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A SIMULATION OF MANAGEMENT STRATEGIES IN THE PACIFIC HALIBUT FISHERY

BY

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FOREWORD

The 1953 Convention between Canada and the United States for the Preservation of the Halibut Fishery of the Northern Pacific Ocean and Bering Sea continued the conservation and development objectives of the three previous conventions, requiring specifically that the stocks of halibut be developed to levels which will permit maximum sustainable yield and be maintained at these levels.

This report continues the research into the dynamics of the Pacific halibut stocks. It presents a simulation of three management strategies applicable to the regulation of the United States and Canadian setline halibut fishery on the grounds south of Cape Spencer, Alaska, in accord with the 1953 Convention. One of the strategies, based on an empirical analysis of the data, depicts the basic management plan pursued by the Commission since 1932. The other two are alternative procedures that the Commission has used to provide a fuller understanding of the reaction of the stocks to fishing.

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INTRODUCTION

Halibut have been fished commercially in the Northeastern Pacific Ocean by United States and Canadian vessels for about 75 years, first off the coasts of Washington, British Columbia and Oregon and later off Alaska.

The history of the fishery can be divided into three periods: first, that of expanding exploitation, from about 1888 to approximately 1912 by which time the previously unfished accumulations of halibut were reduced. The stocks continued to decline under the intensive fishery, and in 1923 the International Fisheries Commission, subsequently named the International Pacific Halibut Commission by the current 1953 Convention, was established to make a thorough investigation into the life history of the Pacific halibut and make recommendations necessary for the preservation and development of the resource.

The second period, one of controlled rehabilitation of the resource, occurred between the early 1930's and the mid-1950's. After a period of investigation and passage of a new treaty the Commission began regulating the fishery in 1932. Annual catch limits which were held slightly below the additions to the weight of the stocks by growth and recruitment were established for the grounds South and West of Cape Spencer. This ensured that the stocks would increase in size and yield greater annual catches.

The third period, with stocks being maintained at levels productive of the maximum sustainable yield, began in the mid-1950's. By this time, the Commission's concern had become the taking of the maximum sustainable yield, a difficult and more complex objective for management. Regulatory areas were opened to fishing during several seasons within a year and fishing was controlled either by fixed period of fishing or by catch limits. By such multiple seasons the fishing power of the fleet was distributed on the grounds more nearly in proportion to the seasonal productivity of the various stock components.

With management directed to maintaining the stocks at an optimum level productive of the maximum sustainable yield it becomes necessary to detect when the stocks may differ from such a level and to initiate the adjustments necessary to restore the stock. Changes in catch and effort statistics as well as age composition data supply the needed information regarding the state of the stocks. The goal then becomes one of utilizing such information in the most efficient manner so that a non-optimal condition can be rectified in the least possible time.

In managing the stocks since 1932, the Commission has attempted to minimize the disturbing effect of regulation upon the economics of the fleets by making any required changes in a gradual manner. In this way the fishery was able to adjust more readily to changes in catch limits or other forms of regulation. The scheme of management, essentially an empirical one, was based primarily on the response of the catch per unit effort to the removals. The interpretation of the changes in catch per unit effort has been qualified by information about the age composition of the catches. In addition, the Commission has employed from time to time the theoretical models described by Thompson and Bell (1934), Schaefer (1954, 1957), Beverton and Holt (1957) and Ricker (1958) in an attempt to explain the reaction of the stocks to fishing and to changes in regulation. However, providing for systematic changes in basic parameters of these models has been difficult and any feedback of information between previous and current states of the model has been discontinuous and accomplished only in part by substituting different values for the parameters in subsequent solutions of the model.

Another approach to studying the reactions of the stocks to fishing, and the one discussed in this report, is to simulate (on a digital computer) by means of a mathematical model the responses of management to the statistics of the fishery and the population. In this way, different management schemes which make use of the feedback of information between the fishery and the population of halibut can be studied and guidelines for more effective management of the resource can be drawn.

A numerical or digital simulation of a management system consists of following step by step the changes in the system. If the system is large and complex, the effects of such changes often are too complicated to be analysed with integral-differential equations by the usual computational techniques since the interactions of the system cannot be conveniently handled.

Adequate treatment of the interactions often requires that the system be expressed in algebraic form. Because of the intricacies of the mathematics, a high-speed digital computer with a large storage capacity is necessary. The algebraic simulation model is designed so that at each step of the analysis the interactions pertinent to the computations can be recalled and treated in the appropriate manner. The initial state of the model is specified and at any given time thereafter its condition is determined by previous circumstances. In a simulation model of a complex system such as a commercial fishery, for instance, the population of fish, the management agency and the fishing fleet would be represented mathematically by distinct sections of the model. The reaction of each section to simulated fishing and the interactions among them must be analogous to the real situation before meaningful inferences can be drawn from the model.

In addition simulation techniques enable consideration of management policies that are either exceedingly difficult or impossible to implement in a real fishery because of time or economic considerations but that may be beneficial in the long run.

The Pacific Coast halibut fishery of Canada and the United States is well adapted to study by simulation because of the long series of biological and statistical data that are available. The International Pacific Halibut Commission has collected statistics of the catch, fishing effort and the area of origin of the catch and has conducted tagging and age and growth studies. The series of catch and effort data are continuous from 1916 to date and some individual records date back to the turn of the century. The series of age composition data is continuous for certain grounds in Area 2, the region reported upon here, from 1932 to date, and is augmented by isolated samples of earlier data collected between 1914 and 1932.

REVIEW OF THE LITERATURE

Simulation techniques with high-speed computers have been utilized in the physical and natural sciences as well as in sociology and economics. The term "simulation" has referred to the numerical solution of stochastic or probabilistic models such as Monte Carlo or Markov Chain models (Hammersley, 1960; Garfinkel, MacArthur and Sack, 1964) as well as to the representation of complex economic systems (Bonini, 1963; Chorafas, 1965; Orcutt, Greenberger, Korbel and Rivlin, 1961). In the former the procedure is to obtain a large number of results from repeated trials of a stochastic model and empirically arrive at an average; in the latter, a decision-making process dependent upon a feedback of factors which interact in varying degrees with each other is usually of prime importance. The decision-making process under study may be based on a model having random variability or on a simpler deterministic model; sometimes the decisions are studied under both conditions. It is in the sense that the decision-making process is dependent on a feedback of information that the term "simulation" is used in this study.

Examples of such simulation in biological situations are found in Wangersky and Cunningham (1957) who studied predator-prey models; in Garfinkel (1962) and Garfinkel and Sack (1964) who simulated a predator-prey problem using the chemical law of mass action described by Lotka (1956); and in Meier, Blakelock and Hinomoto (1964) who developed a simple game called WILDLIFE which is based on an organism-environment interaction; that is, a decision-making process, dependent upon a feedback of information. Watt (1961, 1964) and Holling (1963) discuss a systems approach to simulation of ecological problems.

In fisheries Doi (1957) and Silliman (1967) studied the dynamics of marine fish populations. Royce, Bevan, Crutchfield, Paulik and Fletcher (1963) simulated the Puget Sound net fishery for salmon in the State of Washington to study the economic and biological effects of restricting the number of units of gear in the fishery; and Larkin and Hourston (1964) simulated the population dynamics of five stocks of salmon spawning in a large river system. Paulik and Greenaugh (1966) simulated the management of a salmon resource in which the salmon are fished upon by several different fleets as they migrate from the ocean to their spawning rivers.

GENERAL PLAN OF THE MODEL

A mathematical model of the halibut population and the fishery, consisting of approximately 1500 equations, was written as a computer program. The model, which describes the stock, the fishery and the regulatory procedure controlling the fishery, was divided into three sections. A week is considered as the basic time unit and all instantaneous rates are on a weekly basis.

The biological section simulates the population which consists of 17 age groups. The number of fish at each age at any given time is the result of the unique history of exploitation to which each year class has been subjected. Natural mortality is assumed to be compensatory with population size, and different rates of natural mortality were applied to age groups 4 to 8 and 9 to 20. Fishing mortality is age specific between age 4 and age 10, the age of full vulnerability; above age 10 fishing mortality is constant. Recruitment is assumed to be a function of the number of adults in the stock. Average weight at each age is computed by a Gompertz equation and biomass is obtained by multiplying the average weight by the number of fish at each age.

The fishery section of the model generates the number of vessels fishing each year from information of the previous year's fishery and converts number of vessels into units of effort. It also provides for the entry and exit of vessels from the fleet as well as from the fishing grounds. Fishing mortality coefficients as related to the magnitudes of the fishing effort are applied to the stock in this section.

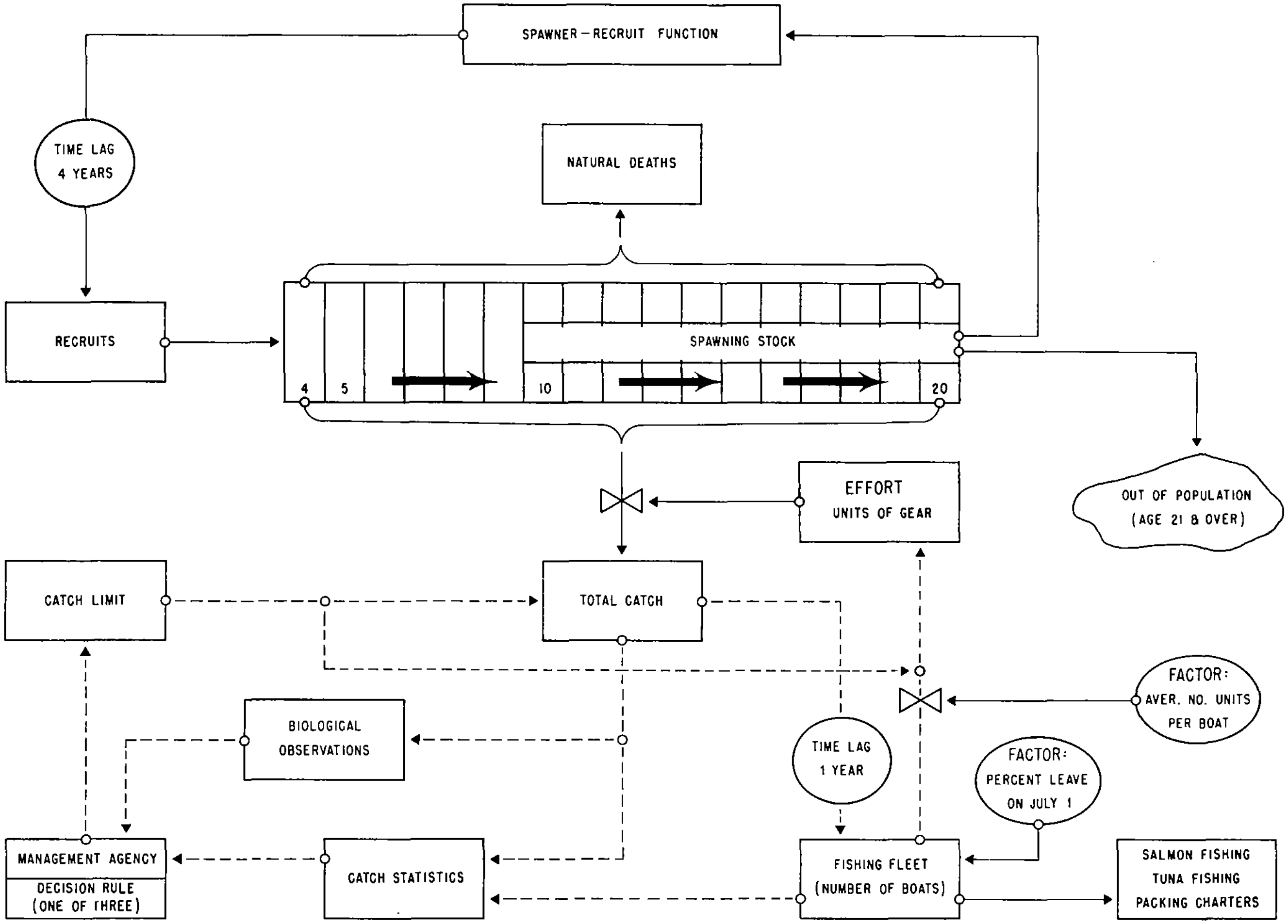

Figure 1. Flow diagram of the simulation model.

