the decrease in individual size at age has been greatest. Constant-age-selectivity estimates of population biomass are lower than constant-length-selectivity estimates. Eight-year-old abundance is greater under the assumption of constant size-specific selectivity.

Retrospective analyses conducted with the new model have shown marked improvement relative to CAGEAN. Comparison of residuals and retrospective patterns obtained using the two formulations (including data up to 1998), however, has not favored one model structure over the other (Clark and Parma, Unpub.³). Both models have problems fitting the early survey age compositions, especially in Area 3A, independently of the assumptions made about selectivity. Early surveys indicate higher relative abundance of young fish in the 1970s than is predicted by either model, while the opposite is true in the 1980s. The cause of this is unknown but density-dependent effects (larger fish excluding the smaller ones) is one possibility.

³ Clark, W. and A. M. Parma. Unpub. Assessment of the Pacific halibut stock in 1998. [In] Int. Pac. Halibut Comm. Rpt. of Assessment and Research Activities 1998: 89-112.

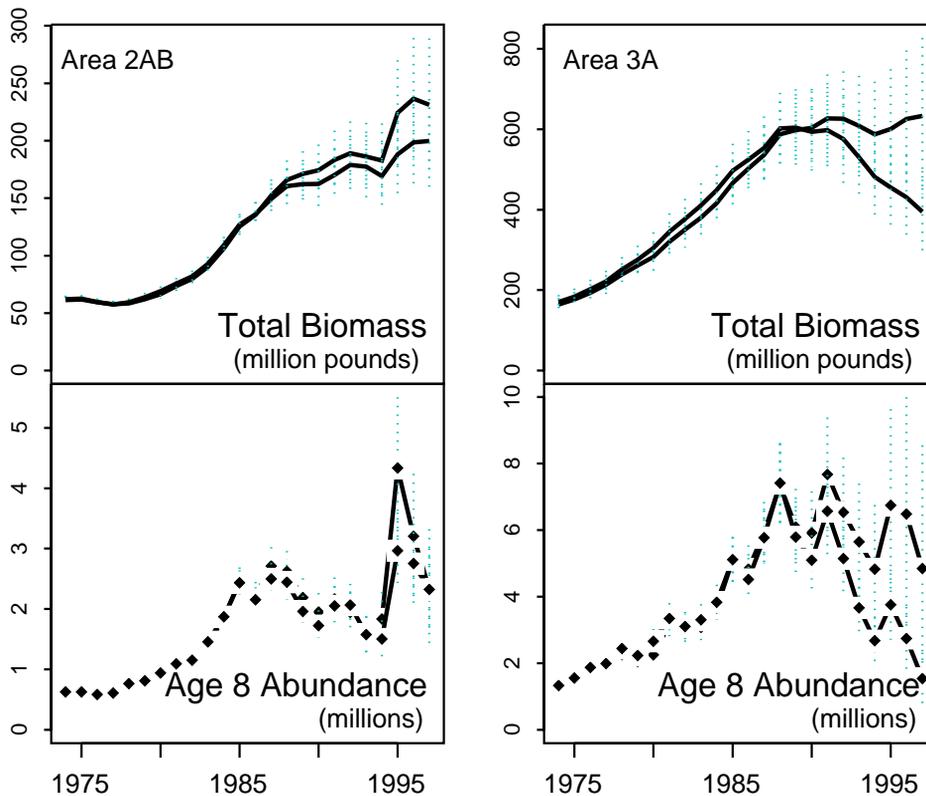


Figure 8. Total biomass (age 8 and older) and eight-year-old abundance for Areas 2AB and 3A for alternate assumptions about survey selectivity. Upper points and lines are from assessments assuming constant selectivity at length in the survey, while lower points and lines are from assessments assuming constant selectivity at age in the survey.

HARVESTING STRATEGY

Background

Until 1985, allowable removals were calculated as a proportion of estimated annual surplus production (ASP), the remaining production being allocated to stock rebuilding. In 1986, once the stocks were considered rebuilt, IPHC adopted a constant harvest rate policy, meaning that allowable removals are determined as a fixed fraction of the estimated exploitable biomass. A fixed-harvest rate strategy was chosen because this kind of strategy has been shown to achieve close to optimal yields in the face of long-term changes in productivity such as exhibited by Pacific halibut (Parma 1990; Walters and Parma 1996). While the intention is to choose a harvest rate and maintain it for a long time, new information provided by the annual assessments led to a series of revisions of the harvest rate. Initially set to 0.26 in 1986, it was soon increased to 0.35 in 1987, and later decreased to 0.30 in 1992. The choice of a 30% harvest rate was based on an analysis of historical trends in biomass using estimates of abundance and growth available at the time (1935-1991). Since then, major changes in the stock

and the assessment methodology have taken place, which required another re-evaluation of the harvest rate. The dramatic reduction in halibut body growth rate observed in recent years implies that future average reproductive contribution per recruit may be lower than assumed in previous analyses if growth rates stabilize or decrease even further. Also, changes in methodology, as explained above, resulted in substantially higher estimates of biomass and recruitment, and lower estimates of selectivity. This section evaluates the performance of different harvest rates using revised models for the relationship between reproductive biomass and subsequent recruitment, new growth rates, and current estimates of selectivity to compute exploitable biomass. In addition, the effect of bycatch on fish below the legal size has been incorporated as pre-recruit mortality.

Table 8. Output from Area 2AB assessment under constant selectivity at age assumption with biomass in millions of pounds net weight and abundance in millions.

Year	Exploitable Biomass w/ Estimated Selectivity	Total Biomass (millions of pounds)	Historical Exploitation Rate	Abundance of Eight-year-old Halibut (millions)
1974	49.51	62.13	0.13	0.63
1975	50.21	62.49	0.17	0.63
1976	47.73	59.61	0.18	0.59
1977	45.05	57.39	0.15	0.61
1978	44.56	59.38	0.12	0.77
1979	46.37	63.83	0.13	0.81
1980	48.04	69.17	0.13	0.94
1981	50.23	75.73	0.13	1.09
1982	52.50	81.90	0.12	1.16
1983	56.62	92.83	0.11	1.46
1984	63.21	109.08	0.17	1.87
1985	67.42	127.17	0.19	2.43
1986	69.34	135.94	0.19	2.15
1987	75.35	149.41	0.20	2.50
1988	84.35	160.38	0.18	2.44
1989	90.31	162.27	0.14	1.96
1990	96.64	162.39	0.12	1.73
1991	102.94	170.14	0.10	2.05
1992	109.19	178.85	0.09	2.07
1993	107.01	177.36	0.13	1.58
1994	100.42	169.28	0.12	1.50
1995	101.91	188.05	0.12	2.96
1996	103.77	198.67	0.11	2.75
1997	105.18	199.68	0.14	2.32
1998	89.74			

Table 9. Summary of Outputs from Area 2AB assessment constant selectivity at length assumption with biomass in millions of pounds net weight and abundance in millions.

Year	Exploitable Biomass w/ Estimated Selectivity	Total Biomass (millions of pounds)	Historical Exploitation Rate	Abundance of Eight-year-old Halibut (millions)
1974	49.21	61.32	0.13	0.63
1975	48.32	61.92	0.18	0.64
1976	44.86	59.33	0.19	0.62
1977	42.30	57.68	0.16	0.64
1978	42.82	58.47	0.13	0.69
1979	45.41	62.09	0.13	0.75
1980	47.57	66.41	0.14	0.87
1981	49.36	73.61	0.14	1.13
1982	51.66	79.72	0.12	1.15
1983	56.11	90.14	0.11	1.41
1984	62.90	106.31	0.17	1.85
1985	67.37	125.25	0.19	2.46
1986	70.19	135.47	0.19	2.22
1987	75.05	152.35	0.20	2.71
1988	82.38	165.78	0.18	2.62
1989	86.81	171.07	0.15	2.19
1990	91.00	174.29	0.12	1.95
1991	93.58	183.39	0.11	2.17
1992	103.89	188.98	0.10	1.95
1993	109.32	186.16	0.12	1.56
1994	106.22	182.77	0.12	1.84
1995	106.70	224.32	0.11	4.33
1996	111.52	236.51	0.10	3.21
1997	117.18	231.27	0.12	2.33

Table 10. Output from Area 3A assessment under constant survey selectivity at age assumption with biomass in millions of pounds net weight and abundance in millions.

Year	Exploitable Biomass w/ Estimated Selectivity	Total Biomass (millions of pounds)	Historical Exploitation Rate	Abundance of Eight-year-old Halibut (millions)
1974	98.62	170.44	0.13	1.33
1975	104.77	184.03	0.13	1.56
1976	112.31	202.55	0.12	1.88
1977	122.68	222.11	0.10	1.99
1978	140.85	251.47	0.09	2.45
1979	159.87	276.07	0.10	2.23
1980	175.31	304.70	0.10	2.66
1981	194.29	345.32	0.10	3.35
1982	211.58	376.91	0.08	3.11
1983	239.19	410.62	0.07	3.30
1984	265.13	449.13	0.08	3.83
1985	291.76	496.96	0.08	5.11
1986	307.73	524.37	0.12	4.52
1987	328.71	553.30	0.11	5.78
1988	354.41	601.86	0.13	7.41
1989	341.87	604.14	0.12	5.80
1990	318.48	593.87	0.11	5.09
1991	295.80	598.01	0.11	6.57
1992	285.21	575.73	0.12	5.15
1993	267.94	529.94	0.11	3.67
1994	275.90	481.48	0.12	2.68
1995	263.60	455.11	0.09	3.76
1996	260.63	430.91	0.10	2.74
1997	251.86	394.76	0.12	1.54

Table 11. Output from Area 3A Assessment constant survey selectivity at length assumption with biomass in millions of pounds net weight and abundance in millions.

Year	Exploitable Biomass w/ Estimated Selectivity	Total Biomass (millions of pounds)	Historical Exploitation Rate	Abundance of Eight-year-old Halibut (millions)
1974	95.24	164.39	0.13	1.29
1975	100.91	176.92	0.13	1.49
1976	107.64	193.68	0.13	1.77
1977	116.06	213.15	0.11	1.98
1978	132.21	239.46	0.10	2.26
1979	149.55	261.00	0.11	2.05
1980	165.44	282.52	0.11	2.25
1981	182.04	319.86	0.10	3.19
1982	195.06	350.51	0.09	3.05
1983	217.82	380.52	0.08	3.08
1984	239.55	416.56	0.09	3.66
1985	264.28	466.66	0.09	5.10
1986	275.51	502.11	0.14	4.83
1987	289.66	536.42	0.13	5.93
1988	314.14	587.42	0.14	7.37
1989	303.68	598.07	0.13	6.09
1990	282.98	603.36	0.13	5.92
1991	263.52	626.79	0.12	7.68
1992	256.15	626.02	0.13	6.53
1993	239.07	608.38	0.13	5.65
1994	246.36	587.84	0.13	4.83
1995	250.16	600.75	0.10	6.74
1996	266.29	625.95	0.10	6.48
1997	280.42	633.46	0.11	4.85

Historical Trends in Abundance

Estimates of recruitment and reproductive biomass for the northeast Pacific (Areas 2 and 3) are shown in Figure 9, together with the corresponding estimates used in the previous analysis of harvesting strategies, which justified the choice of the 30% exploitation rate (dashed lines). The old estimates of abundance were substantially lower than the revised ones produced in 1996 using the new assessment method. In addition, maturity at age was re-evaluated using IPHC research data (Clark and Parma 1995, and analysis of data collected since 1993). This resulted in an age at 50% maturity approximately equal to 11, instead of 12 as used in previous analyses (after data reported in St-Pierre 1984); this change in estimates accounts for the differences prior to 1974. Technical details about methods used to estimate historical trends in abundance are provided in Appendix 4.

The dramatic decline in recruitment of 8-year-olds estimated by CAGEAN during the late 1980s and 1990s coincided with a period of increasing spawning biomass during the early 1980s (Figure 10, top panels). Opposite trends in the number of 8-yr-olds and parental biomass gave support to the hypothesis that the stock-recruitment relationship was strongly density-dependent exhibiting overcompensation.

The new stock assessment results show a very different relationship between spawning biomass and subsequent recruitment in recent years. Instead of declining, recruitment estimates for the last ten years either fluctuate without clear trend (Figure 10, bottom panels) or they increase, depending on whether survey selectivity is assumed to be a function of age or size, respectively. In either case, recruitment

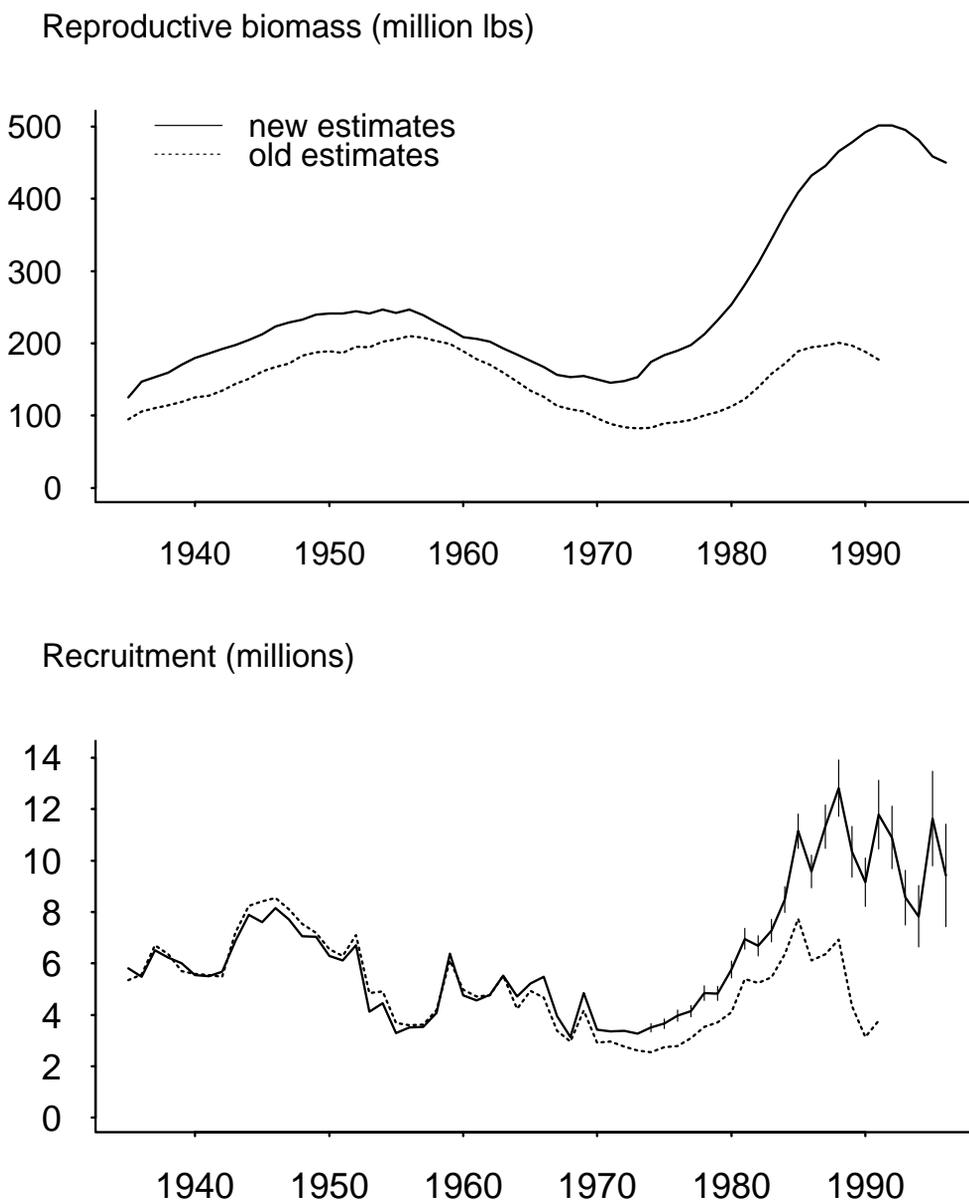
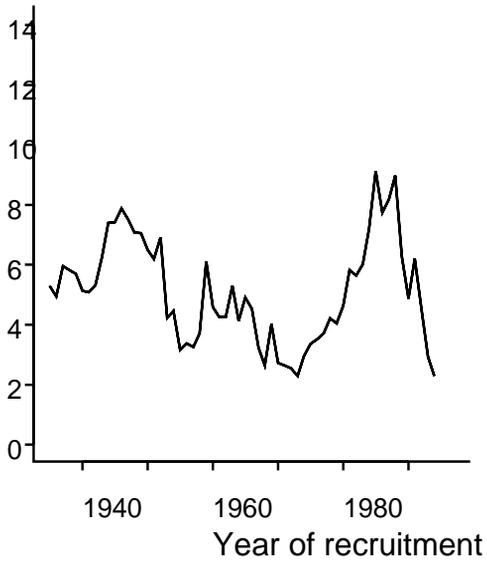


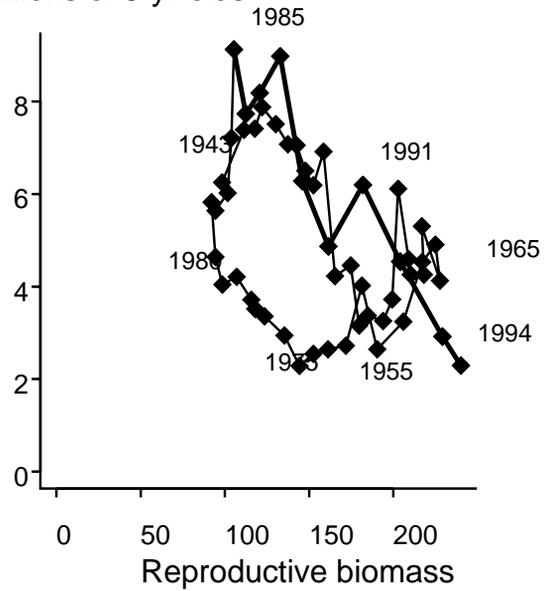
Figure 9. Trends in recruitment and reproductive biomass in the NE Pacific.