The management section of the model collects data each year from the fishery section and samples the catch for age composition data needed for decisions regarding the size of the quota or catch limit and the setting of the opening date of fishing for the following year. The model is designed so that different management schemes can be studied.

The simulated management agency adjusts the yearly catch limit according to a stated decision rule depending upon the reaction of the stock to fishing. Stock reaction is measured by changes in catch per unit effort and age composition of the commercial catch. The yearly removal or catch thus begins the feedback of information that characterizes this model. The catch of the previous year determines the size of the fleet in the present year. The catch of the current year also in part affects the size of the stock the next year, i.e. an unusually large removal will be reflected in a reduced catch per unit effort the following year and vice versa. The catch per unit effort in turn is one of the measures used to determine if an adjustment of the quota for the coming year is needed. This interaction among the catch, fleet size and stock size is supplemented by information concerning the age composition of the commercial catch.

A schematic outline of the model is shown in Figure 1. The percentage of vessels leaving the fleet in order to participate in other fisheries is considered as exogenous information. Initial conditions of the simulated fishery can be altered by varying input data over any range of values that may be of interest, for example, the percentage of vessels leaving the fishery may be altered from run to run. In addition, the biological aspects of the model, that is, the density dependence of the asymptotic weight, or say, the spawner-recruit function may also be varied between runs. Stochastic elements are introduced to simulate the variability inherent in any natural population. In addition to these, other elements are included which impose further variability attributable to the sampling procedures used in gathering statistics and biological data.

COMPUTING LANGUAGE

DYNAMO (Dynamic Models), a computer program for translating mathematical models into tabulated and plotted results, was used in this study (Pugh, 1963; Forrester, 1961). It is the result of modifications of the program SIMPLE (Simulation of Industrial Management Problems with Lots of Equations) developed at the Massachusetts Institute of Technology in 1958.

DYNAMO is designed for dynamic feedback systems comprised of zero- and first-order difference equations. It is particularly suited to biological problems since many biological reactions are described well by the finite time intervals of a difference equation. The notation of the equations is similar to that of a general-purpose scientific compiler such as FORTRAN; however, the equation order is non-sequential, and the equations are ordered by DYNAMO prior to computation (Pugh, 1963, provides a fuller discussion of DYNAMO).

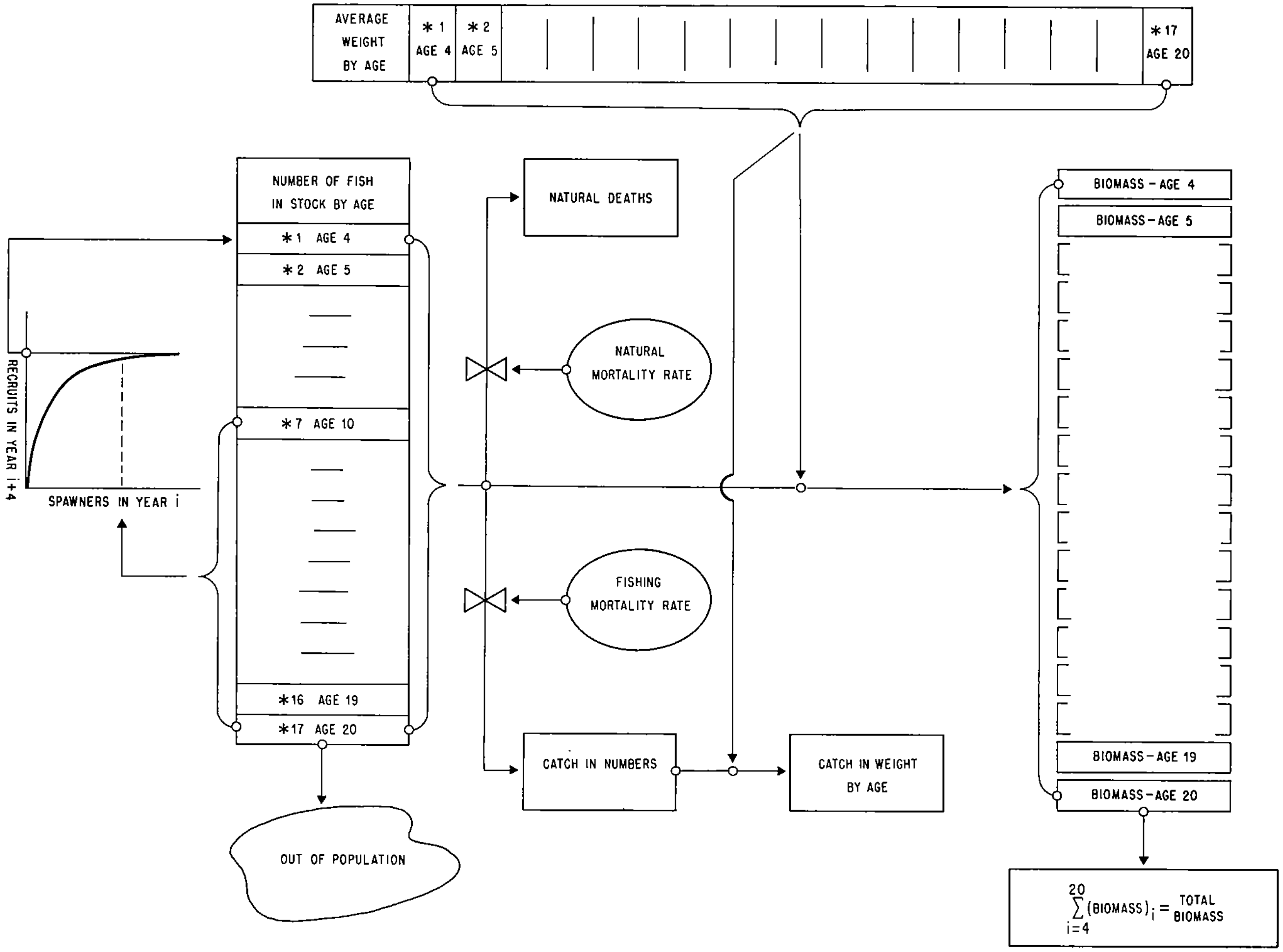

Figure 2. Diagram of the biology section of the model.

BIOLOGY SECTION OF THE MODEL

The biology section simulates a population of halibut with an age distribution extending from age 4 to 20, after which age the fish are no longer considered to be a significant part of the population. Although Pacific halibut live beyond age 20, the objectives of this investigation can be realized without considering the negligible numbers of fish beyond this age. The simulated population is composed of groups of numbers moving through the system. These groups correspond to the age groups in a real population and the numerical value of each group is decreased each time interval according to mortality factors prescribed by the model. Annually, i.e. every 52 weeks, the age designation of the group is advanced by one. The movement of one of these groups is analogous to a year class moving through a real population, being subjected to natural and fishing mortality throughout the year at weekly intervals. Flow through the system is represented diagrammatically in Figure 2. By changing the initial numerical values of the variables and certain parameters, the model can be made to represent different populations. Thus it could depict the stocks of any regulatory area.

Recruitment, which occurs at age 4, is introduced through a spawner-recruit function dependent upon the number of fish 10 years of age and older. The form of the relationship is shown schematically in Figure 3. The average age of maturity of

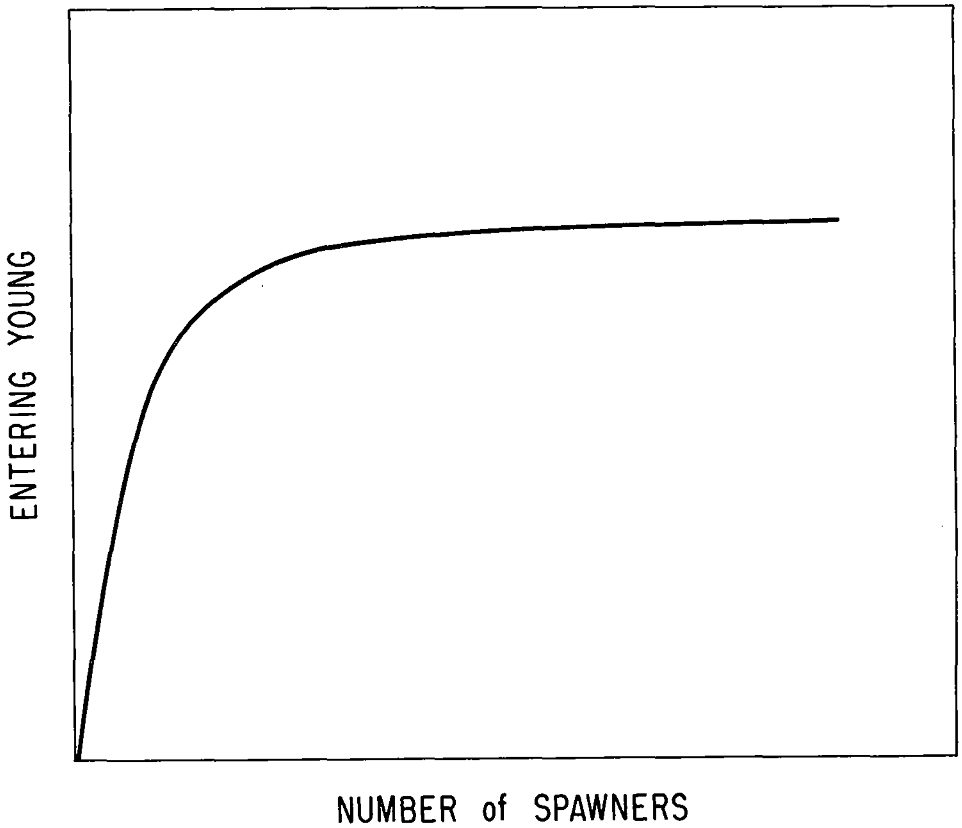

Figure 3. Schematic representations of an asymptotic spawner-recruit function.

female halibut is 12 years; however, because of the large numbers of females below age 12 that would mature and contribute to the spawning, the lower age of the spawning stock is considered to be age 10. The selection of the lower age is arbitrary and there is no evidence to indicate the percentage of the stock included. It does appear reasonable on intuitive grounds, however, to include some of the younger spawning females because under certain conditions the adult population is reduced to a very low level and the stock is maintained, for practical purposes, from spawning derived from fish less than 12 years of age.

Mathematical Form of the Biology Section

An exponential survival relationship is assumed in the population model, and the basic time interval is one week. The decrease in number of halibut at each age is described by the following differential equation:

$$\frac{dN}{dt} = -ZN \qquad (1)$$

where Z is the total removal rate. When integrated, the above gives:

$$N_{(t+\triangle t)} = N_t \exp[-Z_t(\triangle t)] \qquad (2)$$

where $\triangle$ t is the time interval from t to $t+\triangle t$. In this form the number of fish at any time, t, is exactly predicted once the mortality coefficient, Z, is known. Although such a deterministic procedure is useful for some purposes, it does not realistically depict the survival of animals from one age to another since they are subjected to varying environmental conditions.

In order to introduce random variability in the model, an error term has been applied to the total mortality rate. The customary way of introducing a stochastic element is to consider that the error term is additive (Johnston, 1963). Such would be the case if (2) were linearized by taking logarithms of both sides and the error term, u_t, added to the exponent as:

$$\ln[N_{(t+\triangle t)}] = \ln(N_t) - [Z_t(\triangle t) + u_t] \qquad (3)$$

It is assumed that the error term is normally distributed, with mean zero and standard deviation to be specified. Under these conditions it is possible that $(Z_t(\triangle t) + u_t)$ could become negative, and would have no biological meaning. This means that to preclude exponents becoming negative, a value of u_t must be chosen for each value of Z_t. To avoid the rather lengthy programming necessary to accomplish this, u_t is restricted to certain values. The random normal deviate function of DYNAMO generates a sequence of pseudo-random normal deviates which cannot be distinguished statistically from a series of random numbers. However, the sequence does not follow a perfectly normal distribution. In particular it is truncated; no number will exceed 2.4 standard deviations. This truncation facilitates the restriction of u_t. A value for *sigma* is obtained by setting one-half of the range of observed values equal to 2.4 standard deviations. In other words, *sigma* is expressed as a percentage of the range. The resulting normal random deviate is added to one and the quantity $(1+u_t)$ is multiplied by Z. Thus, (3) becomes:

$$\ln[N_{(t+\triangle t)}] = \ln(N_t) - Z_t(\triangle t)(1 + u_t) \qquad (4)$$

and the model is:

$$N_{(t+\triangle t)} = N_t \exp[-Z_t(\triangle t)(1+u_t)] \tag{5}$$

A real population experiences a number of variable environmental pressures the effect of which, while perhaps not susceptible to explicit expression, can be represented in a single random term. Therefore, the introduction of the random error term in (5) is a step closer to depicting the real population.

Initial Population Size

The number of fish at each age on the grounds in regulatory Area 2, extending from Willapa Bay, Washington to Cape Spencer, Alaska, in 1950 is taken as the initial population of the model. This number is approximated by a virtual population calculation, which is a reconstruction of the actual population based on the total commercial catch, taking into account those fish that died naturally (Hardman, Mss). This computation is similar to that of Fry (1949) as extended by Chapman (Mss). The number of fish in any year class at age t is expressed as

$$N_o = \sum_{t=0}^{t=17} \frac{C_t}{(1-n)^t} \tag{6}$$

where n is the known annual natural mortality rate, C the catch at age t, and t is an index referring to the 17 ages in the included span; in this case t=0 corresponds to age 4 years.

Goose Islands grounds is the only locality in Area 2 where there is a historical series of age data of sufficient length to provide the necessary initial values of age composition (IPHC, 1960). The estimated numbers of fish at each age on Goose Islands grounds in 1950 were weighted by the ratio of the Goose Islands catch to the Area 2 catch.

Natural Mortality

Several estimates of natural mortality of Pacific halibut older than about 4 years are given in Table 3 of Report Number 28 of the International Pacific Halibut Commission. These coefficients were estimated from tagging and age composition data. In that report, ". . . a rounded value of 0.20 as a best estimate of the instantaneous rate of natural mortality for the halibut in both Areas 2 and 3" was accepted. In Chapman, Myhre and Southward (1962) it was indicated that a natural mortality coefficient between 0.15 and 0.20 for the fishable stock would best describe the data. Accordingly, the value of 0.20 has been used as the initial value of the instantaneous natural mortality coefficient in this study for ages 4 to 8 and 0.18 was used for ages 9 to 20. In certain runs of the model, natural mortality is varied from year to year in a random manner; such variation is analogous to variations in the environment which affect the survival of all age groups.

The standard deviation of the estimates of natural mortality of 0.20 was not given in Report No. 28; however, a range of estimates for various grounds is given. The range for the Goose Islands ground, which had the longest series of data (21 years) of the grounds in Area 2, was 0.19 to 0.20.* In view of this narrow range,

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*Estimated values of instantaneous natural mortality for Hecate Strait, in Area 2, ranged from 0.07 to 0.33, with a mean of 0.24.

Table 1. Number of fish per 10,000 skates between ages 10 and 20 on the Goose Islands grounds for the years 1935 to 1953.

Year	No. of fish	Year	No. of fish	Year	No. of fish
1935	11,174	1941	11,152	1948	18,402
1936	7,121	1942	13,451	1949	28,209
1937	8,319	1943	7,670	1950	24,803
1938	9,748	1944	14,294	1951	24,544
1939	11,072	1945	5,727	1952	44,793
1940	14,558	1946	14,867	1953	69,558
		1947	23,476		

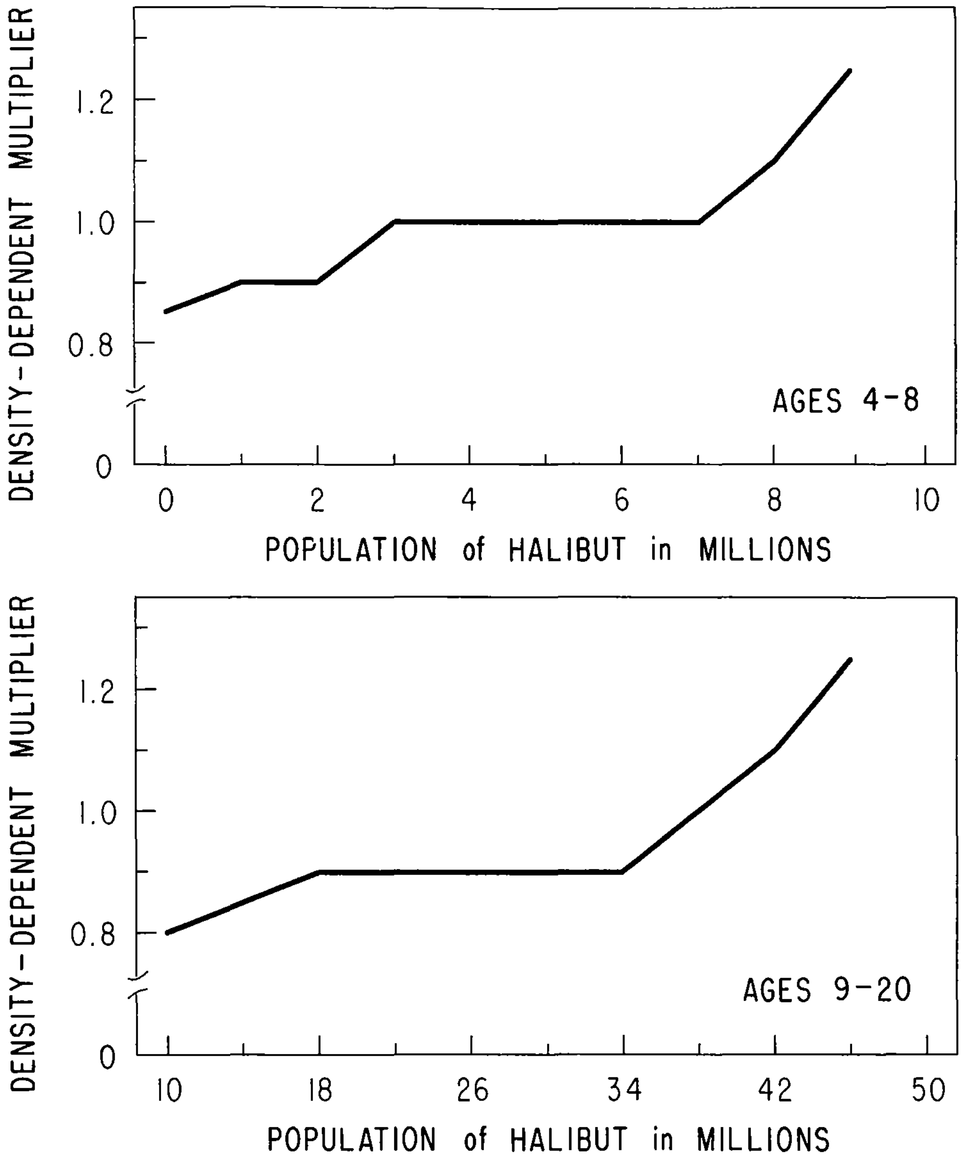

Figure 4. Density-dependent coefficients for age groups 4 to 8 and 9 to 20, which adjust the instantaneous natural mortality coefficients.

which is insufficient to indicate the true variability of the data, it was assumed that the error in the natural mortality coefficient was 10 per cent, that is, *sigma* is 0.02 in the random error function on survival.

Undoubtedly, over a large range of intermediate population sizes, the density of the population does not affect natural mortality, but at very small or very large population sizes, density dependence probably does become operative and natural mortality is respectively either decreased or increased. It is also assumed that the younger fish (ages 4 to 8 years) are responsive to a population density different from that of the older fish (ages 9 to 20 years). Such reaction to density is controlled through a table of multipliers ranging from 0.85 to 1.25 which are functions of numbers of fish in the population (Figure 4) and are applied to the instantaneous natural mortality coefficient. The coefficients for ages 9 to 20 years are adjusted so that over most of the population sizes encountered in this study natural mortality will be 0.18. Inasmuch as observation of the extreme population sizes indicated in Figure 4 have never been made, the values are entirely arbitrary; however, such checks are necessary to prevent the population from either becoming extinct or exploding beyond all reasonable size.

Spawner-Recruit Relationship

Throughout this study the number of age 4 halibut computed by a virtual population method (page 17) is accepted as the measure of recruitment. A plot of the virtual population of a year class, the number of age 4 fish, against the relative number of adults on Goose Islands grounds between ages 10 and 20 from which the year class was derived is given in Figure 5 for the year classes 1935 to 1953 (Table 1).