Results from old model

Millions of 8-yr-olds

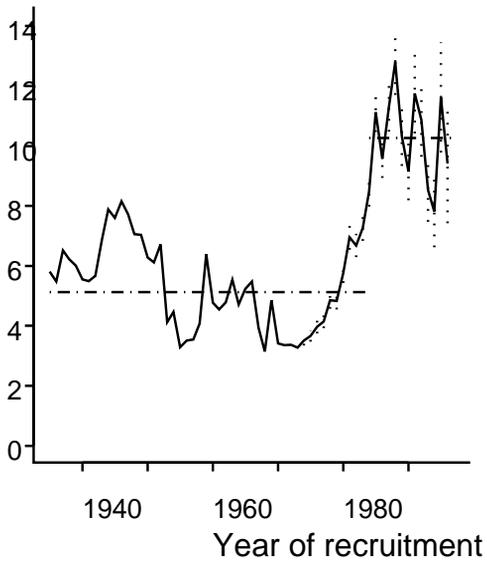


Millions of 8-yr-olds



Results from new model

Millions of 8-yr-olds



Millions of 8-yr-olds

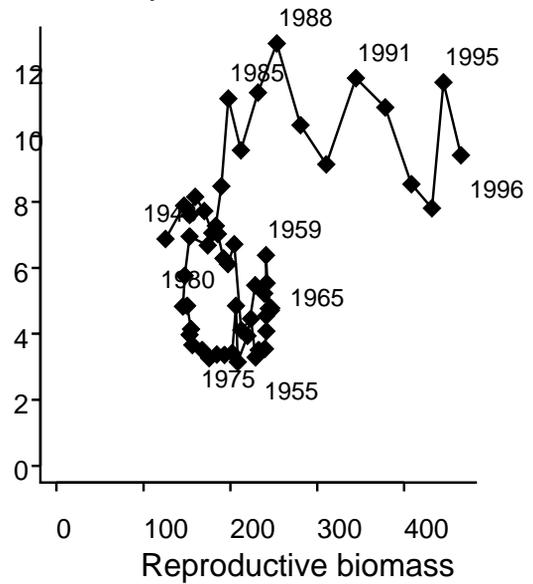


Figure 10. Recruitment trends and stock-recruitment relationships estimated by the old and new assessment model. (Biomass in million pounds.)

on the amount of habitat available in Area 3B and its high catch rates, Area 3B would be expected to produce roughly 2/3 the production in Area 3A. The estimates of biomass derived with the analytical model, however, have been on average 1/3 of those of Area 3A. To the extent that recruitment trends in different areas have been similar, this problem may not have as big an effect on the choice of a harvest rate. Predicted yields, however, would be too low.

Simulation Model

The harvest rate is chosen by simulating stock productivity at a range of harvest rates under the operation of a range of spawner-recruit relationships consistent with historical estimates of spawning biomass and subsequent recruitment. A standard age-structured model was used to simulate the dynamics of the population. The number of fish of age a at the beginning of year t was given by:

$$N_{a,t} = \left[N_{a-1,t-1} e^{-0.5M} - C_{a-1,t-1} \right] e^{-0.5M} \quad \text{for } a=9, \dots, 19,$$

$$N_{20,t} = \left[N_{19,t-1} e^{-0.5M} - C_{19,t-1} + N_{20,t-1} e^{-0.5M} - C_{20,t-1} \right] e^{-0.5M}$$

where $M=0.2$ is the coefficient of natural mortality and $C_{a,t}$ is catch-at-age in numbers. The fishing season was assumed to be short and to take place at the middle of the year.

The recruitment estimates obtained with the new models show weaker density-dependence than earlier estimates, and indicate that the environment has played a major role in driving recruitment variation, at least within the range of stock levels observed. With this new view of historical recruitment patterns, we have considered a number of new stock-recruitment relationships for evaluating alternative harvest strategies. In all of the relationships explored, a great deal of the recruitment variability is induced by the environment, and so is not under management control. Two are described here:

(1) A Ricker model with correlated environmental effects (Figure 12a): number of recruits at age eight is given by

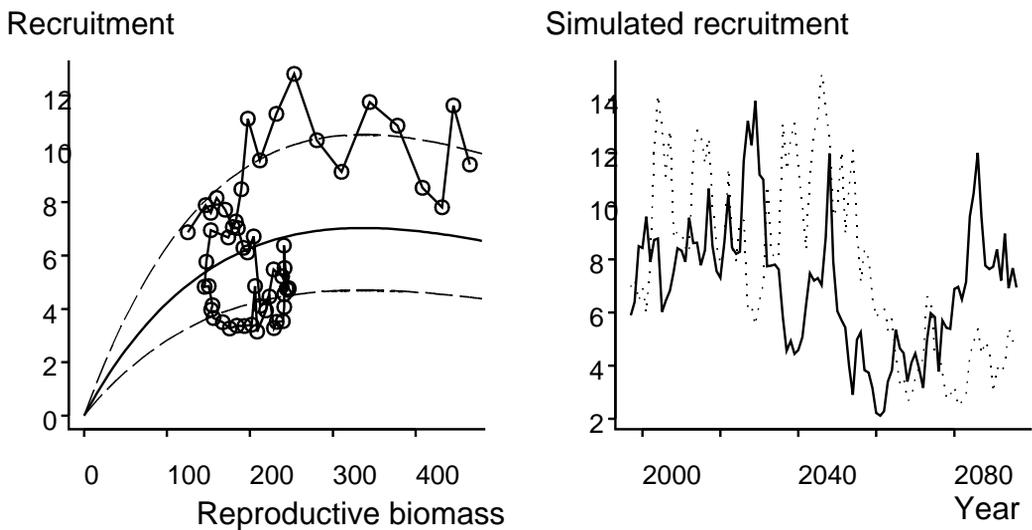
$$R_{t+8} = SB_t \exp(a - bSB_t + {}_r\varepsilon_t),$$

where SB_t is reproductive biomass and $\{{}_r\varepsilon_t\}$ represent random environmental effects. The latter are modeled as an autoregressive (AR) process of order one,

$${}_r\varepsilon_t = \rho {}_r\varepsilon_{t-1} + e_t,$$

where e_t is Gaussian with mean 0 and variance ${}_r\sigma^2(1 - \rho^2)$. The parameter ${}_r\sigma^2$ represents the variance of ${}_r\varepsilon_t$ and ρ corresponds to the correlation between ${}_r\varepsilon_t$ and ${}_r\varepsilon_{t-1}$. Parameters a , b , ρ and ${}_r\sigma^2$ were estimated by maximum likelihood to stock-recruitment data for 1943-1996, assuming that reproductive biomass was observed

a. Ricker model with gradual environmental changes



b. Flat model with shifts in climatic regime

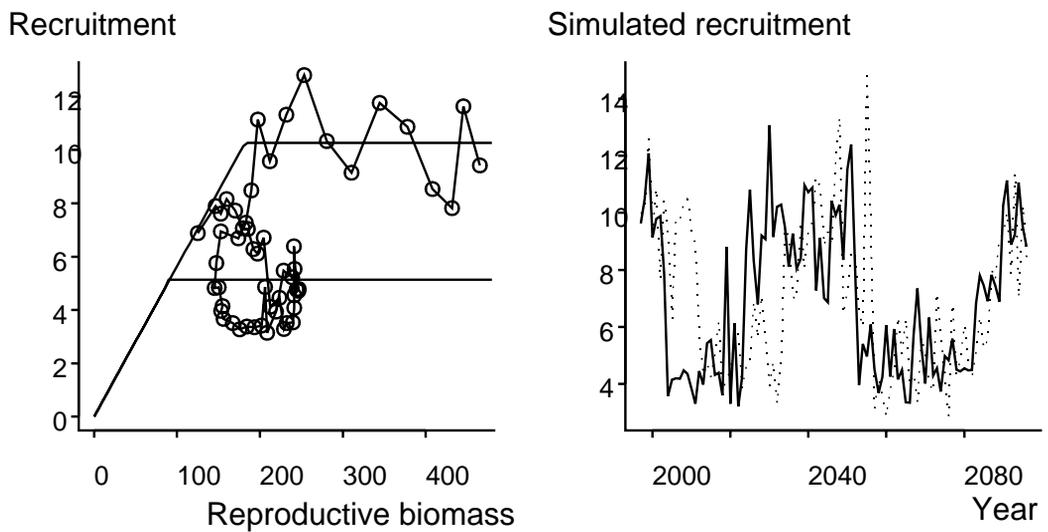


Figure 12. Stock-recruitment models used to evaluate alternative harvest rates. Plots on the right show two recruitment trajectories simulated with each model. (Biomass in million pounds; recruitment in millions of eight-year-olds.)

without error (Table 12). The fit to the Ricker function has strong residual trends (Figure 10a) which resulted in a high autocorrelation coefficient ρ close to 0.9. A few recruitment trajectories simulated with this model (Figure 10a) are included to illustrate how the model reproduces the sort of persistent periods of above- and below-average productivity characteristic of the historical estimates.

Table 12. Parameter values of the different stock-recruitment models used in the simulations (values correspond to biomass expressed in million pounds and recruitment in thousands of 8-yr-olds).

Stock-recruitment model	Parameters of the stock-recruitment function	Parameters controlling unpredictable environmental variability
(1) Ricker with correlated environmental effects	$a = -2.8686$ $b = 2.96982 \times 10^{-3}$	$\rho = 0.89$ $\sigma = 0.40$
(2) Flat with shifts in carrying capacity	$a = 0.784503$ $K_1 = 5130$ $K_2 = 10270$	$\sigma = 0.2$

(2) A flat model with shifts in carrying capacity (Figure 12b): in this scenario expected recruitment increases in proportion to reproductive biomass until carrying capacity (K_i) is reached, and is constant thereafter,

$$R_{t+8} = \min(a SB_t, K_i) e^{r \epsilon_t}.$$

Carrying capacity K_i , is affected by environmental conditions which shifts between two very different regimes every 20-30 years (year is randomly selected) so that K_i alternates between two values, K_1 and K_2 . There is additional process noise represented by $\{r \epsilon_t\}$, a series of independent Gaussian random variables with mean 0 and variance $r \sigma^2$. The slope a was set at the maximum estimated value of R_{t+8} / SB_t ; K_1 and K_2 were set at the exponential of the mean log-recruitment for the periods 1943-1984 and 1985-1996 respectively.

Both models predict that recruitment will decrease gradually if spawning biomass decreases to levels lower than the historical minimum. Predictions made about how juvenile production would change if the stock dropped to unprecedented stock levels are extremely uncertain, as they are based on an extrapolation beyond the range of historical experience. Parameter values used in the simulations are shown in Table 12.

Recruitment (R_t) was simulated according to one of the stock-recruitment models described above, using a random number generator; spawning was assumed to occur at the start of the year. Because recruitment estimates used in the stock-

recruitment analysis were adjusted upwards for pre-recruit mortality due to bycatch, the stock-recruitment models above represent recruitment productivity in the absence of bycatch. In past studies, bycatch was ignored when selecting a harvest rate, and the reproductive loss due to bycatch was later compensated by reducing the setline quota. In the new procedure (Clark and Hare 1998) bycatch of sublegal halibut is accounted for when evaluating alternative harvest rates by assuming that it results in a 10% decrease in recruitment of eight-year-olds.

Harvest rates ranging from $H=0.0$ to $H=0.5$ in intervals of 0.01 were evaluated by Monte Carlo simulations (500 replicates). Total catch in biomass, Y_t , was determined each year as a function of the estimated exploitable biomass, according to the specific harvest rate being evaluated. Thus,

$$Y_t = H\hat{B}_t,$$

provided that $Y_t < B_t \exp(-0.5M)$, where \hat{B}_t is the estimate of exploitable biomass at the beginning of year t , and B_t is the true simulated exploitable biomass. The latter was computed using constant age-specific selectivities s_a and mean weights at age w_a as

$$B_t = \sum_{a=8}^{20} N_{a,t} w_a s_a.$$

Weights-at-age were those estimated from the catch of 1996, so the possibility of future trends in growth was ignored. Age-specific selectivities were the average of the age-specific selectivities estimated for Areas 2AB and 3A in the assessment of 1996, weighted by the respective abundance in those areas. This same fixed-selectivity schedule is used in reality when allowable removals are determined as a fraction of the estimated exploitable biomass. Because it is based on estimates of selectivity obtained with the new assessment model, this fixed-selectivity schedule is lower than the one used until 1995 which was based on a coast-wide assessment done using CAGEAN. As a result, the harvest rate applied to this new definition of exploitable biomass cannot be compared directly to the 30% level used in the past.

The estimate of exploitable biomass was assumed to be a log-normally distributed random variable with median equal to the true (simulated) exploitable biomass, so

$$\hat{B}_t = B_t e^{\eta_t}$$

where η_t is a normal random variable with mean equal to 0 and standard deviation equal to 0.20, which represents assessment error. When computing B_t above, the actual simulated abundance at age eight, was replaced by average abundance at age eight.

Y_t was then partitioned among age classes according to their relative abundance in the exploitable stock, i.e.

$$C_{a,t}w_a = Y_t \frac{N_{a,t}w_a s_a}{\sum_i N_{i,t}w_i s_i}$$

In order to incorporate the uncertainty in the initial stock abundance, the initial conditions for the simulations were selected at random so that each Monte Carlo replicate started with a different “true” stock size. The assumption was that the error in the estimate of abundance for 1997 is a log-normally distributed variable with coefficient of variation equal to 0.20. The actual estimate of exploitable biomass projected for 1997 from the 1996 assessment was used to set the CEY under the postulated harvest rate in the first year of simulation.

Stock trajectories were simulated for 200 years; results from the last 100 years of the runs were used to evaluate long-term performance of the policies in terms of:

- (1) Mean yield (averaged over all years and runs)
- (2) Mean level of reproductive biomass (averaged over all years and runs)
- (3) Mean number of years in which the spawning biomass was below 125 million pounds (approximately the minimum value in the historical time series).

In addition, the probability that the reproductive stock dropped below the historical minimum at least once over the first 20 years of simulation was estimated as the fraction of the Monte Carlo replicates in which that happened.