These data might be interpreted in one of two general ways: first, it might be postulated that recruitment has been essentially constant over a large range in stock size and that the variability around this constant recruitment has been great — the asymptotic spawner-recruit relationship; or secondly, the data might be interpreted

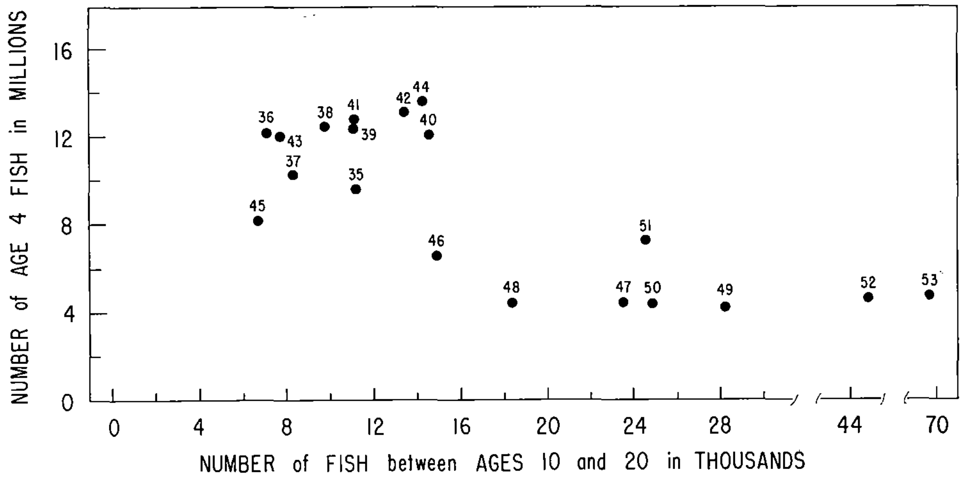

Figure 5. Relationship between the number of age 4 fish and the relative number of fish between ages 10 and 20 on Goose Islands grounds for the years 1935 to 1953. Points are identified by year-class. Classes 1947-1953 inclusive have six or more years of estimated data.

as showing a decline in recruitment beginning about 1945 and continuing until 1953 – the dome-shaped spawner-recruit relationship.

The basic pattern of the data is altered little by changing the measure of spawners from relative number of fish to catch per skate in weight of the same ages or to catch per skate in weight of the trade categories medium (10-60 pounds) and large (over 60 pounds). Similarly, substituting catch per skate in numbers of young fish as a measure of recruitment does not alter the apparent spawner-recruit relationship (Hardman, Mss.; Van Cleve and Seymour, 1953).

The lack of sensitivity of these data to changes in the type of measures of spawning stock indicates either that the spawner-recruit relationship is extremely stable or that some other factor or artifact in the data is producing the suggested relationships. All of the above data have two features in common. First, each data point is the result of a unique history of fishing and also reflects the effect of differential survival from year to year of the egg and larval stages (Thompson and Van Cleve, 1936). Secondly, the number of age 4 fish in the year classes is determined by a virtual population computation in which the commercial catch of each year class was summed.

To permit data covering as great a range as possible to be included in the computation it was necessary to estimate varying numbers of commercial catches for the early and recent year classes for which there have been no real observations. Thus, since a year class remains in the fishery for at least 17 years, with current data it is necessary to estimate the potential contribution to the catch of those age classes that have not yet been caught.

In the estimation, successive ratios of the catch at age i to the catch at age i+1 of the most recently observed catches of the year classes in the fishery were used. The appropriate ratio was multiplied by the last observed catch of the year class in question. The procedure was repeated, the last estimated catch being used, until estimates of all successive catches had been made. The year classes 1947-53 inclusive had six or more years of estimated data.

Bishop (1959) has shown that the estimate of total mortality is biased when it is derived from a ratio of the first two years of a virtual population computation. The ratios of catches described above are similar to the ratio described by Bishop. It can be shown that such a bias does exist in the virtual population computation and can be expressed as the logarithm of the ratio of rate of exploitation in year i+1 to the rate of exploitation in year i. If the length of the season increases or if the amount of fishing effort decreases throughout the period of estimation, the bias is increased.

Beginning in the late 1940's or early 1950's and continuing to date, the amount of fishing effort on Goose Islands grounds has decreased and the length of the season has increased. This action of the fleets introduces the bias, described above, into the virtual population computations. The data points for the years 1947 to 1953, each of which includes six or more estimated catches, all tend to be lower than these points which do not include the estimated values or which have less than six estimated values. These data points contribute to the descending right limb of the dome-shaped spawner-recruit curve (Figure 6). It can also be shown that for similar reasons the low values of virtual population for the years 1921 to 1925 contain the same bias.

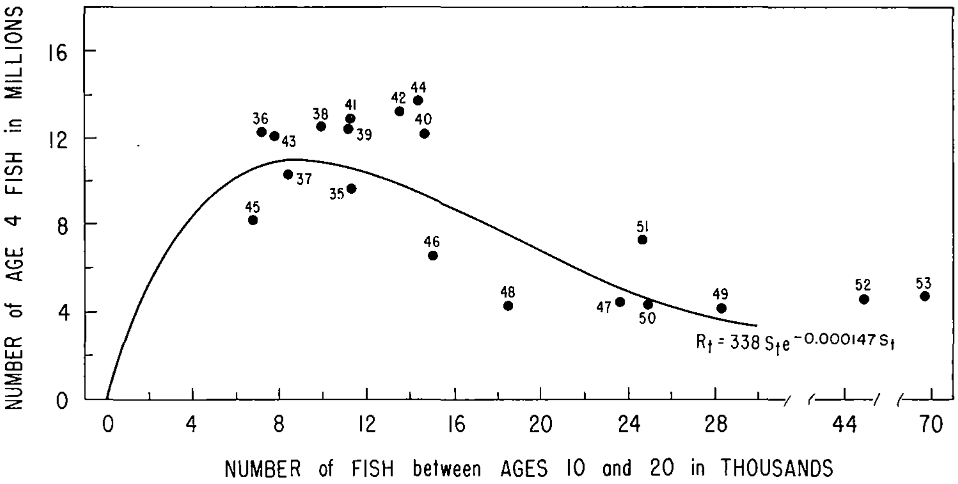

Figure 6. Fitted relationship between the relative number of age 4 fish and the number of fish between ages 10 and 20 on Goose Islands grounds during the period 1935 to 1951.

It is not clear at this time, however, whether the bias discussed above would account entirely for the descending right limb of the dome-shaped spawner-recruit curve. Consequently, in view of the possibility that recruitment has in fact declined in recent years, both the dome-shaped spawner-recruit relationship and the asymptotic relationship are considered as possible relationships in the analysis of management policies applied to the Area 2 population.

The number of age 4 fish and the number of fish between ages 10 and 20 for the years 1935 to 1951 were used to estimate the parameters a and b of the relationship (Ricker, 1958)

$$R_t = aS_t \exp(-bS_t) \tag{7}$$

where R is the number of age 4 fish and S the measure of spawners (Table 2 and Figure 6). Recruitment into the model from this dome-shaped relationship varied as the number of fish between ages 10 and 20 varied. It will be shown that unrealistic results are obtained when this relationship is used to estimate recruitment.

Table 2. Number of age 4 fish, as determined from the relative number of fish between ages 10 and 20.

Number of Spawners	Number of Recruits in 10,000's	Number of Spawners	Number of Recruits in 10,000's
			Equation A*
2,000	538	22,000	594
4,000	852	24,000	528
6,000	1,020	26,000	442
8,000	1,080	28,000	392
10,000	1,070	30,000	330
12,000	1,032	32,000	288
14,000	952	34,000	238
16,000	864	36,000	180
18,000	774	38,000	152
20,000	684	40,000	120

* Equation A: $R_t = 338 S_t e^{-0.0001147 S_t}$

Because the data points for the years 1936 to 1944 are clustered tightly (Figure 6), the asymptotic relationship

$$R_t = \frac{S_t}{a + bS_t} \tag{8}$$

where R and S are recruits and spawners respectively, did not estimate the asymptotic level of recruitment well. In lieu of the fitted relationship, the average of these points is assumed to be the asymptotic level of recruitment and the form of the relationship was determined graphically by trial and error. Adjustments were made in the slope of the curve so that over the range of observed stock sizes recruitment was essentially constant.

Growth Relationship

Average weight of each age group during any week in the year is represented by the Gompertz equation. This growth equation was chosen rather than the Bertalanffy equation since it is a more general form and it appeared frequently as the "best fit" in the data analysed in Southward and Champman (1965). The Gompertz equation is derived as the limiting form of the general growth equation:

$$\frac{dW}{dt} = nW^m - kW \tag{9}$$

where k and m are constants (Richards, 1959). Under certain limiting conditions (9) approaches the Gompertz equation:

$$W_t = W_\infty \exp[-b \exp(-kt)] \tag{10}$$

where b is related to the time scale and becomes unity when time is measured from the point of inflection and k is the rate of change in weight per unit time. The parameters b and k were determined from the data by trial and error. Even though a density-dependent growth relationship was programmed into the model, this feature was bypassed in this study and growth was assumed to be independent of density, notwithstanding its probable importance to the future management of the resource.

The Gompertz growth equation is solved in each time interval, giving a weight at each age. In this form, however, any variability in growth is not retained, that is, if a year class were to grow more than the average amount during a given period, the added growth would not be retained. To provide a more realistic statement of average growth, the average weight at age t is computed as follows:

$$W_{t+1} = W_t + \triangle W_{(t,\,t+1)} \tag{11}$$

where $\triangle W_{(t,\,t+1)}$ is the increment in weight between time t and t+1, and is obtained by subtracting the successive values of the Gompertz equation (10). Random error is introduced through additions of a random normal deviate, u_t. By arguments similar to those given under the discussion of population number Wt+1 becomes:

$$W_{t+1} = W_t + \triangle W_{(t,\,t+1)}(1 + u_t) \tag{12}$$

The correspondence between the computed weights at the 26th time interval and the average weights of fish taken from Goose Islands and Upper Hecate Strait grounds in Area 2 is illustrated in Figure 7. The higher actual weights at ages 4, 5 and 6 reflect the effects of gear selectivity.

The difference between the average weight by age of male and female halibut is great. Unfortunately, however, it is impossible to distinguish the sex of the eviscerated halibut in the commercial landings. Therefore average weight by age developed from samples of unknown sex composition which reflect the average of the landed catch have been used. Accordingly, no distinction is made in the simulation model for differences in average weight between male and female halibut.

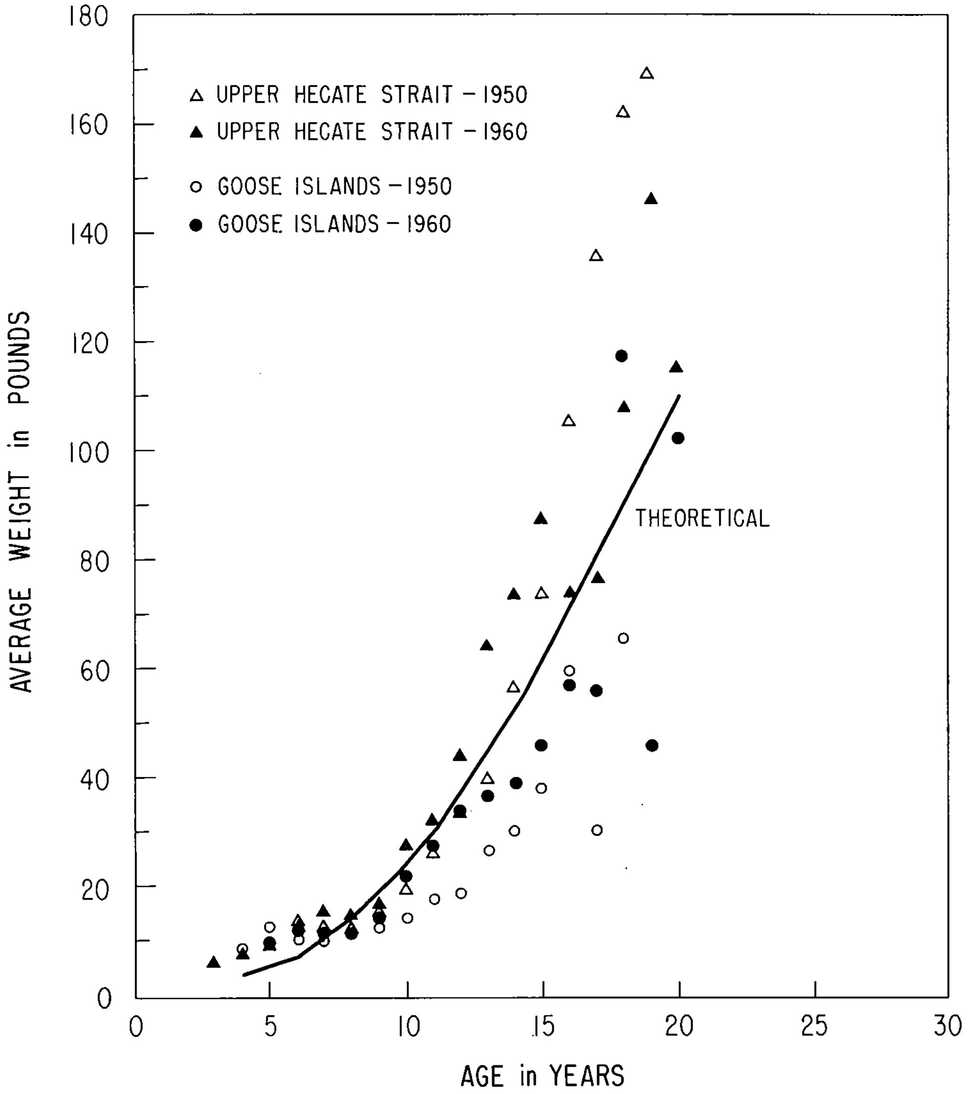

Figure 7. Actual average weight by age for Goose Islands grounds and Upper Hecate Strait grounds for years 1950 and 1960 and average weights calculated by the Gompertz growth equation for the 26th week.

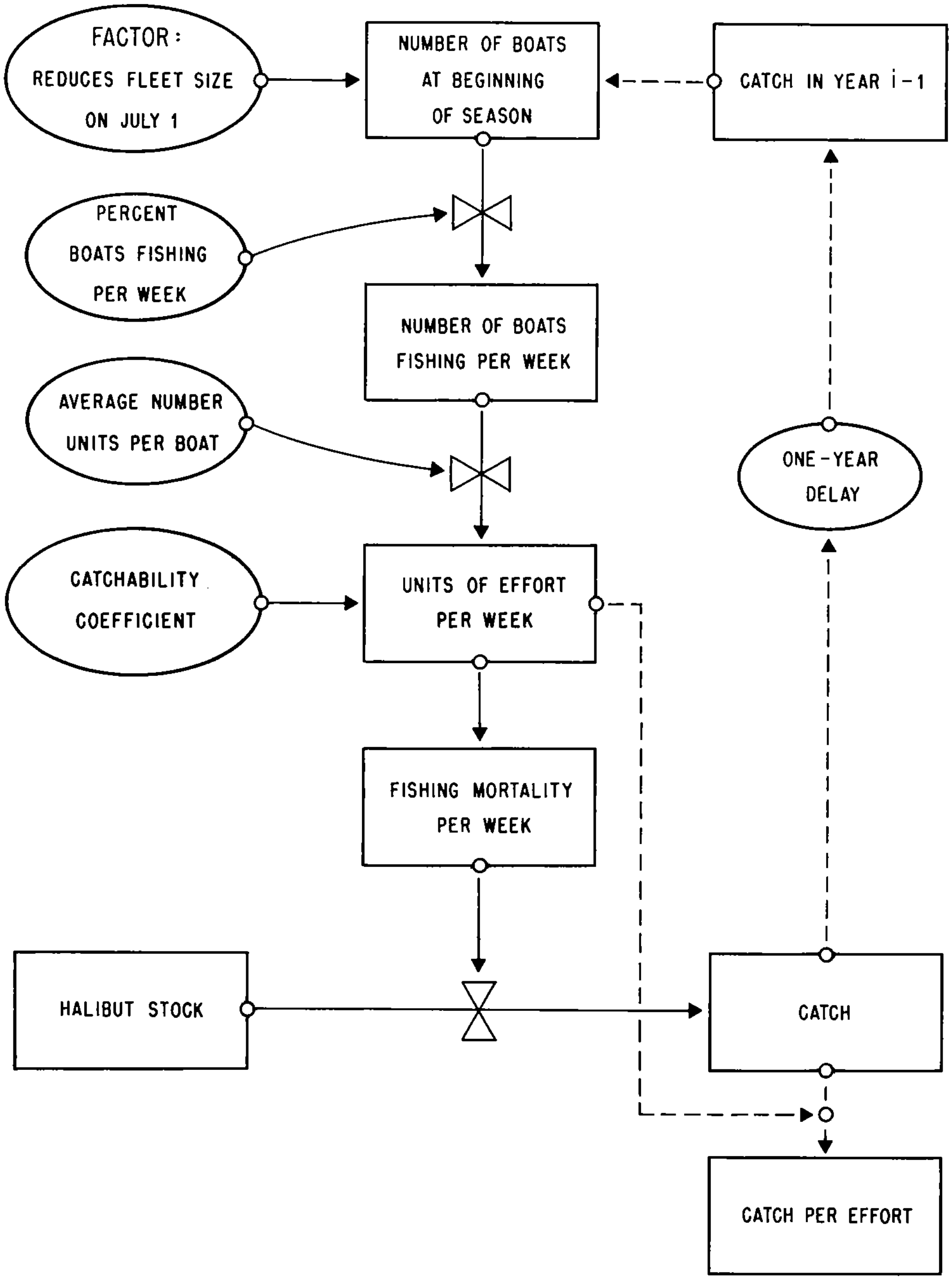

Figure 8. Diagram of the fishery section of the model.

FISHERY SECTION OF THE MODEL

The fishery section of the model simulates the United States and Canadian setline halibut fishery in regulatory Area 2. The amount of effort, the fishing mortality coefficients and the catch from the stock are computed in this section. These operations and the feedback of information connecting them are illustrated diagrammatically in the flow chart in Figure 8. The amount of effort, the number of skates fished in any week, is expressed as the product of the number of vessels on the grounds in the week and the average number of units of effort per week per vessel.

The catching power of the skate* is presumably unaffected by differences in vessel size or power, crew size or time spent in travelling to the fishing grounds. The main difference from one vessel to another results from the number of hooks, which will vary from approximately 75 to 120, the material of the ground line and the bait used. These differences are susceptible to standardization for statistical purposes (Thompson, Dunlop and Bell, 1931, page 28) and a standardized skate has been considered by the Commission as the unit of effort in the halibut fishery statistics.

Total effort as considered here is the number of standard skates fished in a stated period. Throughout this study the term "catch per skate" refers to real data of the halibut fishery and the term "catch per unit effort" refers to values computed by the model.

Fishing mortality coëfficients are determined from the amount of effort and the catchability coefficient, and the catch is the result of applying the fishing mortality to the stock. As is indicated in the diagram, catch is used in determining the catch per unit effort and is also delayed one year in order to provide a basis for determining the number of vessels entering the fleet in the following year.

Fleet Size

The number of vessels which enter the United States and Canadian setline fishery in any year is difficult to express by a simple rule. The decision of an owner to engage in a specified fishery is based on many factors, some of which are: the potential and relative economic attractiveness of the several regional fisheries available, the prospects for chartering the vessel later in the summer, the "carry-over" of the previous year's halibut production as it may affect price prospects, the availability of a crew, and undoubtedly, some personal reasons. All of these factors are extremely variable; many are intangible and none have been well studied. Crutchfield and Zellner (1962) studied some of the economic aspects of the Seattle and Ketchikan halibut fleets. Among other factors, they investigated vessel and crew earnings and estimated daily average prices in several ports. Their study, however, was aimed primarily at an analysis of the existing fishery rather than at the economic reasons which might influence an owner to participate in the halibut fishery. In a later section of this report, a relationship between the number of vessels entering the fishery in year i and the catch in year i—1 is developed.

*The skate (a piece of fishing gear usually 1800 feet in length and with hooks every 18 feet) is the basic unit of effort in the Pacific halibut fishery. The selective characteristics of this gear are such that halibut generally do not enter the catch before 4 years of age. Because of the great difference in size at each age due to different growth rates between the sexes as well as a great range in size of individuals of each sex, some members of each year class do not become fully vulnerable to the fishery until about 10 years of age, and, consequently, the entry of a year class into the fishery is spread over about 6 years.

A large proportion of the vessels which enter the Area 2 halibut fishery at the beginning of the season are primarily salmon fishing vessels which leave by about the first of July to enter the salmon fishery in British Columbia and Southeastern Alaskan waters before the catch limit is obtained. These vessels either engage in actual fishing for salmon, or charter as buying vessels or packers for the various fishing companies. In the Canadian fleet many of the vessels are owned entirely or at least in part by companies interested primarily in salmon and constitute a major portion of their salmon fleet. In addition to salmon fishing, some vessels enter the albacore tuna troll fishery and other pursuits.

The Halibut Commission has recorded the post-halibut season activities of each vessel which initially enters the halibut fishery each year, and these data provide a means of determining the number of vessels leaving the setline fishery by the first of July. The average number of vessels leaving the setline fishery in 1963 was used in this study. The data appear to be fairly consistent over the recent years.

In determining the number of vessels which will enter the setline fleet in any year economic factors must be considered. However, since the economic attractiveness of the different fisheries on the Pacific Coast have not been studied, and in view of the need for some measure determined by the model which reflects the economics of the Pacific halibut fishery, the landed halibut catch in the previous year is used to compute the entry of vessels in the fishery during the current year (Table 3, Figure 9).