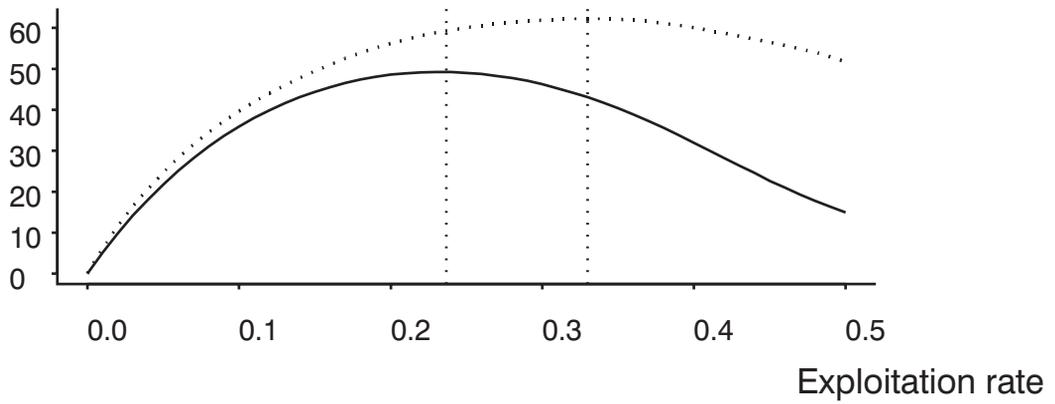
Bycatch of fish of ages 8 and older was not deducted from the simulated CEY, so computed average yields correspond to total removals including legal-sized bycatch mortality.

Performance of Alternative Harvest Rates

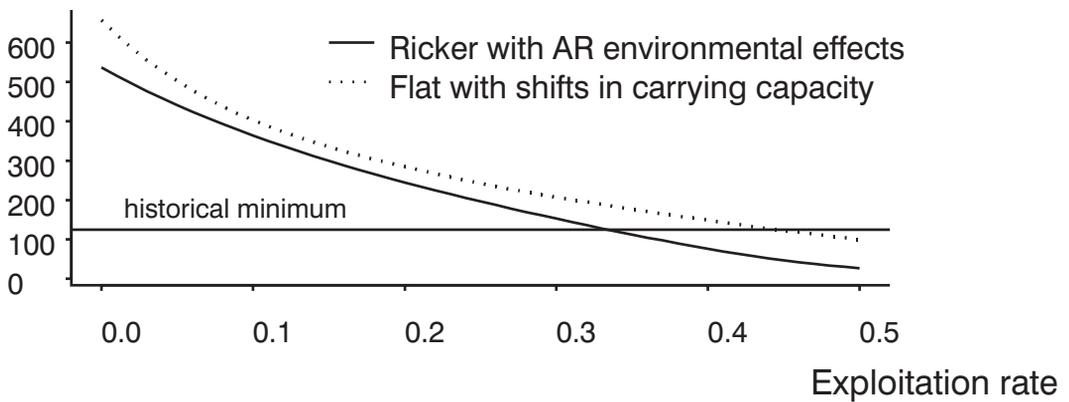
While the exploitation rate that resulted in maximum long-term yield differed for the two models considered (0.23 compared to 0.33, Table 13), a range of harvest rates between 0.20 and 0.30 resulted in average yields that were within 10% of the respective maxima in both cases (Figure 13, Table 13). Even under a very optimistic scenario in which recruitment carrying capacity would remain at the high level indicated by the last 11 estimates (i.e. a flat stock-recruitment model with carrying capacity equal to K_2), yields produced under a 0.25 harvest rate were only 17% lower than the maximum yield obtained with a harvest rate close to 0.50.

Long-term average reproductive biomass for harvest rates between 0.20 and 0.25 ranged between 200 and 280 million pounds, well above the historical minimum of around 125 million pounds attained in the mid 1930s and again in the mid 1970s (Figure 13). Minimum levels of reproductive biomass attained in the simulated trajectories depend on the stock-recruitment model. Under the Ricker model with autocorrelated environmental effects, the probability that the stock dropped below the historical minimum over the first 20 years of simulation was small (less than 10%) for harvest rates of 0.20 and lower, and it increased substantially when the harvest rate was raised above 0.25 (Figure 13, bottom panel). The probability of dropping below the historical minimum increased more slowly for harvest rates exceeding 0.30 under the flat model as carrying capacity could remain high for several years before switching back to the low level. If, instead, high recruitment levels were assumed for the first 20 years of simulation, this probability was close to zero. Results

a. Average long-term yield



b. Average reproductive biomass



c. Probability that biomass < historical minimum over the next 20 yrs

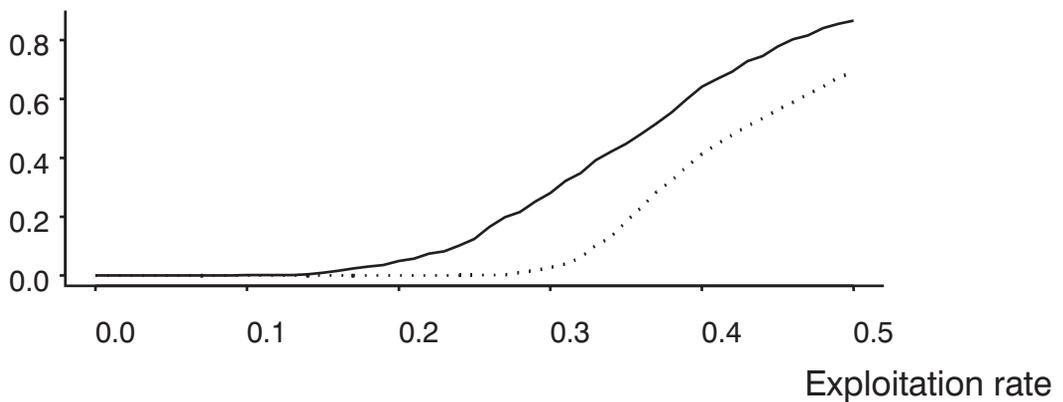


Figure 13. Performance of different harvest rates under two stock-recruitment models. (Biomass in million pounds.)

Table 13. Trade-offs between long-term average yield and the probability that the reproductive biomass drops below the historical minimum over the first 20 years of simulation under different recruitment scenarios. Two stock-recruitment models were used to simulate future trends in abundance: (1) Ricker model with gradual changes in environmental effects, and (2) a flat model with abrupt shifts in carrying capacity. Harvest rates that maximized expected yield are marked with an asterisk. Biomass units are million pounds. Bycatch of legal size fish was not discounted from the catches, so average yields correspond to total removals. Area 4 is not included in the projections.

Model	Harvest rate	Long-term average yield	Long-term average biomass	Prob [biomass < 125 million lbs] over next 20 years
Dome-shaped with gradual changes in environmental effects	0.23*	49.3	215	0.08
	0.20	48.6	245	0.05
	0.25	49.0	196	0.12
	0.30	46.2	153	0.28
	0.35	40.4	112	0.45
Flat with shifts in carrying capacity	0.33*	62.3	188	0.10
	0.20	56.2	284	0.00
	0.25	60.6	242	0.00
	0.30	61.9	206	0.03
	0.35	62.1	177	0.18

of these simulations indicate that, although it is impossible to predict future yields as these will depend on future environmental conditions, harvest rates ranging from 0.20 to 0.25 may achieve close-to-maximum yields under different hypotheses about future stock productivity, while having a high probability that the stock level stays within the range of historical abundance.

Estimated risks are probably optimistic as they are based on assuming that successive errors in the estimates of stock abundance are independent from year to year. Because of the time-series nature of the methods used in the assessment, successive errors are likely to be highly correlated. However, the most serious and persistent errors in fisheries stock assessment in general tend to result from severe misspecification of the assessment model, a problem that is not considered here. Further evaluation of the performance of the new assessment model, involving simulations, needs to be conducted to better characterize the uncertainty associated with the biomass estimates.

DISCUSSION

Uncertainty in fish stock assessments results mainly from three sources: errors and variability in the input data (measurement errors), variability in the population dynamic processes (process error), and errors in the specification of the assessment model (structural uncertainty). Statistical catch-at-age models such as CAGEAN have generally prioritized the first source of errors, while making rather rigid assumptions

about the population dynamics, and generally ignoring process error altogether (beyond, of course, variability in recruitment). Some of the assumptions made by CAGEAN, specifically the assumptions of constant selectivity and constant catchability, resulted in substantial bias of Pacific halibut estimates of biomass. Poor performance manifested as autocorrelated trends in residuals and strong retrospective patterns (Clark et al. 1999). This, together with new evidence from the surveys about major decreases in size at age, prompted a revision of the assessment methodology.

In the new model, we allow for trends in growth, selectivity and catchability by representing them as stochastic processes. Modeling time-varying components as random time series is relatively new in fisheries models. Random walk models with constrained variation have been suggested by Gudmundsson (1994) and Fournier et al. (1998). In contrast, in other fisheries models such as Stock Synthesis (Methot 1990) trends in selectivity or catchability are modeled as parametric functions of time. We prefer to use time-series models because they are more flexible, and the influence of the data on determining time trends is locally restricted. The approach is Bayesian, in the sense that catchability, selectivity, and some of the growth parameters are treated as random variables, the time-series models having the role of prior distributions. Measurement errors are thus represented by the likelihood components of the objective function, while process errors correspond to the penalties. The effect of the different components on the resulting *posterior* distribution depends to some extent on their respective variances, which are specified *a priori*. The choice of variances for both the process and the measurement error components is *ad hoc* and somewhat arbitrary, but represents our best *a priori* understanding of the system. The sensitivity of the results to some of those choices, particularly the ones that are more difficult to rationalize (like the weight assigned to the commercial effort data) needs to be regularly examined.

There is one more subtle technical aspect that deserves consideration. In other state-space models that allow for both stochasticity in the dynamics and measurement errors (see for example Mendelsohn 1988, Sullivan 1992, Schnute 1994), the parameters corresponding to random process error (like all the ε parameters in our model) are integrated out using for example the Kalman filter. The remaining fundamental parameters can then be estimated by maximizing the likelihood in the usual sense. We, instead, maximize the *penalized* likelihood which, in the Bayesian paradigm, corresponds to maximizing the joint posterior distribution for all model parameters (including the ε parameters), that is locating the joint mode of that posterior distribution. A problem with these so called MAP (maximum *a posteriori*) estimators is that they are not *equivariant* to 1-1 transformations (Ripley 1996, pg. 334), in the sense that maximum-likelihood estimators (MLE) are. This means that if, say, $\hat{\theta}$ is the MLE of θ , then $f(\hat{\theta})$ is the MLE of $f(\theta)$. This does not hold for a MAP estimator because when parameters are transformed, the prior density needs to be transformed as well using the *change of variable formula* (Bickel and Doksum 1977, p. 448). For nonlinear transformations, this would change the form of the objective function and likely the location of the maximum, except when the likelihood is very concentrated. As a consequence, if $\tilde{\theta}$ denotes a vector of MAP estimates, the use of $f(\tilde{\theta})$ to estimate derived parameters of management interest (such as current exploitable biomass) is ambiguous. Ideally, we want to describe all parameters of interest using their actual marginal posterior density, which involves integrating the joint posterior density of θ using methods such as Sampling-Importance-Resampling

(SIR) or Markov Chain Monte Carlo (MCMC) (Punt and Hilborn 1997). AD Model Builder includes an efficient implementation of MCMC, which seems to work well for the halibut assessment model, although it takes two to three days to converge after about one million MCMC steps.

The use of SIR or MCMC also has advantages for evaluating alternative harvesting strategies. These methods readily provide samples from the joint posterior distribution of all fundamental and derived model parameters (e.g. abundance at age in year $T + 1$) needed to simulate future stock trajectories under different policies. In addition, if parameters of the stock-recruitment models are estimated jointly with the rest, their uncertainty and correlation with other model parameters are directly carried into the policy evaluation.

These sophisticated approaches may still leave out some of the main sources of uncertainty, as they depend on the structure of the model being correct. Perhaps the most difficult and potentially most consequential issue in the assessment of Pacific halibut and fish stocks in general is that of model misspecification. This may result from poor choice of input parameter values, or from any structural assumption being wrong. Retrospective analysis, which was a powerful method of diagnosing structural problems in the old assessment model, has improved considerably with the new models. However, some important structural uncertainties still remain. One is the choice of a working value for the coefficient of natural mortality. For the 1997 assessment discussed here, we assume a constant rate of natural mortality of 0.2 over all ages and years. Natural mortality is confounded with fishing mortality in age- and size-structured models, and so commonly fixed at an assumed value rather than estimated internally. The value of 0.2 has historically been used in Pacific halibut assessments, and justification for it can be found in the catch-curve analysis of Robson and Chapman (1961) and in Chapman et al. (1962). More recent work by Clark (1999) on the effect of assumed values of natural mortality in assessments and on choice of harvest rate for Pacific halibut suggests that values towards the lower end of the range of estimates are safer for the stock without much yield being forfeited. A value of $M=0.15$ has been used since 1998.

After the value of M , the next main source of uncertainty in our estimates comes in the modeling of survey selectivity. We cannot attempt to estimate simultaneous trends in both growth and commercial selectivity without assuming that survey selectivity is either constant at size or constant at age. We have postulated those two alternative hypotheses, which in fact represent extremes in how the process might be modeled. In reality, size and age-dependent effects are both likely, but would be difficult to separate with the data at hand. It is not clear how many more years of data will be required to discern which of the two effects is most important, or whether an intermediate, more realistic model that allows for both size and age effects is possible. The latter conceivably could be developed using tagging data to explore age and size effects on fish availability on the grounds, and information on hooking efficiency to determine size-dependent effects on vulnerability. For example, direct underwater observations of hooking behavior (Kaimmer 1999), albeit limited, show that hooking success increases with fish size. Results of this type could be used as inputs to constrain model parameters, such as was done with the experiments on the relative fishing power of circle and J hooks. The latter were very important to interpret correctly observed trends not only in catch rates but also in size at age. The switch to circle-hooks took place at a time when the apparent decrease in size at age

was most dramatic. The experimental results show a shift in hook selectivity toward smaller fish sizes, which must account for some of the observed decline.

Exploitable biomass has been used traditionally to represent estimated trends in stock abundance. This was the index of choice because it represents the portion of the stock available to fishing and it could serve as a rough proxy for spawning biomass. One reason it worked was because selectivity was assumed to remain constant with time, thus providing a consistent statistic across years for comparison. It now seems clear that selectivity at age changes with time, at least in the commercial fishery. This makes exploitable biomass difficult to use to track stock trends even though to some extent it still represents the stock available for fishing.

Changes in selectivity also have implications on the evaluation and actual implementation of fixed-harvest rate policies. Stock projections used to select a target harvest rate have been made assuming a fixed selectivity schedule derived from current selectivity estimates. Estimated selectivities, however, will likely change in the future in response to real trends in the fishery or, if nothing else, because assessment models will continue to evolve. This is further complicated by regional differences in selectivities, where it appears that fish become selected when they are younger in the southern regions than in the central Gulf. In an attempt to equalize exploitation rates across areas, a common fixed-selectivity schedule is used to determine allowable removals in all areas. This is, indeed, the same schedule used in the analysis that justifies the choice of the target harvest rate. While this is internally consistent, it entails working with two definitions of exploitable biomass, one based on a fixed schedule and the other based on time- and region-specific selectivities. Policies that attempt to equalize the reproductive contribution per recruit across areas (i.e. constant egg-per-recruit policies), instead of equalizing the harvest rate, may be a reasonable alternative to deal with these problems. Such policies may be more robust not only to changes in selectivity but also to changes in weight at age, as they would automatically adjust the harvest fraction in a compensatory way.

A final problem of either constant harvest rate or constant egg-per recruit policies is that they translate the year-to-year variability in the assessment directly into variability in the quotas. This is especially problematic when variability between successive assessments is high and mostly a result of changes in assessment methodology, as has been the case of the last few halibut assessments. Alternatives to fixed-harvest rates aimed at increasing the stability of quotas are now being considered by the staff.

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APPENDIX 1. RELATIVE FISHING POWER OF CIRCLE-HOOKS AND J-HOOKS

In the early 1980s both the commercial halibut fishery and the IPHC setline surveys changed from the traditional J-shaped hook (“J-hook”) to a circle-shaped hook (“circle-hook”), because catch rates were substantially higher with circle-hooks. In 1984 the Commission surveyed Areas 2B and 3A twice, fishing the same stations with both hook types. In the aggregate, circle-hooks caught 2.2 times as much in weight of legal sized halibut as J-hooks. This factor has been used ever since to convert commercial and survey J-hook catch rates in early data to the present circle-hook standard. For example, commercial J-hook catch per effort in the late 1970s is multiplied by 2.2 to make it comparable with present circle-hook data, and survey catch rates likewise.

This conversion factor clearly has a large effect on estimates of trends in relative abundance. Its importance was recognized by the scientists who conducted the peer review of the IPHC stock assessment in September 1997. As a result of their questions, the staff realized that we should have calculated a different conversion factor when we began to fit the assessment model to survey catch rates in number rather than in weight. In the 1984 surveys, circle-hooks caught 2.4 times as much as J-hooks in number of legal sized halibut, and 2.6 times as much in total numbers. After the review, the staff re-analyzed the 1984 data to develop a more appropriate conversion. Results are summarized below.