The basis of the relation between the entry of vessels into the halibut fishery and the size of the catch in the previous year takes into account the fact that during the 12-year period from 1952 to 1963 the catch was for practical purposes not controlled by a fixed catch limit. During the period 1952 to 1960 there were additional non-catch limit seasons in Area 2. In 1961 and 1962 the season extended into early September and in 1963 the season was closed upon the statutory closing date without attainment of the catch limit.

The earnings per unit effort or the price per pound of halibut would be preferable in determining the entry of vessels in the fishery, but were not inherent information contained in the model. The magnitude of the landed catch in the previous year is available and in the real fishery is determined with precision; consequently it is accepted here for the period in question as representative of the economic attractiveness of setline fishing for the current year.

Table 3. Catch in million of pounds and number of vessels starting the Area 2 fishery the following year for the period 1952 to 1963.

Year	Catch in millions	Number of vessels	Year	Catch in millions	Number of vessels
1952	30.6	528	1958	30.6	417
1953	30.8	502	1959	30.6	378
1954	33.0	513	1960	30.8	335
1955	36.7	470	1961	31.8	345
1956	28.7	440	1962	28.9	365
1957	35.4	552	1963	28.7	353

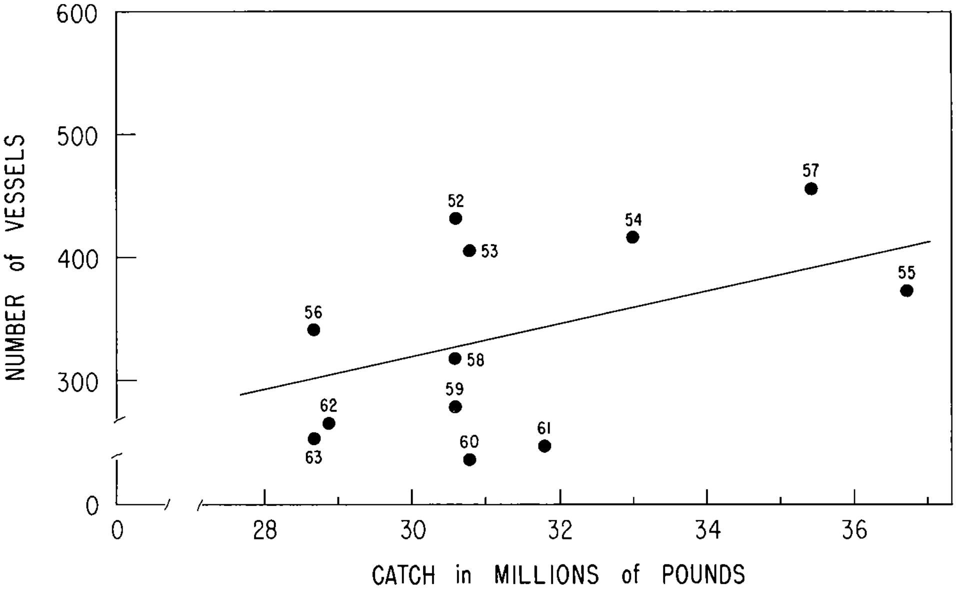

Figure 9. Number of vessels in year i regressed on the catch in year i-1.

A multiple regression using catch, the catch per unit effort of the previous year and the number of vessels in the last two years might give a better explanation of the entry pattern of vessels. Without doubt the entry of vessels into the setline fleet needs additional study from an economic viewpoint. It is also possible that the simple linear regression postulated has affected the behavior of the model; such a relationship might induce a cyclic reaction in the entry of vessels.

Calculation of Effort

As mentioned earlier, the number of United States and Canadian vessels on the grounds in any week varies periodically with the amplitude of the period, decreasing as the season progresses. This concept is expressed as a percentage of the vessels fishing each week (Figure 10). The percentages and timing approximate the behavior of the fleet in past years.

The number of vessels fishing each week is the product of the initial fleet size and the percentage fishing in any given week. The number of standardized skates fished each week is the product of a proportionality factor, *gamma,* and the number of vessels fishing. *Gamma* is the ratio of the average number of calculated skates during the first week of fishing and the average number of vessels during the period 1952 to 1963 (Table 3).

Catchability Coefficients

Catchability coefficients were estimated for each year from catch statistics of the Area 2 Pacific halibut fishery by the methods of DeLury (1947) and Leslie and Davis (1939). In each of these methods the slope of the line, the logarithm of catch per unit effort on cumulative effort in the case of the DeLury method and catch per unit effort on cumulative catch for the Leslie-Davis method, estimates the catchability coefficient. Weekly values of catch per skate were plotted. For a portion of each

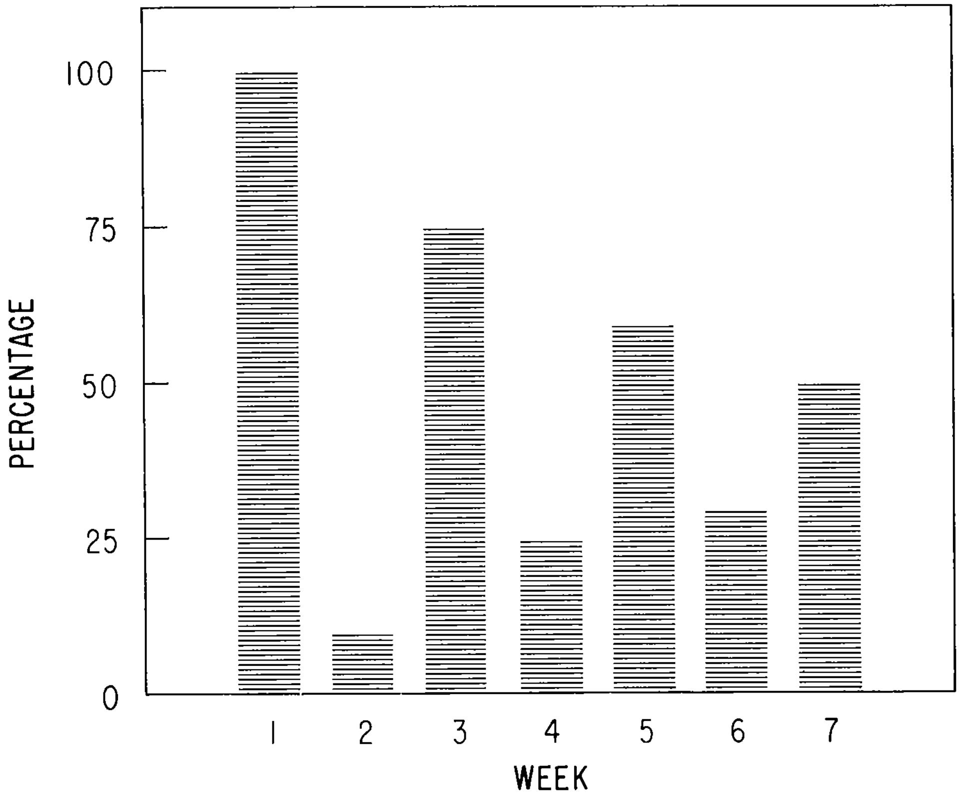

Figure 10. Percentage of available Area 2 fleet in the fishing grounds by week.

season the change in population size is due to removals by the fishery and the weekly values will lie on a straight line. Afterwards, the population changes size because of either immigration or emigration of fish as well as a result of fishing and the weekly values will no longer lie on a straight line. Thus, only those values which lie reasonably well in a straight line were used to estimate the coefficients. The estimates of catchability for the two methods are given in Table 4. In view of the similarity of the 14-year average values obtained by each method ($1.14\text{x}10^{-6}$ and $1.23\text{x}10^{-6}$), they were averaged and a value of $1.18\text{x}10^{-6}$ was taken as the estimate of catchability.

Table 4. Estimated catchability coefficients for the period 1950 to 1963.

	Delury Method	Leslie Method		Delury Method	Leslie Method
1950	$.749 \times 10^{-6}$	$.747 \times 10^{-6}$	1957	1.090×10^{-6}	1.164×10^{-6}
1951	$.687 \times 10^{-6}$	$.668 \times 10^{-6}$	1958	$.467 \times 10^{-6}$	$.465 \times 10^{-6}$
1952	1.288×10^{-6}	1.204×10^{-6}	1959	$.518 \times 10^{-6}$	$.603 \times 10^{-6}$
1953	1.148×10^{-6}	1.118×10^{-6}	1960	$.750 \times 10^{-6}$	$.666 \times 10^{-6}$
1954	$.287 \times 10^{-6}$	$.292 \times 10^{-6}$	1961	$.134 \times 10^{-6}$	$.136 \times 10^{-6}$
1955	1.211×10^{-6}	1.124×10^{-6}	1962	5.023×10^{-6}	5.386×10^{-6}
1956	$.328 \times 10^{-6}$	$.349 \times 10^{-6}$	1963	2.335×10^{-6}	3.062×10^{-6}
			Average	1.144×10^{-6}	1.213×10^{-6}

Calculation of Fishing Mortality Coefficients

Instantaneous fishing mortality coefficients are determined for the fully vulnerable ages from

$$F_t = qf_t \tag{13}$$

where q is the catchability coefficient and f the number of standardized units of gear.

Fishing mortality coefficients for each age between 4 to 9, those ages not fully vulnerable to the fishery, are expressed as the coefficient of the fully vulnerable ages times an age-of-entry factor. The method of obtaining the latter is described in Chapman, Myhre and Southward (1962): ". . . . the logarithms of the catch in numbers per unit effort for successive years are plotted against age. A straight line is fitted by eye to the right-hand side of the curve passing through the maximum point and extending to the youngest age. Successive differences between the curve and the extrapolated line are computed. The antilogarithm of any difference multiplied by 100 is the estimated percentage selection at that age." The only departure from this method was to fit the line by the method of least squares. In the above report a variable age of entry was considered; however, in this study a constant selection ogive empirically determined has been assumed.

Age composition data for the years 1950, 1953, 1956, 1959 and 1962 from the Goose Islands and Upper Hecate Strait grounds in Area 2 (Table 5) were averaged and form the bases for determining the fishing mortality coefficient for ages 4 to 9. The percentages of entry by age are as follows:

Age	Cumulative Percentage
4	1.0
5	5.0
6	9.0
7	22.0
8	54.0
9	85.0
10	100.0

Calculation of Catch

The rate of exploitation, u, is computed from F and M and is expressed as

$$u = \frac{F_t(1 - \exp[-(M + F_t)(\triangle t)])}{M + F_t} \tag{14}$$

where F is the fishing mortality coefficient and M is the natural mortality coefficient. These rates were expressed in weekly units. Age specific u's were computed for ages less than 10 years.

The weekly catch in numbers of fish is computed for each age in the population. It is the product of the rate of exploitation, the available population and a zero-one multiplier which designates whether or not the season is open or closed.

Table 5. Number of fish per 10,000 skates for the years 1950, 1953, 1956, 1959 and 1962 for the Goose Islands and Upper Hecate Strait grounds, used in estimating age of entry.