THE 1984 SURVEY DATA

In 1984 a total of 174 fixed survey stations in Area 2B (northern British Columbia) and Area 3A (Kodiak) were fished twice, once with J-hooks and once with circle-hooks. In fact a slightly larger number of stations were fished with both hook types, but at a few stations the number of hooks fished differed between hook types. There were some other stations that were fished only with J-hooks or only with circle-hooks in 1984, so the data have to be compiled carefully to allow for a straightforward comparison of catch rates.

COMPARISON OF LENGTH FREQUENCIES

In both Area 2B and Area 3A, circle-hooks caught a significantly higher proportion of small fish than J-hooks (Figure A1.1). This indicates that the relative fishing power of circle-hooks is not the same for all sizes of fish. But the differences in length frequencies between the two hook types are in the same direction and about the same size in Area 2B and Area 3A, which means that the data can be pooled for purposes of analysis and a single conversion procedure can be used for all areas.

RELATIVE FISHING POWER AS A FUNCTION OF LENGTH

The absolute length frequencies of halibut caught at the same stations show clearly that circle-hooks were far more effective than J-hooks for all sizes of fish (Figure A1.2). The ratio of circle-hook to J-hook catches, smoothed by running a data smoother through the length frequencies, suggests that the relative power of

circle-hooks increases from about 2 among the smallest fish in the catch (50-60 cm) to over 3 among fish near legal size (60-90 cm) and then decreases gradually back to about 2 among the largest fish (over 120 cm; Figure A1.2a). These are ratios of catches in number, so the estimates of relative power refer to catch in number rather than weight.

These estimates of relative fishing power are of course subject to sampling error. The variance of the sample ratios was estimated by bootstrapping the survey data. Each bootstrap trial consisted of drawing a sample of 174 stations with replacement from the 174 stations in the dataset and calculating the relative fishing power of circle-hooks relative to J-hooks for each 10-cm length interval (Figure A1.3). The variance of the bootstrap sample ratios is an estimate of the variance of the point estimate calculated directly from the whole dataset. Both the point estimates (not the bootstrap means, although they are almost the same) and the bootstrap standard deviations are shown in Table A1.1. The ratios are quite variable for the smallest and largest fish, but for intermediate sizes the standard deviation is about 0.2, which indicates (as did the Kolmogorov-Smirnov test in Figure A1.1) that the decrease from over 3 among near-legal-sized fish to about 2 among large fish is significant.

DISCUSSION

The relative fishing power of circle-hooks and J-hooks is not a simple matter. Evidently it depends on size, the superiority of circle-hooks being greatest among fish near the legal size limit (81 cm) and somewhat less among the smallest and largest fish. The reasons for this variation are not known, but presumably result from the different ways in which fish of different sizes are hooked by the two hook types. It is known, for example, that circle-hooks almost always hook fish around the mouth, while J-hooks are sometimes swallowed. J-hooks often snag large fish; circle-hooks do not.

Fortunately, the differences between circle-hooks and J-hooks were the same in Area 2B and Area 3A in 1984, and presumably are the same now as well. There has been a large decline in size at age in Area 3A since 1984, with the result that fish in Area 3A are now about the same size at age as fish in 2B. It seems reasonable to suppose that the effect of hook type has not been influenced by the change in growth since 1984 because it was not influenced by the difference in growth in 1984.

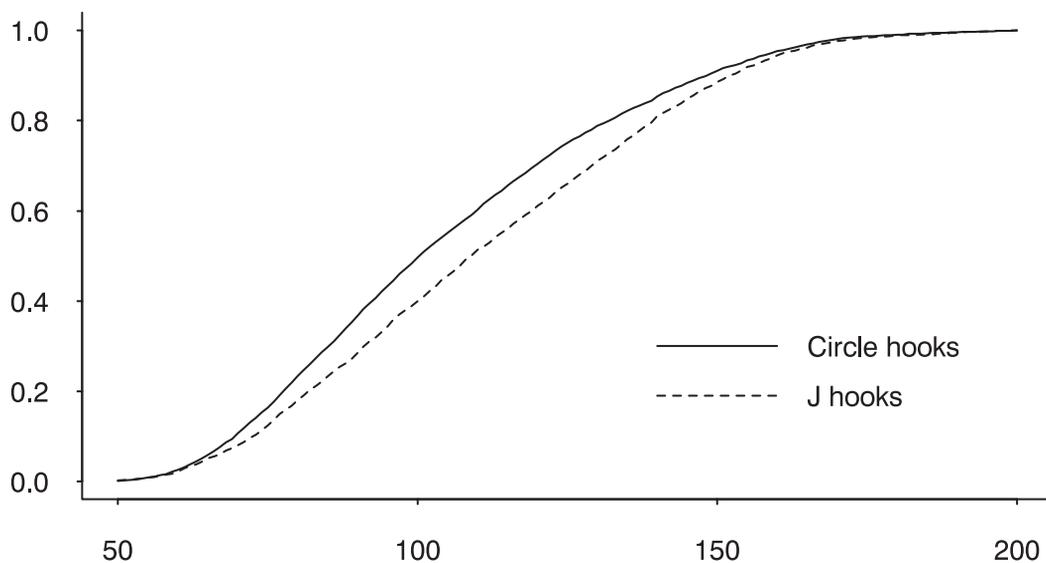


Figure A1.1a. Cumulative length frequencies of catches in Area 2B in 1984. Kolmogorov-Smirnov statistic = 0.085; 95% point =0.055.

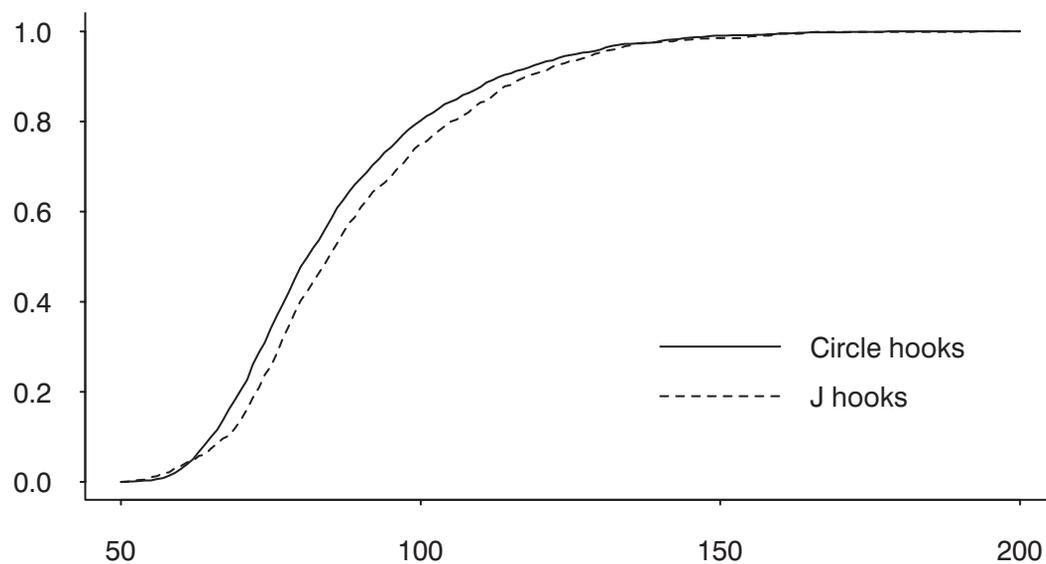


Figure A1.1b. Cumulative length frequencies of catches in Area 3A in 1984. Kolmogorov-Smirnov statistic = 0.100; 95% point =0.027.

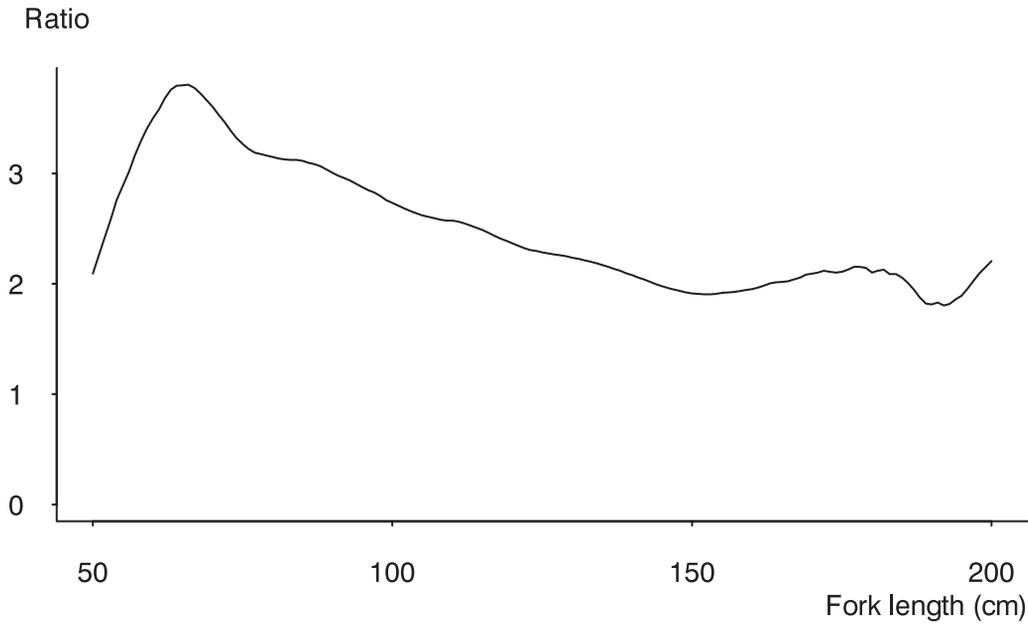


Figure A1.2a. Fishing power of circle hook relative to J hooks as a function of length. (Ratio of smoothed length frequencies in Figure A1.2a.)

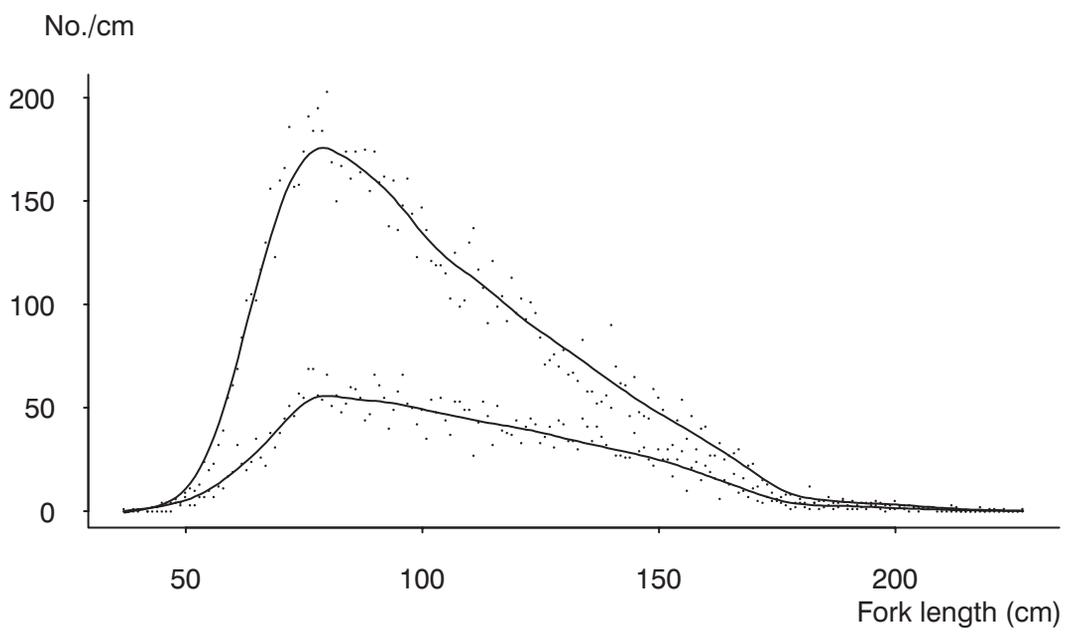


Figure A1.2b. Length frequencies of circle hook (upper) and J-hook (lower) catches at the same stations in 1984. (Lines are data smoothers.)

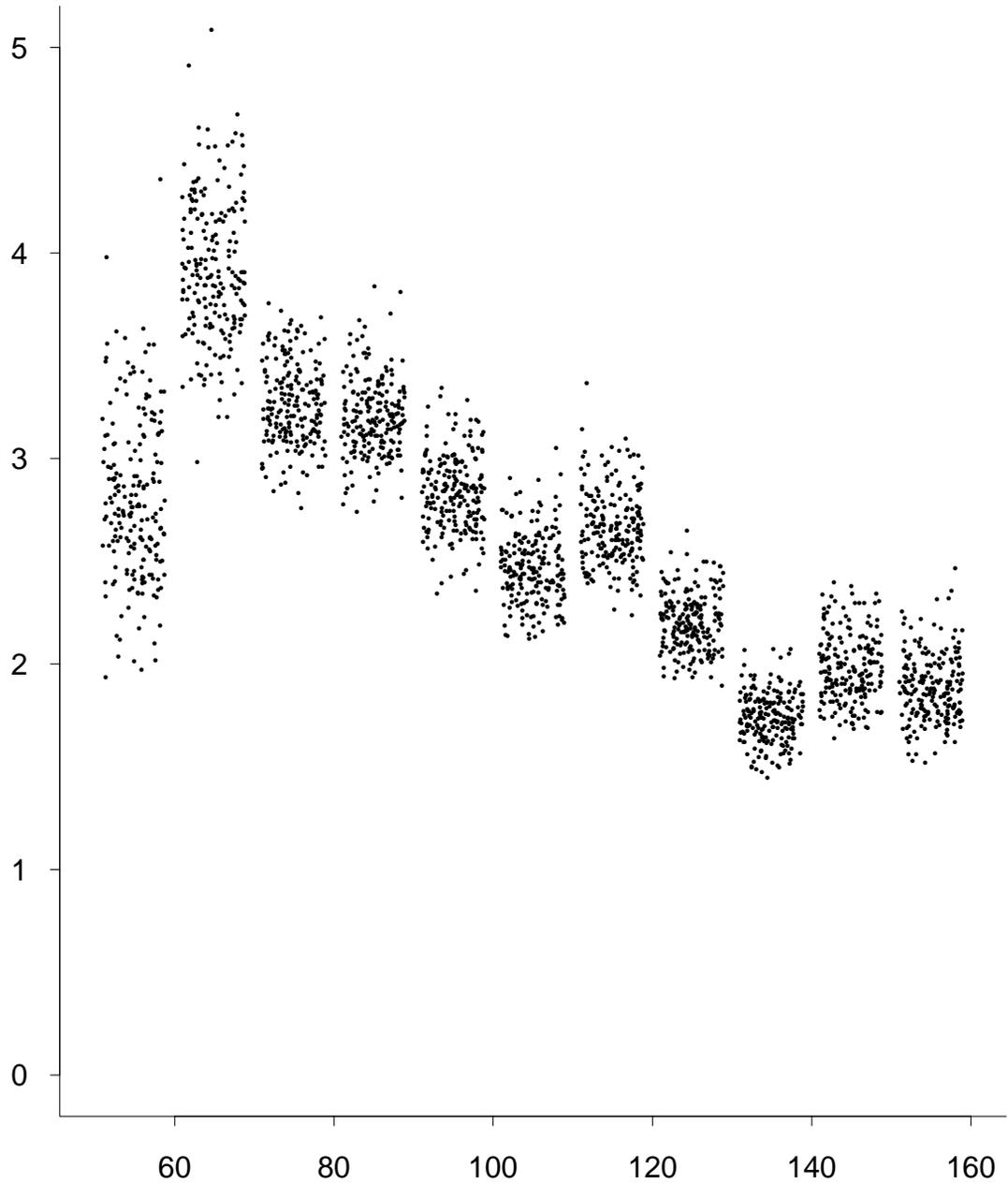


Figure A1.3. Distribution of 200 bootstrap estimates of circle hook relative to J-hook fishing power by 10 cm length interval. (Each estimate calculated by sampling the 174 stations iwth replacement; points jittered.)

Table A1.1. Point estimates and bootstrap standard deviations of the fishing power of circle-hooks relative to J-hooks, by 10-cm length interval.

Length interval (cm)	Point estimate	Standard deviation
50-60	2.81	0.39
60-70	3.92	0.35
70-80	3.27	0.22
80-90	3.18	0.19
90-100	2.83	0.18
100-110	2.47	0.17
110-120	2.67	0.19
120-130	2.21	0.14
130-140	1.75	0.14
140-150	2.00	0.17
150-160	1.92	0.16
160-170	2.10	0.20
170-180	2.19	0.39

APPENDIX 2. NOTATION

Prescripts:

b	Bycatch
c	Commercial
s	Survey

Subscripts:

a	Age
t	Time
l	Length category

Superscripts:

obs	Observation variable
+	Time immediately after fishing

Catch and Abundance:

a, b	Parameters of stock-recruitment functions used in forward simulations
${}^c C_{a,t}$	Commercial catch at age and time
${}^c C_{a,t}^{\text{obs}}$	Observed commercial catch at age and time
${}^s P_{a,t}$	Age composition in survey catches at time t
${}^s P_{a,t}^{\text{obs}}$	Observed age composition in survey catches at time t
${}^b C_{l,t}^{\text{obs}}$	Observed bycatch at size category and time
${}^s CPUE_t$	CPUE at time for survey
${}^s CPUE_t^{\text{obs}}$	Observed CPUE at time for survey
${}^c E_t$	Fishing effort at time for commercial fishery
${}^c E_t^{\text{obs}}$	Observed fishing effort at time for commercial fishery
F_t	Instantaneous fishing mortality at time
${}^b H_{a,t}$	Finite rate of bycatch mortality at age and time
${}^b H_{l,t}$	Finite rate of bycatch mortality at size category and time
${}^c H_t$	Harvest fraction of fully selected fish = $1 - \exp(-F_t)$
K_t	Carrying capacity for the flat stock-recruitment model
$N_{a,t}$	Population numbers at age and time
M	Instantaneous natural mortality
R_t	Recruitment of eight-year-olds in Areas 2 and 3
${}^c Q_t$	Catchability for commercial fishery
${}^s Q$	Catchability for survey
${}^c s_t(X)$	Selectivity of fish of log-length X in the commercial fishery in year t
${}^s s_t(X)$	Selectivity of fish of log-length X in the survey in year t
${}^c S_{a,t}$	Selectivity of fish of age a in the commercial fishery in year t
${}^s S_{a,t}$	Selectivity of fish of age a in the survey in year t
SB_t	Reproductive biomass in year t
V_t	Variance-like parameter of size-based selectivity function $s_t(X)$
X_t^{full}	Log-length beyond which fish are fully-selected

Size and Growth:

α_t	Intercept of the recursive growth equation
β	Slope of the recursive growth equation
c	Intercept of standard deviation relative to mean log-length at age
d	Slope of standard deviation relative to mean log-length at age
$f_{l a,t}$	Distribution of size (by l categories) at age a in year t
$f_{all,t}$	Distribution of age for fish in size category l in year t
$m_{a,t}$	Median length at age ($= \exp(\mu_{a,t})$) in the population in year t
$m^+_{a,t}$	Median length of fish of age a surviving the fishing season in year t
$\mu_{a,t}$	Mean log-length at age in the population in year t
$\mu^+_{a,t}$	Mean log-length of fish of age a surviving the fishing season in year t
${}_c\mu_{a,t}$	Mean log-length at age in the commercial catch in year t
${}_s\mu_{a,t}$	Mean log-length at age in the survey catch in year t
${}_c\mu_{a,t}^{\text{obs}}$	Observed mean log-length at age and time in the commercial catch
${}_s\mu_{a,t}^{\text{obs}}$	Observed mean log-length at age and time in the survey catch
$\phi(X \mu, \sigma^2)$	Gaussian probability density function with mean μ and variance σ^2
$\sigma^2_{a,t}$	Variance of log-length at age in the population in year t
${}_c\sigma^2_{a,t}$	Variance of log-length at age in the commercial catch in year t
${}_s\sigma^2_{a,t}$	Variance of log-length at age in the survey catch in year t
${}_c\sigma^2_{a,t}^{\text{obs}}$	Observed variance of log-length at age and time in the commercial catch
${}_s\sigma^2_{a,t}^{\text{obs}}$	Observed variance of log-length at age and time in the survey catch
X	Log-length
Y_t	Simulated catch in biomass

Weighting factors and variances of random components:

$\lambda_{c:c}$	Weight for commercial log-catch-at-age residuals
$\lambda_{e:c}$	Weight for commercial effort residuals
$\lambda_{c:\mu}$	Weight for commercial mean log-length residuals
$\lambda_{c:\sigma}$	Weight for commercial variance of log-length residuals
$\lambda_{s:c}$	Weight for survey log-catch-at-age residuals
$\lambda_{s:e}$	Weight for survey CPUE residuals
$\lambda_{s:\mu}$	Weight for survey mean log-length residuals
$\lambda_{s:\sigma}$	Weight for survey variance of log-length residuals
$\lambda_{c:j}$	Weight for C-J catch ratios
${}_{\mu}\sigma^2$	Variance of ${}_{\mu}\epsilon_t$, the time-series deviations affecting $\mu_{6,t}$
${}_q\sigma^2$	Variance of ${}_q\epsilon_t$, the time-series deviations of log-catchability
${}_{\text{sel}}\sigma_a^2$	Variance of selectivity deviations ${}_{\text{sel}}\epsilon_{a,t}$
${}_v\sigma^2$	Variance of ${}_v\epsilon_t$, the time-series deviations affecting selectivity parameter v_t
${}_r\sigma^2$	Variance of ${}_r\epsilon_t$, the recruitment deviations
ρ	Autocorrelation of recruitment deviations
η_t	Simulated assessment error used in policy evaluation

APPENDIX 3. INPUT DATA FOR AREA 3A.

Table A3.1 Smoothed weight at age by year in the commercial catch (Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	13.4	16.1	19.7	23.7	31.2	41.1	48.9	58.6	70.5	78.4	83.5	89.0	101.7	121.0	149.7
1975	13.5	15.9	19.0	23.8	31.5	40.8	48.2	58.5	69.3	79.5	82.2	91.1	107.8	120.6	148.3
1976	13.5	15.7	18.6	23.8	31.7	40.5	47.6	58.4	68.2	80.0	81.0	92.2	111.8	120.2	146.0
1977	13.6	15.7	18.3	23.9	31.7	40.2	47.3	58.1	67.3	79.9	79.8	92.3	113.9	119.5	142.8
1978	13.6	15.7	18.2	24.0	31.6	39.9	47.1	57.8	66.5	79.1	78.7	91.5	114.3	118.8	139.4
1979	13.5	15.8	18.3	24.2	31.3	39.6	47.0	57.5	65.8	77.6	77.5	89.6	112.6	117.9	134.6
1980	13.3	16.0	18.7	24.3	30.9	39.3	47.3	57.6	66.3	75.0	77.8	87.7	109.4	118.4	132.3
1981	13.2	16.3	19.1	24.4	30.4	38.8	47.6	57.5	66.6	72.0	78.2	85.5	105.1	118.9	131.0
1982	13.0	16.3	19.4	24.2	30.0	38.1	47.8	57.0	66.1	68.5	78.2	83.3	99.7	117.8	131.6
1983	13.0	16.1	19.5	23.8	29.6	37.5	47.5	55.8	64.3	65.7	76.9	80.9	94.1	117.4	132.0
1984	13.1	15.9	19.5	23.2	29.2	36.8	46.8	54.4	61.9	63.3	75.2	78.1	88.4	116.3	131.8
1985	13.2	15.5	19.0	22.6	28.8	36.3	45.2	52.3	58.7	61.9	72.2	75.0	83.1	112.7	128.5
1986	13.1	15.1	18.3	22.0	28.1	35.6	43.2	50.3	56.3	61.3	69.3	71.3	78.6	106.2	124.9
1987	13.0	14.7	17.6	21.3	27.2	34.6	40.9	48.0	53.9	60.6	66.2	67.4	74.8	98.5	119.8
1988	12.9	14.5	16.9	20.5	26.0	32.8	38.6	45.6	51.9	59.0	64.3	64.3	72.4	89.3	113.2
1989	12.8	14.5	16.6	19.8	24.6	30.6	36.2	42.7	49.5	56.3	61.6	61.3	70.6	81.4	103.0
1990	12.8	14.4	16.4	19.1	23.3	28.2	33.9	39.7	46.7	52.9	58.4	58.1	68.6	73.5	91.7
1991	12.8	14.4	16.4	18.5	22.0	25.9	31.4	35.8	42.6	47.9	53.1	53.7	65.1	66.4	79.3
1992	13.0	14.4	16.3	17.9	21.1	24.2	29.0	32.8	38.4	43.1	47.8	48.8	60.2	59.9	69.8
1993	13.1	14.4	16.2	17.5	20.4	23.1	27.1	30.4	34.7	38.7	43.0	44.2	55.3	54.1	61.5
1994	13.0	14.6	15.9	17.4	20.2	22.7	26.2	29.5	32.6	35.8	39.7	40.8	51.1	48.9	55.5
1995	12.9	14.9	15.7	17.5	20.2	23.0	26.0	29.5	31.5	33.8	37.6	38.0	47.3	44.3	51.3
1996	12.7	15.3	15.3	17.8	20.5	23.8	26.7	30.5	31.4	32.9	36.5	36.1	44.0	40.4	48.7
1997	12.4	15.9	14.9	18.2	21.1	25.3	28.2	32.5	32.4	33.2	36.7	35.0	41.4	37.4	47.8

Table A3.2. Total directed removals at age by year in numbers (thousands, Area 3A).

1973	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	0.7	3.8	12.1	14.7	22.5	25.2	21.4	27.9	9.9	8.3	9.7	4.2	2.2	1.7	1.0
1975	0.5	7.8	20.1	30.7	28.1	33.3	31.9	20.7	24.5	9.7	8.1	6.6	2.4	0.9	2.3
1976	0.9	4.6	24.3	32.9	29.7	26.1	28.2	21.6	20.0	16.4	5.2	4.7	3.7	1.7	2.8
1977	0.6	5.3	13.8	27.7	30.2	21.6	22.5	21.5	15.8	13.4	10.6	4.4	3.4	1.8	2.7
1978	3.3	10.0	30.8	32.3	40.9	34.4	28.6	19.1	17.1	11.8	9.7	4.5	2.2	0.8	2.1
1979	3.4	17.2	27.1	42.0	47.4	38.6	36.2	19.4	19.2	10.6	8.4	4.0	2.4	0.8	1.7
1980	1.8	9.1	26.5	30.4	48.2	43.8	39.3	27.0	21.8	13.0	9.9	5.8	4.8	3.1	3.6
1981	3.9	9.7	26.3	46.4	42.9	57.5	45.8	37.2	23.3	14.9	9.9	7.9	4.9	1.7	6.2
1982	1.4	6.9	13.8	29.5	37.9	39.2	47.0	35.7	27.5	15.9	8.4	9.1	3.4	2.7	4.6
1983	2.0	7.0	27.0	40.9	64.6	48.4	52.5	34.6	20.9	16.4	12.9	6.8	3.9	2.5	4.4
1984	2.2	24.5	47.7	63.8	74.1	104.1	69.9	43.4	34.2	21.5	10.5	9.1	4.3	2.2	5.1
1985	3.1	11.1	49.2	79.0	89.2	93.2	96.4	51.6	37.5	28.4	20.9	6.4	5.9	3.5	8.3
1986	0.9	22.0	58.4	128.1	146.4	152.9	145.7	112.2	62.1	40.2	30.7	11.4	10.8	5.4	7.4
1987	1.4	29.8	79.2	116.0	175.1	159.4	126.9	95.1	72.8	37.5	23.1	11.6	10.1	4.6	8.3
1988	4.2	41.2	138.6	200.0	177.2	239.7	137.2	81.6	81.7	58.3	24.9	16.8	9.1	5.8	9.0
1989	4.4	16.2	79.9	198.2	190.1	150.8	186.8	97.9	72.1	48.5	35.3	15.8	12.5	4.7	9.2
1990	3.4	17.2	38.1	98.0	221.8	158.7	140.8	142.2	69.4	43.2	41.8	20.8	7.8	4.0	7.5
1991	1.7	21.3	61.8	50.2	106.8	179.3	157.7	124.2	102.2	58.6	29.0	19.2	11.9	4.2	9.1
1992	0.4	6.9	44.7	121.3	90.8	164.4	187.8	118.2	89.3	79.3	40.8	21.9	17.7	7.7	7.7
1993	0.4	3.9	24.4	62.2	131.1	111.7	120.5	154.8	123.7	88.4	61.1	40.6	19.1	8.1	13.4
1994	1.7	7.8	13.1	29.2	80.5	155.4	156.7	157.6	168.5	125.8	86.2	50.5	30.5	11.8	17.0
1995	0.8	7.1	30.6	32.7	63.0	110.9	149.9	109.3	105.9	76.1	45.8	25.9	20.7	9.4	22.8
1996	1.5	2.7	23.0	68.8	63.3	91.8	141.2	150.9	103.0	66.7	62.7	36.4	21.5	10.9	22.1
1997	1.1	6.1	17.7	62.5	106.8	96.3	145.6	145.6	130.1	85.8	60.3	52.6	31.5	23.2	26.6

Table A3.3. Coefficient of variation of catch at age by year (Area 3A).

1973	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	0.15	0.09	0.04	0.05	0.04	0.03	0.05	0.04	0.05	0.07	0.06	0.11	0.11	0.15	0.23
1975	0.40	0.14	0.10	0.08	0.07	0.06	0.07	0.07	0.08	0.13	0.11	0.16	0.16	0.16	0.19
1976	0.22	0.13	0.06	0.05	0.05	0.08	0.06	0.05	0.06	0.06	0.13	0.13	0.14	0.19	0.18
1977	0.26	0.09	0.07	0.05	0.04	0.07	0.06	0.05	0.05	0.06	0.05	0.08	0.15	0.20	0.14
1978	0.18	0.11	0.04	0.04	0.04	0.05	0.04	0.08	0.07	0.08	0.08	0.18	0.20	0.37	0.17
1979	0.19	0.08	0.07	0.05	0.06	0.07	0.05	0.07	0.10	0.14	0.14	0.16	0.24	0.40	0.25
1980	0.20	0.07	0.06	0.05	0.04	0.03	0.04	0.05	0.04	0.08	0.09	0.12	0.11	0.13	0.11
1981	0.21	0.16	0.08	0.09	0.07	0.06	0.06	0.06	0.07	0.12	0.18	0.14	0.21	0.30	0.18
1982	0.24	0.12	0.08	0.05	0.03	0.04	0.04	0.04	0.06	0.08	0.13	0.08	0.14	0.14	0.15
1983	0.34	0.18	0.08	0.05	0.06	0.07	0.07	0.06	0.09	0.11	0.12	0.18	0.26	0.28	0.24
1984	0.31	0.10	0.08	0.07	0.06	0.05	0.05	0.08	0.09	0.08	0.15	0.12	0.22	0.27	0.18
1985	0.24	0.11	0.06	0.05	0.04	0.04	0.04	0.06	0.06	0.08	0.09	0.16	0.20	0.23	0.14
1986	0.38	0.11	0.07	0.04	0.05	0.04	0.04	0.04	0.07	0.09	0.08	0.17	0.15	0.24	0.15
1987	0.33	0.08	0.05	0.04	0.02	0.03	0.03	0.04	0.05	0.07	0.08	0.09	0.13	0.16	0.16
1988	0.19	0.08	0.05	0.04	0.04	0.03	0.05	0.06	0.05	0.08	0.09	0.14	0.20	0.21	0.21
1989	0.30	0.15	0.06	0.03	0.03	0.03	0.03	0.04	0.06	0.06	0.08	0.11	0.13	0.28	0.14
1990	0.31	0.21	0.12	0.07	0.03	0.04	0.05	0.06	0.07	0.10	0.09	0.17	0.22	0.34	0.20
1991	0.45	0.13	0.07	0.08	0.05	0.04	0.04	0.05	0.06	0.08	0.11	0.13	0.17	0.29	0.20
1992	1.00	0.24	0.09	0.05	0.06	0.04	0.04	0.05	0.06	0.07	0.10	0.13	0.15	0.22	0.22
1993	1.00	0.30	0.12	0.07	0.05	0.05	0.05	0.04	0.05	0.06	0.07	0.09	0.14	0.21	0.16
1994	0.50	0.24	0.18	0.12	0.07	0.05	0.05	0.05	0.05	0.06	0.07	0.09	0.12	0.19	0.16
1995	0.58	0.19	0.09	0.09	0.06	0.04	0.04	0.04	0.05	0.06	0.07	0.10	0.11	0.17	0.11
1996	0.45	0.33	0.11	0.06	0.07	0.05	0.04	0.04	0.05	0.06	0.07	0.09	0.12	0.17	0.12
1997	0.71	0.30	0.18	0.09	0.07	0.07	0.06	0.06	0.06	0.08	0.09	0.10	0.13	0.15	0.14

Table A3.4. Number of effective skates (Area 3A).