	Goose Islands grounds				
Age	1950	1953	1956	1959	1962
3					43
4	65	377	94	1,054	796
5	120	1,130	1,432	16,213	3,652
6	3,129	2,322	6,108	7,990	5,425
7	7,555	9,886	10,654	9,030	9,148
8	19,166	19,208	9,726	19,880	12,514
9	23,200	33,205	6,382	7,799	9,153
10	12,679	27,619	6,526	4,091	6,860
11	6,138	19,522	4,191	2,613	5,187
12	3,609	10,953	6,034	1,218	1,306
13	1,134	5,492	3,767	903	558
14	720	3,390	2,671	753	300

	Upper Hecate Strait grounds				
Age	1950	1953	1956	1959	1962
3					
4			918	854	946
5	264	246	4,791	6,582	3,866
6	709	2,194	10,195	5,324	2,883
7	3,233	5,079	10,594	5,863	4,579
8	6,813	6,755	10,262	10,851	5,795
9	13,444	13,904	8,545	6,728	5,286
10	8,066	12,253	11,419	4,055	4,485
11	5,328	10,034	5,364	2,336	4,754
12	3,134	7,495	6,349	1,078	2,053
13	1,600	5,572	3,780	798	859
14	1,188	1,750	2,036	764	415

MANAGEMENT SECTION OF THE MODEL

In order to regulate a fishery in a knowledgeable manner, a management agency must have, in addition to statistics of the catch, effort and length of season, an understanding of the life history of the species. Knowledge of the seasonal distribution of the fish within its geographical range, age of maturity, season and location of spawning, age of entry into the fishery, and age composition of the catch, are needed to understand the significance of changes in catch and effort statistics. An appreciation of the economics of the industry must also accompany the biological background. Often when two alternative ways of accomplishing a biological purpose are available, one will impose an economic hardship and the other will not, and good judgment would dictate the choice.

Since its inception the Halibut Commission has observed the stock reaction to fishing and has collected biological and statistical data to provide a background of knowledge upon which to base a scheme of regulation. Decisions regarding regulation are expressed in terms of catch and effort statistics, but the judgments regarding changes in the stocks are tempered by the background of knowledge of the species and economics of the fishery.

In the management section of the model, catch and the amount of effort expended are tabulated, and the catch per unit effort is computed. The age distribution of the catch is computed from periodic samples of the catch. These items of information become the basis for management decisions concerning the opening date of the coming season and the allowable removals.

The flow of information through the management section is shown in Figure 11. It starts with the management scheme or policy to be simulated. Statistics of catch and effort are utilized in each scheme but the distribution of ages in the catch may or may not be utilized depending upon the scheme chosen. In each, the decision-making process or rule requires a comparison or testing of data of the fishery with values estimated by the management scheme. The outcome of the comparison controls the adjustment of the catch limit the following year.

Through a feedback of information the self-adjusting model attempts to achieve a May 1 to September 1 fishing season. Initially the opening date of the fishing

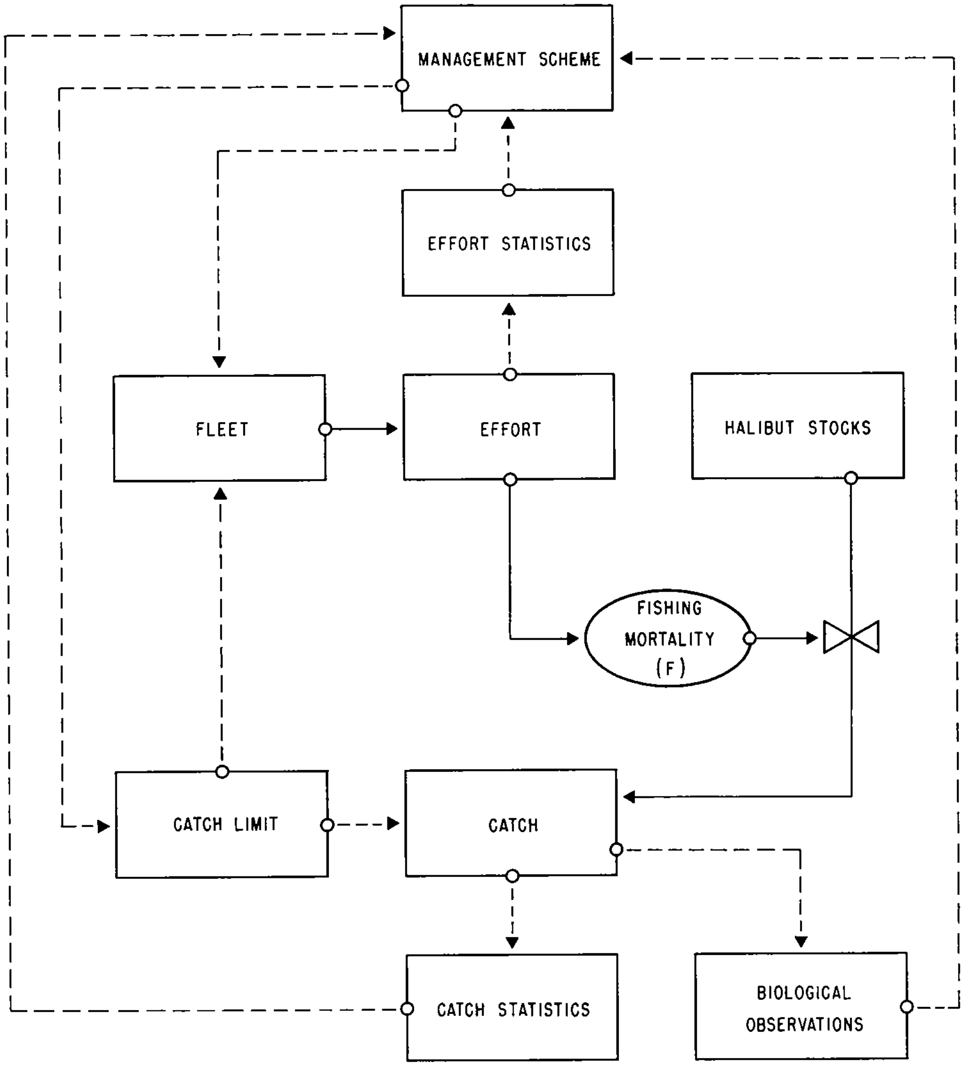

Figure 11. Diagram of the management section of the model.

season for year one is May 1; the opening date in any subsequent year i depends upon whether or not the catch limit has been obtained by the first of September in the previous year. If not, the opening date for the subsequent year is set one week earlier so that additional fishing intensity is applied to the stock. As soon as the catch limit is attained by September 1, the opening date for subsequent years is moved in weekly steps to the first of May.

Normally, the season is closed when the catch limit is reached; if, however, the catch limit is not taken by the first week in November, the season is automatically closed until the following year.

Throughout the season a comparison is made between the catch and the catch limit. As soon as the accumulated weekly catches equal or exceed the catch limit, the season is closed until the following year. The catch limit for the following year is adjusted, increased, decreased or not changed, depending upon the catch and the management decision rule.

Sampling the Catch for Age Composition Data

The age distributions resulting from the sampling by the Commission of the commercial landings are weighted by the annual catch per skate and the data are expressed in terms of abundance by age (Hardman and Southward, 1965). From these, estimates of growth and mortality rates as well as estimates of strengths of entering year classes are obtained. By necessity, in the real fishery sampling is by fishing grounds and any data from individual grounds must be weighted in some manner before it will represent a regulatory area. In the model this weighting procedure is not necessary since the model has no spatial characteristics.

In the model, the catch in weight and the catch in numbers are sampled at two-week intervals. The abundance by age is computed in the same manner as in the real fishery. The number of fish at each age in the sample of the season's catch is weighted by the ratio of the sum of the weights by age in the sample to the catch per unit for the season, resulting in abundance by age data.

Catch and Effort Statistics of the Simulated Fishery

Seasonal total catches and yearly effort values are stored for subsequent use.

Values of weekly catch per unit effort are computed and the cumulative average catch per unit effort, which is the total catch divided by the total effort, is computed and stored.

Management Schemes

Management of a fishery by a regulatory agency depends upon decisions which control the amount of fishing and hence the fishing mortality in response to population reaction to previous fishing. These may be formally stated as decision rules or they may, as is usually the case, be followed in an intuitive manner. Three different management schemes, each of which requires different information and is based on a different analysis, were incorporated in the model. The analyses are:

(1) empirical analysis

(2) potential-yield curve analysis

(3) yield-per-recruitment analysis

Empirical Analysis

In the empirical analysis no attempt is made to define maximum sustainable yield; rather the strategy is to regulate so that when the catch per unit effort is high and the abundance of the classes entering the fishery is high, removals are gradually increased. As the stock responds to the removals and abundance falls, or if a series of weak year classes enters the fishery, removals are gradually reduced. This analysis is based on a combination of trends over the last 5 years in (1) catch per unit effort, (2) number of fish 12 years old and older and (3) the number of fish between 6 and 11 years of age; it requires catch and effort statistics as well as knowledge of the age distribution in the catches. The decision rule based on the combination of these three trends is summarized in Table 6.

Table 6. Decision rule for the empirical analysis.

DIRECTION OF TREND			DECISION
Catch per unit effort	Number of age 6-11 fish	Number of age 12-20 fish	
+	+	+	increase limit 1 million pounds
+	+	—	no change
+	—	+	
—	+	+	
—	—	+	
—	+	—	
+	—	—	
—	—	—	decrease limit 1 million pounds

The amount of the catch-limit change is specified at the beginning of each simulation or "computer run" and can be varied from run to run if desired. A change of one million pounds, which is approximately 3 percent of the maximum equilibrium yield, was used since changes actually made in the limits have usually been of that amount. The Commission's purpose in making one-million-pound changes is twofold: (1) experience has shown that the catch per unit effort in the fishery will increase or decrease in response to changes in removals of this magnitude; (2) a succession of such small alterations in the catch limit over a period of years is less disruptive to the industry than large changes in single years.

Potential-Yield Curve Analysis

The equilibrium yield function described by Schaefer (1954, 1957) was next considered as a basis for a management scheme. In this approach to management an estimate of maximum sustainable yield and the associated optimum stock size are obtained from statistics of fishing effort and catch, as well as an estimate of either present potential yield or stock size.

The Schaefer analysis is based on the assumption that the rate of population change can be represented by the equation

$$\frac{dP}{dt} = k_1P(L-P)-k_2FP \tag{15}$$

where k_1 is the rate of population increase, k_2 is the catchability coefficient, L the maximum population size, F is fishing effort, and P is the current population size.

Further, it is assumed that at level P_i in year i, equilibrium yield, Y_e is estimated by $\triangle P +$ Catch, and that

$$\triangle P = \frac{\overline{P}_{t+1} - \overline{P}_{t-1}}{2} = \frac{C_{t+1}/F_{t+1} - C_{t-1}/F_{t-1}}{2} \qquad (16)$$

where C is catch. To use these equations it is necessary to relate P and u, catch per unit effort, that is

$$\overline{P} = k_2 u \qquad (17)$$

If P in equation 15 is replaced by $\overline{P}$, then all three parameters k_1, k_2 and L can be estimated from a series of data on catch and catch per unit effort. This 1957 procedure of Schaefer's was first tried as a basis for a decision rule.