Year	Skates
1974	57520
1975	72958
1976	83986
1977	65659
1978	61572
1979	61931
1980	44765
1981	45685
1982	38183
1983	NA
1984	41967
1985	45085
1986	70583
1987	69279
1988	84785
1989	83757
1990	94593
1991	91736
1992	80111
1993	73373
1994	92386
1995	59161
1996	56039
1997	69358

Table A3.5. Mean log length (ln(cm)) at age in the commercial catch (Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	4.46	4.51	4.57	4.62	4.70	4.78	4.83	4.89	4.94	4.98	4.99	5.01	5.06	5.10	5.16
1975	4.45	4.51	4.56	4.62	4.68	4.77	4.83	4.87	4.94	4.97	4.99	5.03	5.08	5.14	5.20
1976	4.47	4.51	4.59	4.64	4.71	4.78	4.85	4.90	4.95	4.99	4.99	5.04	5.10	5.10	5.19
1977	4.47	4.48	4.54	4.59	4.67	4.75	4.81	4.85	4.92	4.93	4.96	4.99	5.02	5.03	5.16
1978	4.46	4.50	4.55	4.61	4.70	4.76	4.82	4.89	4.92	4.97	4.98	5.03	5.10	5.12	5.16
1979	4.46	4.51	4.53	4.62	4.70	4.78	4.82	4.90	4.92	4.98	5.01	5.01	5.09	5.13	5.12
1980	4.46	4.50	4.54	4.59	4.68	4.76	4.81	4.86	4.91	4.92	4.96	5.01	5.01	5.07	5.12
1981	4.46	4.52	4.57	4.63	4.69	4.75	4.81	4.86	4.91	4.96	4.96	4.97	5.08	5.10	5.15
1982	4.45	4.51	4.56	4.62	4.67	4.76	4.82	4.89	4.94	4.96	4.98	5.02	5.11	5.13	5.18
1983	4.45	4.52	4.55	4.62	4.68	4.74	4.82	4.87	4.92	4.89	4.97	4.94	5.03	5.09	5.14
1984	4.46	4.50	4.58	4.61	4.68	4.74	4.81	4.86	4.90	4.89	4.98	4.99	5.02	5.06	5.15
1985	4.46	4.49	4.56	4.59	4.66	4.74	4.77	4.83	4.86	4.90	4.91	4.93	4.98	5.11	5.08
1986	4.46	4.50	4.55	4.59	4.66	4.74	4.79	4.81	4.85	4.88	4.91	4.92	4.96	5.04	5.12
1987	4.47	4.49	4.53	4.59	4.65	4.71	4.77	4.82	4.84	4.89	4.92	4.95	4.99	5.11	5.11
1988	4.44	4.48	4.52	4.58	4.66	4.71	4.78	4.83	4.86	4.88	4.92	4.89	4.93	4.99	5.06
1989	4.44	4.47	4.51	4.56	4.63	4.69	4.73	4.80	4.85	4.90	4.89	4.89	4.93	4.93	5.10
1990	4.45	4.49	4.50	4.55	4.60	4.66	4.72	4.74	4.81	4.84	4.90	4.89	4.95	4.90	5.10
1991	4.46	4.49	4.52	4.57	4.58	4.61	4.67	4.70	4.75	4.82	4.83	4.85	4.89	4.95	4.96
1992	4.51	4.54	4.52	4.54	4.61	4.64	4.68	4.73	4.77	4.78	4.80	4.83	4.89	4.92	4.92
1993	4.39	4.48	4.50	4.53	4.56	4.60	4.64	4.67	4.72	4.73	4.78	4.77	4.85	4.83	4.84
1994	4.64	4.47	4.52	4.53	4.57	4.58	4.62	4.65	4.66	4.68	4.72	4.74	4.80	4.76	4.88
1995	4.45	4.49	4.48	4.53	4.58	4.62	4.64	4.68	4.68	4.70	4.74	4.72	4.78	4.79	4.84
1996	4.43	4.49	4.50	4.54	4.57	4.62	4.64	4.68	4.69	4.71	4.72	4.74	4.79	4.79	4.82
1997	4.45	4.51	4.49	4.55	4.58	4.63	4.67	4.70	4.70	4.71	4.75	4.73	4.80	4.73	4.80

Table A3.6. Standard error of mean log length at age in the commercial catch (hundredths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	1.25	0.84	0.67	0.76	0.74	0.72	0.75	0.63	0.98	1.05	0.96	1.44	1.82	2.01	2.55
1975	1.78	0.79	0.66	0.71	0.82	0.79	0.79	0.93	0.73	1.25	1.25	1.40	1.94	2.87	1.71
1976	1.08	0.85	0.66	0.66	0.80	0.90	0.80	0.88	0.83	0.87	1.59	1.55	1.34	2.33	1.45
1977	1.70	0.66	0.71	0.65	0.73	0.93	0.88	0.90	0.91	1.05	1.13	1.59	1.71	2.60	1.65
1978	1.00	0.87	0.63	0.73	0.77	0.88	0.96	1.24	1.26	1.53	1.62	2.37	2.85	5.58	3.03
1979	2.59	1.31	1.09	0.92	0.98	1.34	1.30	1.72	2.02	2.49	2.31	4.61	4.48	8.92	8.35
1980	1.83	1.03	0.77	0.73	0.70	0.74	0.96	1.41	1.28	2.21	1.74	2.61	3.01	2.99	3.33
1981	2.95	1.54	1.21	1.02	1.31	1.33	1.78	1.31	1.77	2.08	2.58	4.52	3.05	5.12	2.42
1982	1.91	1.31	1.06	0.86	0.78	0.89	0.82	0.97	1.19	1.46	2.19	1.91	2.86	4.05	2.23
1983	3.11	3.03	1.23	1.09	1.03	1.47	1.54	2.00	2.21	3.37	3.08	3.12	4.73	4.99	2.92
1984	4.28	1.26	1.15	0.92	0.98	0.99	1.03	1.65	1.57	2.59	2.69	4.09	3.43	6.28	4.58
1985	3.21	1.39	1.00	0.87	0.84	0.81	1.10	1.50	1.50	1.96	2.55	3.80	3.98	5.31	3.20
1986	9.40	1.71	0.91	0.83	0.75	0.79	0.84	0.99	1.41	1.66	2.06	2.95	4.12	3.88	4.55
1987	4.83	0.64	0.54	0.73	0.60	0.57	0.81	1.01	0.88	1.57	1.92	2.53	2.20	3.34	3.17
1988	2.75	0.99	0.54	0.60	0.70	0.71	0.91	1.16	1.21	1.32	2.12	2.84	4.58	5.44	4.71
1989	2.40	1.30	0.63	0.46	0.65	0.70	0.80	1.25	1.07	1.86	1.37	2.98	2.86	7.83	2.81
1990	2.85	1.61	1.53	0.91	0.74	0.93	1.04	0.86	1.39	2.57	2.69	2.64	7.28	7.70	7.04
1991	2.50	0.95	0.77	1.11	0.73	0.65	0.81	0.91	1.05	1.40	2.09	2.42	3.38	5.74	3.57
1992	NA	2.96	0.86	0.64	0.89	0.66	0.70	0.95	1.16	1.25	1.84	2.27	2.79	3.59	4.08
1993	NA	2.22	1.10	0.79	0.60	0.74	0.84	0.79	0.94	1.11	1.34	1.68	2.48	4.26	3.26
1994	11.40	1.74	1.54	1.18	0.83	0.66	0.76	0.79	0.79	0.94	1.13	1.44	2.11	3.09	2.56
1995	4.55	1.17	0.63	0.86	0.78	0.65	0.56	0.76	0.77	0.95	1.25	1.61	1.72	2.61	1.90
1996	0.89	4.58	0.82	0.64	0.78	0.74	0.62	0.66	0.82	1.13	1.09	1.58	2.03	2.65	1.84
1997	1.73	2.70	1.39	1.01	0.83	1.03	0.94	1.04	1.08	1.29	1.62	1.75	1.99	2.56	2.54

Table A3.7. Variance of log length at age in the commercial catch (hundredths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	0.39	0.77	1.19	1.56	2.08	2.17	2.08	2.18	2.30	2.35	2.34	2.15	1.93	1.93	1.76
1975	0.27	0.70	1.14	1.53	2.09	2.11	2.20	2.29	2.18	2.45	2.39	2.32	2.29	2.64	1.58
1976	0.36	0.73	1.41	1.71	2.15	2.50	2.31	2.41	2.13	2.37	2.73	2.73	1.73	2.65	1.66
1977	0.46	0.48	1.10	1.52	2.07	2.40	2.54	2.73	2.57	2.84	3.08	2.49	2.41	2.78	2.14
1978	0.35	0.69	1.14	1.62	2.24	2.51	2.42	2.73	2.61	2.55	2.47	2.36	1.87	2.49	2.21
1979	0.40	1.00	1.07	1.76	2.20	2.67	2.51	2.48	2.98	2.46	2.14	2.38	1.94	1.58	4.61
1980	0.31	0.66	1.18	1.41	2.06	2.26	2.56	2.38	2.46	3.29	2.42	2.57	2.63	2.13	2.14
1981	0.36	0.83	1.38	1.65	2.34	2.45	2.86	2.22	2.46	2.15	2.16	2.91	1.41	2.10	1.43
1982	0.21	0.72	1.26	1.77	1.84	2.39	2.40	2.62	2.50	2.67	2.50	2.65	1.65	2.87	2.11
1983	0.20	0.86	1.11	1.59	2.31	2.47	2.97	2.98	2.60	3.39	3.01	2.53	2.64	2.16	1.54
1984	0.26	0.75	1.53	1.56	2.17	2.68	2.52	2.97	2.63	2.68	2.24	2.86	2.00	2.92	3.16
1985	0.33	0.71	1.39	1.48	2.19	2.40	2.75	3.00	3.37	3.31	3.01	3.90	2.51	3.4	2.19
1986	0.22	0.68	1.10	1.54	2.02	2.33	2.67	2.96	3.30	2.95	2.94	2.46	2.69	2.02	2.78
1987	0.45	0.53	0.98	1.55	2.06	2.29	2.51	2.50	2.44	2.59	2.58	3.12	1.81	2.23	2.63
1988	0.22	0.48	0.93	1.32	1.89	2.39	2.54	2.69	2.74	2.64	3.02	3.04	2.71	3.67	2.54
1989	0.16	0.42	0.76	1.21	1.74	2.44	2.64	2.63	2.67	2.85	2.07	2.65	2.72	4.29	2.52
1990	0.18	0.47	0.99	1.26	1.58	2.09	2.65	2.49	2.24	2.62	2.56	2.17	3.73	3.22	3.77
1991	0.31	0.55	1.00	1.79	1.61	2.11	2.96	2.97	3.22	3.31	3.62	3.13	3.85	3.95	3.31
1992	NA	1.58	0.84	1.20	1.87	1.86	2.27	2.75	3.13	3.22	3.59	2.90	3.54	2.56	3.32
1993	NA	0.54	0.83	1.09	1.33	1.73	2.43	2.75	2.98	3.07	3.05	3.25	3.33	4.18	4.03
1994	5.20	0.54	0.69	0.94	1.26	1.54	2.06	2.28	2.42	2.41	2.49	2.40	3.05	2.57	2.55
1995	0.62	0.36	0.47	0.90	1.44	1.73	1.69	2.44	2.38	2.64	2.73	2.49	2.30	2.45	3.14
1996	0.04	1.89	0.51	0.87	1.25	1.56	1.78	2.10	2.29	2.79	2.42	3.01	2.92	2.53	2.47
1997	0.06	0.80	0.61	1.16	1.33	1.84	2.32	2.83	2.67	2.59	2.80	2.84	2.25	2.71	3.07

Table A3.8. Standard error of variance of log length at age in the commercial catch (thousandths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	2.35	1.43	1.94	3.67	3.87	4.49	5.93	5.45	4.26	6.11	5.99	11.55	0.75	5.11	3.01
1975	2.33	1.44	1.95	3.69	3.87	4.52	5.94	5.45	4.25	6.12	6.00	11.55	0.76	5.12	3.02
1976	2.35	1.44	1.95	3.68	3.87	4.50	5.94	5.47	4.24	6.11	5.98	11.55	0.76	5.11	3.02
1977	2.36	1.43	1.96	3.68	3.87	4.51	5.94	5.46	4.24	6.11	6.01	11.55	0.75	5.12	3.00
1978	2.36	1.45	1.95	3.68	3.88	4.52	5.95	5.46	4.23	6.12	5.98	11.56	0.77	5.10	3.01
1979	1.90	1.56	2.03	2.13	2.46	4.41	4.67	4.79	3.01	5.90	4.58	7.38	3.30	4.30	2.17
1980	1.90	1.57	2.03	2.13	2.45	4.41	4.66	4.78	3.00	5.92	4.60	7.38	3.29	4.28	2.18
1981	1.90	1.56	2.03	2.13	2.46	4.41	4.66	4.80	3.00	5.93	4.60	7.36	3.29	4.28	2.19
1982	1.89	1.56	2.02	2.12	2.45	4.40	4.66	4.79	3.00	5.90	4.60	7.39	3.30	4.28	2.19
1983	1.90	1.57	2.02	2.13	2.45	4.42	4.66	4.80	2.99	5.90	4.61	7.36	3.27	4.28	2.19
1984	1.91	1.55	2.02	2.12	2.45	4.40	4.66	4.78	3.00	5.92	4.59	7.38	3.28	4.29	2.18
1985	1.88	1.56	2.02	2.13	2.45	4.42	4.68	4.77	3.00	5.92	4.61	7.37	3.29	4.28	2.19
1986	1.88	1.56	2.02	2.13	2.46	4.40	4.67	4.80	3.00	5.93	4.59	7.38	3.28	4.28	2.20
1987	1.89	1.55	2.03	2.12	2.47	4.42	4.67	4.80	3.00	5.91	4.59	7.39	3.29	4.28	2.18
1988	1.87	1.57	2.01	2.11	2.46	4.42	4.67	4.79	3.01	5.91	4.59	7.39	3.28	4.29	2.18
1989	1.96	1.56	2.01	2.12	2.45	4.42	4.67	4.79	3.02	5.93	4.60	7.39	3.29	4.29	2.19
1990	1.87	1.55	2.02	2.12	2.45	4.41	4.66	4.78	3.00	5.92	4.61	7.38	3.28	4.28	2.19
1991	2.13	2.54	1.88	2.88	2.38	2.19	2.49	2.58	3.16	3.57	7.46	6.67	11.78	27.93	12.81
1992	NA	11.38	2.16	2.32	2.73	1.79	2.02	3.36	3.26	2.96	5.28	9.37	8.96	16.79	11.65
1993	NA	2.72	7.10	3.59	1.74	2.13	2.77	2.09	2.77	3.50	3.90	4.84	8.89	19.9	15.35
1994	41.08	10.51	5.17	4.77	2.55	2.88	3.09	2.55	2.57	2.77	3.24	5.30	10.28	15.16	10.53
1995	3.58	1.05	1.61	2.88	2.07	2.08	1.74	2.42	1.98	2.80	4.94	3.64	7.31	9.24	8.26
1996	0.20	28.24	1.91	1.74	2.24	1.81	1.78	1.76	2.75	3.32	2.59	4.21	5.61	6.10	12.55
1997	0.30	4.09	6.38	1.97	2.43	2.72	2.64	2.89	3.79	4.25	4.73	3.83	6.32	16.8	9.00

Table A3.9. Mean log length at age in the survey catch (ln(cm), Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA														
1975	NA														
1976	NA														
1977	4.15	4.27	4.38	4.56	4.57	4.72	4.86	4.94	4.99	5.01	5.05	5.20	5.03	5.06	5.09
1978	4.19	4.30	4.45	4.53	4.68	4.77	4.87	4.94	5.00	5.05	5.11	5.08	5.13	NA	5.20
1979	4.16	4.31	4.47	4.60	4.61	4.74	4.80	4.89	4.96	5.01	5.04	5.05	5.13	5.13	5.16
1980	4.14	4.26	4.44	4.54	4.66	4.74	4.81	4.88	4.93	5.00	4.98	5.02	5.00	5.01	5.18
1981	4.15	4.27	4.42	4.53	4.64	4.70	4.80	4.88	4.91	5.01	5.07	5.13	5.12	5.17	5.18
1982	4.21	4.36	4.45	4.54	4.67	4.79	4.85	4.88	4.89	4.94	5.00	5.01	5.14	5.04	5.18
1983	4.18	4.28	4.40	4.52	4.62	4.72	4.83	4.88	4.93	4.95	4.93	5.07	5.15	5.13	5.22
1984	4.14	4.26	4.41	4.49	4.61	4.70	4.75	4.86	4.90	4.92	5.00	5.10	5.05	5.18	5.22
1985	4.15	4.28	4.40	4.49	4.59	4.65	4.73	4.78	4.95	5.01	4.94	5.00	5.03	5.10	5.24
1986	4.19	4.28	4.39	4.47	4.60	4.67	4.71	4.80	4.83	4.94	4.98	4.93	5.05	5.08	5.23
1987	NA														
1988	NA														
1989	NA														
1990	NA														
1991	NA														
1992	NA														
1993	4.13	4.26	4.31	4.37	4.42	4.47	4.54	4.57	4.62	4.70	4.81	4.83	4.92	4.98	5.02
1994	4.12	4.27	4.31	4.36	4.44	4.48	4.57	4.59	4.63	4.69	4.74	4.82	4.85	4.73	4.97
1995	4.20	4.25	4.34	4.40	4.46	4.50	4.58	4.60	4.66	4.71	4.73	4.70	4.82	4.82	4.72
1996	4.16	4.26	4.32	4.39	4.45	4.51	4.56	4.58	4.65	4.68	4.72	4.75	4.77	4.87	4.94
1997	NA														

Table A3.10. Standard error of mean log length at age in the survey catch (hundredths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA	NA	NA												
1975	NA	NA	NA												
1976	NA	NA	NA												
1977	1.57	1.41	1.74	1.79	2.46	2.69	3.07	2.55	2.79	2.54	2.89	1.97	11.34	7.18	12.96
1978	1.57	2.20	2.17	2.82	2.82	3.81	2.97	3.85	4.35	2.26	2.45	9.27	8.14	NA	NA
1979	1.52	1.47	1.54	1.33	1.67	1.88	2.17	2.26	2.71	4.91	3.64	3.98	2.78	2.12	5.76
1980	2.06	1.78	1.55	1.56	1.48	1.54	1.53	1.39	1.70	2.31	2.84	2.37	4.38	4.42	2.73
1981	1.47	1.36	1.27	1.17	1.30	1.14	1.22	1.16	1.61	1.54	1.57	1.25	2.47	3.97	2.23
1982	1.62	1.59	1.28	1.14	1.22	1.31	1.29	1.59	2.05	2.82	3.32	5.70	2.99	3.47	5.20
1983	1.53	1.46	1.62	1.67	1.53	1.36	1.45	1.46	1.86	2.21	3.04	3.08	2.83	3.84	3.10
1984	1.63	1.40	1.64	1.43	1.67	1.42	1.72	1.82	1.85	2.39	4.22	3.12	3.71	3.13	2.24
1985	2.27	2.09	1.27	1.42	1.47	1.73	1.47	2.02	2.06	2.85	3.62	3.92	9.65	3.16	1.85
1986	1.45	1.81	1.96	1.48	1.60	1.80	1.87	1.60	2.30	2.67	2.70	3.68	4.08	4.03	1.81
1987	NA	NA	NA												
1988	NA	NA	NA												
1989	NA	NA	NA												
1990	NA	NA	NA												
1991	NA	NA	NA												
1992	NA	NA	NA												
1993	2.88	1.98	1.43	1.02	0.76	1.13	1.23	1.21	1.85	2.12	3.09	3.61	4.20	4.75	6.37
1994	1.19	1.58	1.32	1.07	0.96	0.94	1.37	1.40	1.52	2.31	3.06	5.58	5.69	3.46	9.91
1995	1.59	1.02	0.82	0.93	0.89	0.95	1.05	1.28	1.40	2.21	2.08	4.59	6.40	6.69	3.16
1996	1.31	1.75	1.04	0.77	1.05	1.09	0.99	1.08	1.19	1.47	1.80	1.96	3.10	3.20	3.25
1997	NA	NA	NA												

Table A3.11. Variance of log length at age in the survey catch (hundredths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA														
1975	NA														
1976	NA														
1977	1.57	2.64	3.76	3.55	4.34	3.91	2.36	1.49	1.56	1.10	2.01	0.35	5.14	2.06	6.72
1978	2.03	3.11	4.11	4.37	3.25	4.80	2.91	2.22	1.70	0.51	0.54	3.44	2.65	NA	NA
1979	2.05	2.59	2.44	2.23	2.73	2.75	3.26	1.38	1.62	3.37	1.72	1.74	0.54	0.09	2.32
1980	1.36	2.59	2.85	3.02	2.84	3.09	2.56	1.60	2.12	1.81	1.94	1.35	2.49	2.15	1.19
1981	1.82	1.80	2.68	2.48	2.93	2.04	2.30	1.71	2.18	1.52	1.23	0.50	1.53	1.89	1.39
1982	1.91	2.31	2.41	3.30	3.09	2.83	2.47	2.73	2.85	3.02	3.30	2.60	1.16	1.69	3.25
1983	2.02	2.18	2.97	3.51	3.64	2.63	2.03	2.51	2.34	2.64	2.59	1.52	0.72	1.03	1.63
1984	0.90	1.52	2.70	2.24	3.55	3.35	3.08	2.25	2.27	2.81	4.80	2.44	1.93	1.27	1.55
1985	1.86	2.84	2.47	2.72	2.77	3.39	3.12	4.22	2.41	3.17	2.62	2.61	6.52	1.10	0.96
1986	0.93	1.74	2.53	2.68	2.73	3.14	3.62	2.73	3.98	4.22	3.07	3.65	3.00	2.11	1.90
1987	NA														
1988	NA														
1989	NA														
1990	NA														
1991	NA														
1992	NA														
1993	1.08	0.98	1.53	1.71	1.80	2.47	3.38	2.94	3.37	3.88	4.86	4.43	2.82	1.58	4.46
1994	0.44	1.43	1.16	1.92	2.43	2.18	3.08	2.57	2.49	2.98	3.64	5.61	3.56	0.72	9.82
1995	0.91	1.09	1.39	1.93	2.11	2.50	3.16	2.65	2.28	3.36	2.42	3.16	3.28	2.24	0.40
1996	0.36	1.35	1.44	1.70	2.17	3.34	3.47	3.79	3.69	4.13	4.29	3.60	3.56	3.88	4.44
1997	NA														

Appendix 3.12. Standard error of variance of log length at age in the survey catch (thousandths, Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1975	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1976	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1977	2.78	3.24	4.78	4.77	7.23	7.52	6.68	4.39	4.93	3.77	5.80	1.65	36.35	14.57	47.52
1978	3.17	5.50	6.23	8.33	7.18	11.82	7.16	8.11	8.01	2.28	2.55	24.32	18.74	NA	NA
1979	3.07	3.34	3.40	2.81	3.90	4.40	5.55	3.76	4.88	12.74	6.75	7.42	2.89	0.90	12.40
1980	3.40	4.04	3.71	3.84	3.54	3.83	3.45	2.48	3.51	4.39	5.60	3.90	9.77	9.17	4.21
1981	2.81	2.58	2.93	2.61	3.15	2.30	2.62	2.14	3.36	2.69	2.46	1.25	4.33	7.72	3.71
1982	3.16	3.42	2.82	2.92	3.04	3.11	2.86	3.72	4.89	6.93	8.52	13.00	4.55	6.39	13.27
1983	3.08	3.05	3.95	4.42	4.12	3.12	2.93	3.27	4.01	5.08	6.92	5.37	3.39	5.51	5.59
1984	2.18	2.43	3.82	3.02	4.45	3.68	4.27	3.86	3.95	5.68	13.06	6.90	7.29	4.98	3.94
1985	4.38	4.98	2.82	3.32	3.45	4.51	3.68	5.88	4.51	7.18	8.29	8.95	34.85	4.69	2.57
1986	1.98	3.38	4.40	3.43	3.75	4.51	5.02	3.73	6.50	7.77	6.70	9.93	10.00	8.28	3.53
1987	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1988	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1989	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1990	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1991	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1992	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1993	4.24	2.77	2.50	1.89	1.44	2.51	3.21	2.93	4.81	5.92	9.62	10.74	9.97	8.45	19.02
1994	1.12	2.68	2.00	2.09	2.12	1.97	3.39	3.18	3.39	5.63	8.24	18.70	15.18	4.16	43.92
1995	2.14	1.50	1.37	1.82	1.84	2.12	2.65	2.95	2.98	5.72	4.57	11.54	16.40	14.17	2.83
1996	1.11	2.88	1.77	1.42	2.20	2.81	2.61	2.98	3.24	4.22	5.28	5.25	8.28	8.90	9.69
1997	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table A3.13. Proportion at age in the survey catch (Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA														
1975	NA														
1976	NA														
1977	0.09	0.19	0.18	0.16	0.10	0.08	0.04	0.03	0.03	0.02	0.03	0.01	0.01	0.01	0.01
1978	0.18	0.14	0.19	0.12	0.09	0.07	0.07	0.03	0.02	0.02	0.02	0.01	0.01	0.00	0.00
1979	0.11	0.15	0.13	0.16	0.12	0.10	0.09	0.03	0.03	0.02	0.02	0.01	0.01	0.00	0.01
1980	0.03	0.08	0.12	0.12	0.13	0.13	0.11	0.08	0.07	0.03	0.02	0.02	0.01	0.01	0.02
1981	0.06	0.07	0.12	0.13	0.12	0.11	0.11	0.09	0.06	0.04	0.03	0.02	0.02	0.01	0.02
1982	0.05	0.07	0.11	0.19	0.15	0.12	0.11	0.08	0.05	0.03	0.02	0.01	0.01	0.01	0.01
1983	0.08	0.09	0.10	0.11	0.14	0.12	0.08	0.10	0.06	0.05	0.02	0.01	0.01	0.01	0.01
1984	0.03	0.08	0.10	0.11	0.13	0.16	0.10	0.07	0.07	0.05	0.03	0.02	0.01	0.01	0.03
1985	0.03	0.06	0.14	0.13	0.12	0.11	0.14	0.10	0.05	0.04	0.02	0.02	0.01	0.01	0.03
1986	0.04	0.05	0.07	0.12	0.11	0.10	0.10	0.11	0.08	0.06	0.04	0.03	0.02	0.01	0.06
1987	NA														
1988	NA														
1989	NA														
1990	NA														
1991	NA														
1992	NA														
1993	0.01	0.02	0.05	0.11	0.21	0.13	0.15	0.13	0.06	0.06	0.03	0.02	0.01	0.00	0.01
1994	0.02	0.04	0.05	0.12	0.19	0.18	0.12	0.10	0.08	0.04	0.03	0.01	0.01	0.00	0.01
1995	0.02	0.06	0.11	0.12	0.14	0.15	0.16	0.09	0.06	0.04	0.03	0.01	0.00	0.00	0.00
1996	0.01	0.02	0.05	0.12	0.08	0.12	0.15	0.13	0.11	0.08	0.05	0.04	0.02	0.02	0.02
1997	NA														

Table A3.14. Standard deviation of proportion at age in the survey catch (Area 3A).

	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20+
1974	NA														
1975	NA														
1976	NA														
1977	0.12	0.08	0.08	0.09	0.11	0.13	0.20	0.20	0.22	0.24	0.20	0.33	0.50	0.50	0.50
1978	0.10	0.12	0.10	0.13	0.15	0.17	0.17	0.25	0.33	0.31	0.33	0.50	0.50	NA	1.00
1979	0.10	0.08	0.09	0.08	0.09	0.11	0.11	0.19	0.21	0.26	0.28	0.30	0.38	0.71	0.38
1980	0.17	0.11	0.09	0.08	0.08	0.08	0.09	0.11	0.11	0.17	0.20	0.20	0.28	0.30	0.25
1981	0.11	0.10	0.07	0.07	0.07	0.08	0.08	0.08	0.11	0.12	0.14	0.17	0.20	0.29	0.19
1982	0.11	0.10	0.08	0.06	0.06	0.07	0.08	0.09	0.12	0.16	0.18	0.35	0.28	0.27	0.29
1983	0.10	0.09	0.09	0.08	0.07	0.08	0.10	0.09	0.12	0.13	0.19	0.25	0.33	0.38	0.24
1984	0.17	0.11	0.09	0.09	0.08	0.07	0.09	0.12	0.12	0.14	0.19	0.20	0.27	0.28	0.18
1985	0.16	0.12	0.07	0.08	0.08	0.09	0.08	0.09	0.13	0.16	0.22	0.24	0.38	0.30	0.19
1986	0.15	0.13	0.12	0.08	0.09	0.10	0.09	0.09	0.11	0.13	0.15	0.19	0.23	0.28	0.13
1987	NA														
1988	NA														
1989	NA														
1990	NA														
1991	NA														
1992	NA														
1993	0.28	0.20	0.11	0.07	0.05	0.07	0.06	0.07	0.10	0.10	0.14	0.17	0.25	0.38	0.30
1994	0.18	0.13	0.12	0.07	0.06	0.06	0.07	0.08	0.09	0.13	0.16	0.23	0.30	0.41	0.32
1995	0.17	0.09	0.07	0.06	0.06	0.06	0.05	0.08	0.09	0.12	0.13	0.26	0.35	0.45	0.50
1996	0.22	0.15	0.08	0.06	0.07	0.06	0.05	0.05	0.06	0.07	0.08	0.10	0.16	0.16	0.15
1997	NA														

Table A3.15. Bycatch mortality by length group in numbers (thousands) in Area 3A; groups represent 10 cm length intervals, e.g. 80 is 80-90cm.

	80	90	100	110	120	130	140	150	160	170	180	190	200	210+
1974	25.7	19.9	19.0	13.3	11.9	8.7	8.3	5.8	3.4	1.7	0.7	0.8	0.2	0.1
1975	24.7	17.2	13.4	8.0	7.3	3.8	3.7	3.0	1.6	0.8	0.4	0.3	0.1	0.1
1976	32.9	25.2	15.6	8.5	5.9	3.6	2.7	2.7	1.3	0.7	0.4	0.3	0.3	0.1
1977	30.1	22.3	20.4	12.9	8.2	6.3	3.5	3.0	2.0	1.0	0.5	0.4	0.1	0.1
1978	24.9	27.3	22.4	6.7	10.9	4.1	0.7	1.0	0.3	0.3	0.1	0.1	0.0	0.0
1979	50.5	36.5	27.2	12.9	11.9	9.1	3.8	2.4	2.4	1.0	1.9	0.1	0.2	0.0
1980	46.9	38.4	26.8	19.9	14.0	9.2	5.6	3.5	2.2	1.5	0.8	0.3	0.2	0.1
1981	46.9	35.5	28.1	16.8	11.7	6.9	4.7	2.1	0.6	0.5	0.3	0.2	0.1	0.0
1982	34.0	29.5	20.9	14.8	9.1	6.4	3.1	1.9	0.9	0.2	0.4	0.1	0.1	0.0
1983	30.7	21.7	13.8	7.5	6.2	3.0	2.1	1.2	0.4	0.2	0.1	0.0	0.0	0.0
1984	20.5	15.5	10.3	5.7	4.6	2.2	1.5	0.9	0.3	0.1	0.1	0.0	0.0	0.0
1985	10.2	8.4	6.0	3.0	2.5	1.3	0.8	0.4	0.1	0.0	0.0	0.0	0.0	0.0
1986	7.9	6.8	5.1	2.6	2.2	1.2	0.7	0.3	0.1	0.0	0.0	0.0	0.0	0.0
1987	26.4	17.9	11.1	5.5	3.9	1.8	1.2	1.0	0.3	0.2	0.1	0.0	0.0	0.0
1988	40.5	23.1	15.5	7.0	5.9	1.6	1.5	1.1	0.5	0.1	0.2	0.0	0.0	0.1
1989	33.3	15.8	9.6	6.5	3.6	2.3	1.5	0.8	0.9	0.8	0.2	0.1	0.1	0.1
1990	50.9	33.8	16.6	7.3	5.5	2.4	2.4	0.9	0.8	0.5	0.3	0.1	0.0	0.1
1991	63.6	40.7	22.5	11.0	6.1	3.1	1.6	1.2	0.6	0.2	0.2	0.1	0.1	0.1
1992	60.0	37.8	18.1	10.3	4.8	1.4	1.2	0.9	0.1	0.1	0.2	0.0	0.0	0.0
1993	44.5	20.5	13.5	5.3	5.2	2.1	1.7	0.7	0.2	0.0	0.1	0.0	0.0	0.0
1994	46.5	34.4	16.1	8.5	6.2	2.1	1.3	0.3	0.1	0.0	0.1	0.2	0.0	0.0
1995	30.9	15.9	8.7	5.7	3.5	1.9	1.7	1.1	0.4	0.0	0.1	0.1	0.0	0.0
1996	30.5	13.6	7.0	3.9	1.5	1.1	0.6	0.2	0.3	0.0	0.1	0.0	0.2	0.0
1997	30.4	13.6	7.0	3.9	1.5	1.1	0.6	0.2	0.3	0.0	0.1	0.0	0.2	0.0

APPENDIX 4. ESTIMATION OF HISTORICAL TRENDS IN ABUNDANCE

In order to produce long-term trends in abundance, the results of the annual stock assessment were extended back in time using data on catch at age and fishing effort available since 1935. The assessment conducted in 1996, based on the size-age model described above, was used for this purpose. Because long-term series of survey data are not available for the earlier period, a simpler and less data-demanding model was developed as explained below. Abundance at age for each IPHC regulatory area (except Area 4) was estimated using catch and weight at age data (ages 8 to 17+) for the period 1935-1973. These data are the same data used in previous historical catch-at age analyses (Quinn et al. 1985), originally compiled by applying a missing-data algorithm as described in Quinn et al. (1983). Data quality varies over time as sampling coverage was very poor in some area-period strata. Data were fitted using a procedure similar to CAGEAN, based on assuming independent log-normally distributed measurement errors. The variance of the catch-at-age data was assumed to be half of that corresponding to the effort data, as was done in the past. Estimates were constrained so that the abundances at age projected for 1974 matched those estimated using the new stock assessment method. Because final abundances were constrained, the usual assumptions of constant selectivity and catchability made by CAGEAN could be relaxed without causing estimability problems. The coefficient of catchability Q_t linking observed effort to predicted full-recruitment fishing mortality was assumed to change over time according to a random walk model identical to that assumed for the size-age assessment model (equation (4)), with standard deviation equal to 0.03.

Selectivities for ages 13 through 17+ were assumed to be constant and selectivity for the pooled age group 17+ was set to one. Selectivity for younger ages was allowed to change by letting

$$\ln(S_{a,t+1}) = \ln(S_{a,t}) + \varepsilon_{a,t} ,$$

where ε is Gaussian with zero mean and standard deviation equal to 0.014. These values are admittedly arbitrary, but they seem to allow sufficient variability to accommodate trends in catchability and selectivity that could have resulted from historical trends in size at age and changes in the conduct of the fishery. The fact that separate analyses are done for the periods 1935-1973 and 1974-1996 implies a discontinuity in the time series of catchability and selectivity at the break point. The year 1974 was chosen as the break point because that is when the commercial size limit was raised from 60 cm to the current limit of 81 cm (32 inches) (Clark and Parma 1995). As could be anticipated, this change in regulations resulted in an abrupt reduction in the selectivity of the younger age classes, and a corresponding increase in their average weight in the catch.

Estimates of initial abundances, recruitment of 8-yr-olds, full-recruitment fishing mortalities (F_t) and catchabilities for years 1935-1973, and selectivity parameters were estimated by minimizing the function

$$f = 0.5 n \ln \left(\sum_{a,t} (\ln(C_{a,t}^{\text{obs}}) - \ln(C_{a,t}))^2 + \frac{1}{2} \sum_t (\ln(E_t^{\text{obs}}) - \ln(Q_t) - \ln(F_t))^2 \right) + 0.5 \left(1000 \sum_t \varepsilon_t^2 + 5000 \sum_{a,t} \varepsilon_{a,t}^2 \right)$$

subject to the constraint that predicted 1974 abundance at age matches the initial abundances estimated by the 1996 assessment. Here n denotes the total number of catch and effort observations, $C_{a,t}^{\text{obs}}$ and $C_{a,t}$ are observed and predicted catches at age a in year t , and E_t^{obs} is observed fishing effort in year t .

Estimates of abundance of 8-year-olds were adjusted by adding the estimated number of recruits lost due to bycatch. Reproductive biomass was computed as

$$SB_t = \sum_a N_{a,t} f_a w_{a,t},$$

where $w_{a,t}$ represents smoothed average weights at age and year estimated by sampling commercial landings, and f_a represents the fraction of females that mature at age, which was assumed to be constant and equal to

$$f_a = \frac{1}{1 + \exp(-0.8(a-11))}.$$

While major changes in female maturity at size have been observed in specimens collected during setline surveys, maturity fractions at age have remained relatively stable throughout the period when growth rates were dramatically changing in the central Gulf of Alaska. Age at 50% maturity has changed by at most one year (Clark and Parma 1995, Clark et al. 1999, Schmitt and Skud 1978) while length at 50% maturity has decreased from about 125 cm in the 1980s to about 90 cm in the 1990s in Area 3A, and from 110 cm to 98 cm in Area 2B. Schmitt and Skud (1978) report an increase in the size at maturity between the 1950s and the 1960s and 1970s during a period of increasing size at age. Parameters used here roughly correspond to the average maturity schedule for females in the northeast Pacific. Note that no attempt has been made to estimate actual female spawning biomass using weights at age for females and estimates of sex ratio at age. In the context of evaluating exploitation rates, the above index was considered more appropriate as estimates of commercial selectivity for females are not available at this time. Thus, while historical trends in female spawning biomass could be approximated from sex-specific data collected during research cruises, evaluating the effect of changes in target harvest rate on female biomass would require further assumptions about sex-specific selectivity in the fishery which are hard to substantiate in the absence of information about sex ratio in the commercial catch.

APPENDIX 5. MISCELLANEOUS MATHEMATICAL NOTES

1. The estimated variance of s^2

The model parameters include the variances of length at age in commercial and survey catches. These variances are estimated from the sample data using the usual formula for the sample variance s^2 , but every such variance estimate has its own variance that needs to be estimated for the purpose of weighting the observation.

Snedecor and Cochran (6th edition, p. 89) give the exact variance of s^2 as:

$$V(s^2) = \frac{2\sigma^4}{f} \cdot \left\{ 1 + \frac{f}{f+1} \cdot \frac{\gamma_2}{2} \right\}$$

where σ^4 is the square of the variance, f is the number of degrees of freedom (i.e., $n-1$) and γ_2 is the kurtosis of the distribution. The factor outside the brackets is the variance of s^2 when the distribution is normal ($\gamma_2 = 0$). The kurtosis can be estimated by the sample kurtosis g_2 :

$$g_2 = m_4/m_2^2 - 3$$

where m_2 and m_4 are the second and fourth sample moments about the mean (e.g., $m_4 = \sum_i (x_i - \bar{x})^4/n$).

It is not possible to use $(s^2)^2$ directly as an estimate of σ^4 because

$$E[(s^2)^2] = [E(s^2)]^2 + V(s^2) = \sigma^4 + V(s^2)$$

This formula provides a way to get an unbiased estimate. For the case of the normal distribution:

$$\begin{aligned} E\left[\frac{2 \cdot (s^2)^2}{f}\right] &= \frac{2\sigma^4}{f} + \frac{2 \cdot V(s^2)}{f} \\ &= V(s^2) + \frac{2 \cdot V(s^2)}{f} \\ &= V(s^2)[1 + 2/f] \\ E\left[\frac{2 \cdot (s^2)^2}{f \cdot (1 + 2/f)}\right] &= V(s^2) \end{aligned}$$

The denominator on the left reduces to $(f + 2)$, so for the normal distribution we have

$$\hat{V}(s^2) = \frac{2 \cdot (s^2)^2}{f + 2} \text{ and for the general case } \hat{V}(s^2) = \frac{2 \cdot (s^2)^2}{f + 2} \cdot \left\{ 1 + \frac{f}{f + 1} \cdot \frac{g_2}{2} \right\}.$$

This estimator is quite variable; trials with samples of size 25 from a normal distribution show that its coefficient of variation exceeds 50%. The correction factor (the term on the right) is also quite variable because it depends on the second and fourth sample moments about the mean. This suggests using the normal form whenever possible.

2. Conversions between moments of log-length and moments of length

Let $E[L]$, $V[L]$, and $CV[L]$ denote, respectively, the mean, variance, and coefficient of variation of length. If L has a lognormal distribution, then

$$X = \log(L) \sim N(\mu, \sigma^2)$$

The following conversions apply:

$$E[L] = e^{\mu + \sigma^2/2}$$

$$V[L] = (E[L])^2 \cdot (e^{\sigma^2} - 1)$$

$$CV[L] = \sqrt{e^{\sigma^2} - 1}$$

and

$$\sigma^2 = \log \left(\frac{V[L]}{(E[L])^2} + 1 \right)$$

$$\mu = \log(E[L]) - \sigma^2/2$$

3. Integration across the distribution of log length-at-age

The distribution of length at age is modeled as a lognormal:

$$X = \ln(L) \sim N(\mu_a, \sigma_a^2)$$

with density function

$$\varphi(X|\mu_a, \sigma_a^2) = \frac{1}{\sigma_a \sqrt{2\pi}} \exp\left\{-\frac{(X - \mu_a)^2}{2\sigma_a^2}\right\}$$

For log lengths above the log length at full selection X_{full} , selectivity $s(X)$ is one. For log lengths below X_{full} , selectivity is modeled as a half-normal without the multiplier $\frac{1}{\sigma\sqrt{2\pi}}$:

$$\begin{aligned} s(X) &= \exp\left\{-\frac{(X - X_{\text{full}})^2}{2\sigma_s^2}\right\} \\ &= \sigma_s \sqrt{2\pi} \cdot \varphi(X|X_{\text{full}}, \sigma_s^2) \end{aligned}$$

For a given age group, the age-specific selectivity $s(a)$ is obtained by integrating $s(X)$ over $\varphi(X|\mu_a, \sigma_a^2)$:

$$s(a) = \int_{X_{\text{min}}}^{X_{\text{full}}} s(X) \cdot \varphi(X|\mu_a, \sigma_a^2) dX + 1 - \Phi\left(\frac{X_{\text{full}} - \mu_a}{\sigma_a}\right)$$

In this expression, X_{min} is the minimum size limit, i.e., $\ln(81)$ for the commercial fishery or $-\infty$ for the survey. $\Phi(\cdot)$ is the cumulative standard normal distribution function; the term following the integral is the proportion of the age group larger than X_{full} .

Expanding the terms in the integral and completing the square in the exponent puts the exponent in the form:

$$\text{exponent} = \frac{-(X - \theta)^2}{2\eta^2} + \frac{\theta^2}{2\eta^2} - \frac{X_{\text{full}}^2}{2\sigma_s^2} - \frac{\mu_a^2}{2\sigma_a^2}$$

where

$$\theta = \frac{\sigma_a^2 \cdot X_{\text{full}} + \sigma_s^2 \cdot \mu_a}{\sigma_a^2 + \sigma_s^2}$$

$$\eta^2 = \frac{\sigma_a^2 \cdot \sigma_s^2}{\sigma_a^2 + \sigma_s^2}$$

The expression for age-specific selectivity can then be written as:

$$\begin{aligned}
s(a) &= \frac{1}{\sigma_a \sqrt{2\pi}} \cdot \frac{\eta \sqrt{2\pi}}{\eta \sqrt{2\pi}} \cdot \exp\left\{\frac{\theta^2}{2\eta^2} - \frac{X_{\text{full}}^2}{2\sigma_s^2} - \frac{\mu_a^2}{2\sigma_a^2}\right\} \cdot \int_{X_{\text{min}}}^{X_{\text{full}}} \exp\left\{-\frac{(X-\theta)^2}{2\eta^2}\right\} dX + 1 - \Phi\left(\frac{X_{\text{full}} - \mu_a}{\sigma_a}\right) \\
&= \frac{\eta}{\sigma_a} \cdot \exp\left\{\frac{\theta^2}{2\eta^2} - \frac{X_{\text{full}}^2}{2\sigma_s^2} - \frac{\mu_a^2}{2\sigma_a^2}\right\} \cdot \left[\Phi\left(\frac{X_{\text{full}} - \theta}{\eta}\right) - \Phi\left(\frac{X_{\text{min}} - \theta}{\eta}\right)\right] + 1 - \Phi\left(\frac{X_{\text{full}} - \mu_a}{\sigma_a}\right) \\
&= I \cdot \left[\Phi\left(\frac{X_{\text{full}} - \theta}{\eta}\right) - \Phi\left(\frac{X_{\text{min}} - \theta}{\eta}\right)\right] + 1 - \Phi\left(\frac{X_{\text{full}} - \mu_a}{\sigma_a}\right)
\end{aligned}$$

where the integrating factor I can be calculated as shown on the second line above or alternatively as:

$$I = \sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_a^2}} \cdot \exp\left\{\frac{-(X_{\text{full}} - \mu_a)^2}{2(\sigma_s^2 + \sigma_a^2)}\right\}$$

The mean log-length of fish in the catch is:

$$\begin{aligned}
{}_c\mu_a &= \frac{\int_{X_{\text{min}}}^{\infty} X \cdot s(X) \cdot \varphi(X|\mu_a, \sigma_a^2) dX}{s(a)} \\
&= \frac{1}{s(a)} \cdot \left[I \cdot \int_{X_{\text{min}}}^{X_{\text{full}}} X \cdot \varphi(X|\theta, \eta^2) dX + \int_{X_{\text{full}}}^{\infty} X \cdot \varphi(X|\mu_a, \sigma_a^2) dX \right]
\end{aligned}$$

Similarly the variance is:

$$\begin{aligned}
{}_c\sigma_a^2 &= \frac{\int_{X_{\text{min}}}^{\infty} X^2 \cdot s(X) \cdot \varphi(X|\mu_a, \sigma_a^2) dX}{s(a)} - {}_c\mu_a^2 \\
&= \frac{1}{s(a)} \cdot \left[I \cdot \int_{X_{\text{min}}}^{X_{\text{full}}} X^2 \cdot \varphi(X|\theta, \eta^2) dX + \int_{X_{\text{full}}}^{\infty} X^2 \cdot \varphi(X|\mu_a, \sigma_a^2) dX \right] - {}_c\mu_a^2
\end{aligned}$$

The definite integrals of X and X^2 over normal densities in these equations can be evaluated using the formulas:

$$\int_{X_1}^{X_2} X \cdot \varphi(X|\mu, \sigma^2) dX = \sigma^2 \cdot [\varphi(X_1) - \varphi(X_2)] + \mu \cdot \left[\Phi\left(\frac{X_2 - \mu}{\sigma}\right) - \Phi\left(\frac{X_1 - \mu}{\sigma}\right) \right]$$

$$\int_{X_1}^{X_2} X^2 \cdot \varphi(X|\mu, \sigma^2) dX = \sigma^2 \cdot [(X_1 + \mu) \cdot \varphi(X_1) - (X_2 + \mu) \cdot \varphi(X_2)] +$$

$$(\mu^2 + \sigma^2) \cdot \left[\Phi\left(\frac{X_2 - \mu}{\sigma}\right) - \Phi\left(\frac{X_1 - \mu}{\sigma}\right) \right]$$

The mean log length of survivors is a function of the harvest rate. Lets H denote the exploitation rate of fully vulnerable fish. Then the mean log length of fish of age a in the sea after the commercial catch has been taken is:

$$\mu_a^+ = \frac{\int_{-\infty}^{\infty} X \cdot [1 - s(X) \cdot H] \cdot \varphi(X|\mu_a, \sigma_a^2) dX}{1 - s(a) \cdot H}$$

$$= \frac{\mu_a - s(a) \cdot H \cdot {}_c\mu_a}{1 - s(a) \cdot H}$$

In the second equation above, ${}_c\mu_a$ is of course the mean log length in the commercial catch. This equation can also be obtained by noting that the mean length before harvest is just a weighted average of the mean lengths of fish in the catch and survivors:

$$\mu_a = s(a) \cdot H \cdot {}_c\mu_a + [1 - s(a) \cdot H] \cdot \mu_a^+$$