Initially a 15-year series of data was divided into three equal parts, that is, 1 to 5, 6 to 10, and 11 to 15 years. The three parameters were estimated from the three sets of data by solving the simultaneous equations of the form

$$\frac{1}{n_i} \sum_{j=1}^{n_i} \triangle u_i = k_1 L \sum_{j=1}^{n_i} \frac{\overline{u}_j}{n_i} - \frac{k_1}{k_2} \sum_{j=1}^{n_i} \frac{\overline{u}_j^{\,2}}{n_i} - k_2 \sum_{j=1}^{n_i} \frac{f_{jt}}{n_i} \qquad (18)$$

where k_1, k_2 and L are parameters, $\triangle u_i$ is the change in catch per unit effort, $\overline{u_i}$ is the average catch per unit effort, $\overline{u}_i^2$ is the average catch per unit effort squared, f_i the number of units of effort and n_i the length of the period in years.

In this analysis average values of catch per unit effort and effort were used, since earlier work by the Commission (IPHC, No. 28) dealing with the estimation of fishing mortality using this method had given estimates much in excess of the total mortality rate as determined from age composition. It was expected that the average values would produce more meaningful results than earlier attempts to use the logistic model. However, the successive estimates of maximum stock size were absurdly low and the signs of all parameters alternated in a meaningless manner from positive to negative.

Since Schaefer's 1957 model did not produce a workable scheme, the estimation procedure outlined in Schaefer (1954) was next considered. Maximum stock size is estimated from the intercept of the regression of equilibrium yield per unit effort on effort. This estimation procedure is more straightforward than the one for the three parameters but requires an additional estimate of the catchability, q.

In using Schaefer's 1954 procedure as a management rule the optimum stock is expressed in terms of catch per unit effort multiplied by the factor 0.8635 (Chapman, Myhre and Southward, 1962, page 16), and the observed catch per unit effort is compared to it. If the observed catch per unit effort is larger than the optimum, the implication is that the stock is too large and, therefore, the catch limit is increased to reduce the stock. If the observed value is smaller than the optimum, the implication is that the stock is too small and the catch limit is reduced.

As will be shown later, Schaefer's 1954 logistic population model also failed to give meaningful results in this simulation study. Reasons for this failure will be discussed under the results of simulation runs. An alternative formulation of the potential-yield curve analysis is now given.

Schaefer (1957) states that the "simplest assumption we can make about the form of $\overline{f(P)}$ is that it is linear with $\overline{P}$ . . ." The next stage of complexity would be to consider an exponential function,

$$Y_e = a\,P\,\exp(-bP) \tag{19}$$

where a and b are constants, in place of the above mentioned linear function. Such a treatment results in a curve which agrees with the skewed appearance of the true equilibrium curve (page 69). This function was used by Ricker (1958) to represent a spawner-recruit relationship; however, it is not used here in that context.

Differentiating this expression with respect to population size, P, results in an estimate of maximum sustainable yield of

$$Y_{max} = \frac{a}{be}, \tag{20}$$

Estimates of a and b can be obtained from the data by making the usual assumption that the catch per unit effort is proportional to stock size:

$$u = qP \tag{21}$$

Substituting and taking logarithms of both sides give

$$\ln\left(\frac{Y_e}{u}\right) = \ln\left(\frac{a}{q}\right) - \left(\frac{b}{q}\right)u \tag{22}$$

which is a linear equation in equilibrium yield and catch per unit effort.

In using this approach it is assumed that an estimate of the catchability coefficient, q, is available. The constants a and b are estimated from the least-squares equation (22). An estimate of the optimum catch per unit effort is obtained through a Newton-Raphson iteration.

The management decision rule is based on the estimates of optimum stock size and maximum equilibrium yield. Lower and upper limits were set to these estimates so that minor changes in the variables would not cause a change in regulation. These limits are considered fixed in this study, but may be changed from one run to another. In the following table (Table 7) which summarizes the management decisions, the plus and minus signs indicate an increase or decrease, respectively, of one million pounds; the zero indicates no change in catch limit.

Table 7. Summary of decision rule based on the potential-yield curve analysis.*

Catch	Catch Per Unit Effort		
	Less than 95% of optimum CPU	Within ±5% of optimum CPU	Greater than 105% of optimum CPU
Less than 95% of maximum sustainable yield	—	+	+
Within ±5% of maximum sustainable yield	—	0	+
Greater than 105% of maximum sustainable yield	—	—	+

*Plus and minus signs refer to a 1,000,000-pound increase or decrease respectively in the catch limit; 0 indicates no change.

To reach a decision to change the catch limit, the observed catch and catch per unit effort are compared with the estimated maximum sustainable yield and the optimum catch per unit effort. If, for example, the catch of the year was five per cent less than the estimated maximum sustainable yield and at the same time the catch per unit effort was five per cent less than the estimated optimum catch per unit effort, i.e., the catch per unit effort associated with the maximum sustainable yield, then the catch limit for the coming season would be reduced by one million pounds.

Yield-Per-Recruitment Analysis

In the analysis based on yield-per-recruitment calculated from vital rates (Ricker, 1958), equilibrium yield is expressed as

$$Y_e = \sum_{t=4}^{t=20} \frac{F_t \, W_{(4)t} \, [1 + e^{(g-Z)_t}]}{2} \qquad (23)$$

In analyzing a real fishery these rates would be estimated from age and growth studies based on samples of the commercial catch taken at the time of landing and at sea as well as tagging studies. The usefulness of the yield-per-recruitment model

Table 8. Actual and computed catch* for regulatory Area 2 by year for the period 1922 to 1951.

		Computed catch in million of pounds		
Year	Actual catch in millions of pounds	Asymptotic recruitment function		Dome-shaped recruitment function
1922	30.5	60.5		60.4
1923	28.0	49.6		49.2
1924	26.2	38.4		38.0
1925	22.6	30.3		29.8
1926	24.7	30.1		29.5
1927	22.9	28.1		29.1
1928	25.4	30.7		34.9
1929	24.6	32.2		40.9
1930	21.4	29.6		41.1
1931	21.6	25.0		36.8
1932	22.0	22.1		32.9
1933	22.5	23.4		34.3
1934	22.6	23.5		33.6
1935	22.8	22.5		31.5
1936	24.9	29.1		40.3
1937	26.0	27.1		37.8
1938	25.0	23.5		33.8
1939	27.4	29.6		44.0
1940	27.6	28.1		43.0
1941	26.0	27.0		42.0
1942	24.3	24.5		38.5
1943	25.3	23.5		37.1
1944	26.5	22.7	22.8**	36.0
1945	24.4	23.3	23.8	37.0
1946	29.7	27.9	28.7	41.2
1947	28.7	26.6	29.2	42.0
1948	28.4	25.4	30.7	39.7
1949	26.9	24.9	30.2	38.6
1950	27.0	24.4	28.3	36.7
1951	30.6	28.2	31.0	41.1

*Catches determined from asymptotic and dome-shaped spawner-recruit functions.

**Catches in this column result from doubling the recruitment of 4-year-old fish in 1941.

in describing the reaction of the stocks to fishing greatly depends upon how well the estimated rates reflect the true rates in the population. There are two sources of variability in the estimation of these rates. The first is the variability inherent in the population, and the second the variation due to the sampling and estimating procedures. In this study no attempt has been made to simulate individually the various sampling and estimating procedures; rather, basic population rates to which error terms have been added are assumed. Thus, the vital rates of the biology section have had an error term added to them and have been used to compute equilibrium yield. Recruitment in the Ricker analysis is considered to be knife-edged at age 8.

The same decision rule, a simultaneous consideration of the estimates of optimum stock size and maximum equilibrium yield, used in the potential-yield curve analysis is used in this analysis (see Table 8).

A yield-per-recruitment approach to management of the halibut resource requires more information than the other two decision rules and would be more expensive to implement. However, from a mathematical as well as a biological viewpoint the assumptions appear to be realistic. The fact that the seasonal aspect of the Pacific Coast halibut fishery can be duplicated by this means of analysis lends realism to such an analysis. Also, the yield-per-recruitment approach treats biological information such as growth in a more flexible manner than the potential-yield-curve approach, thereby allowing for the incorporation of changes that might occur in the life of a long-lived species.

RESULTS OF SIMULATION

Deterministic Version of the Simulation Model

The simulation model was initially treated as a deterministic model by setting the error terms to zero. Since the simulation model is complex and some of the relationships described are gross approximations, one means of validating the model was to hindcast a historical series of data. Yearly values of known effort were entered into the model and the computed series of catch was compared with the corresponding series of actual catch. After this test, equilibrium yield was determined for a set of fixed conditions and, finally, the model was controlled by one of three management schemes; namely, the empirical analysis, the potential-yield analysis and the yield-per-recruitment analysis. The maximum equilibrium yield and the associated optimum catch per unit effort served as criteria in making adjustments in removals depending on the management scheme, the aim being to bring the stock to the size producing the maximum equilibrium yield and to maintain it there.

Validation of the Model

Effort data beginning with 1922 and the empirically fitted asymptotic spawner-recruit relationship were entered in the model for the hindcast of the series of catch records from Area 2. It also would have been desirable to include the age composition of the stock on the grounds in 1922, but since such data were not available, the composition of one-half the number of fish computed to be on the grounds in 1950 was arbitrarily taken as a starting point. It takes approximately 10 years before the effects of the initial condition become negligible; therefore, the comparison between theoretical and actual values in Figure 12 (Table 8) is more meaningful for the period subsequent to 1932.

Even though the initial conditions influenced the behavior of the system prior to 1932, the trends in catch from 1926 to 1932 are nearly the same, though at a much higher level. By 1932, the magnitude of the calculated catch is nearly that of the actual catch. From 1932 to 1942 the actual catch is well represented by the calculated catch; in some years the computed removals are above the actual catch and in some below it, but the major trends are reproduced. In 1943 and particularly in 1944 the two lines are noticeably different, the actual catch goes up and the calculated catch falls. After 1944, the trends are again nearly the same; however, the calculated catch is consistently below the actual catch for the remainder of the period.

It is noteworthy that in 1943 and 1944 the catch per skate of the trade category of chicken halibut* for the grounds in Southeastern Alaska increased sharply (Bell, Mss), suggesting an increase in recruitment due to a strongly entering year class or classes. Chicken halibut are usually six or seven years of age, so it is possible that the higher yield of the actual catch could be due to the contribution of the same strong year class which caused the high abundance of chicken halibut. If it is assumed that they were six years old in 1944, this class would be only 13 years old by 1951 and still would be contributing significantly to the catch. The effect of the increase in recruitment was simulated with the model by doubling the recruitment of 4-year-old fish in 1941. The catch with the simulated increase in recruitment is shown in Figure 12 as the dotted line.

*Halibut which weigh at least 5 pounds but less than 10 pounds with the head off and eviscerated.

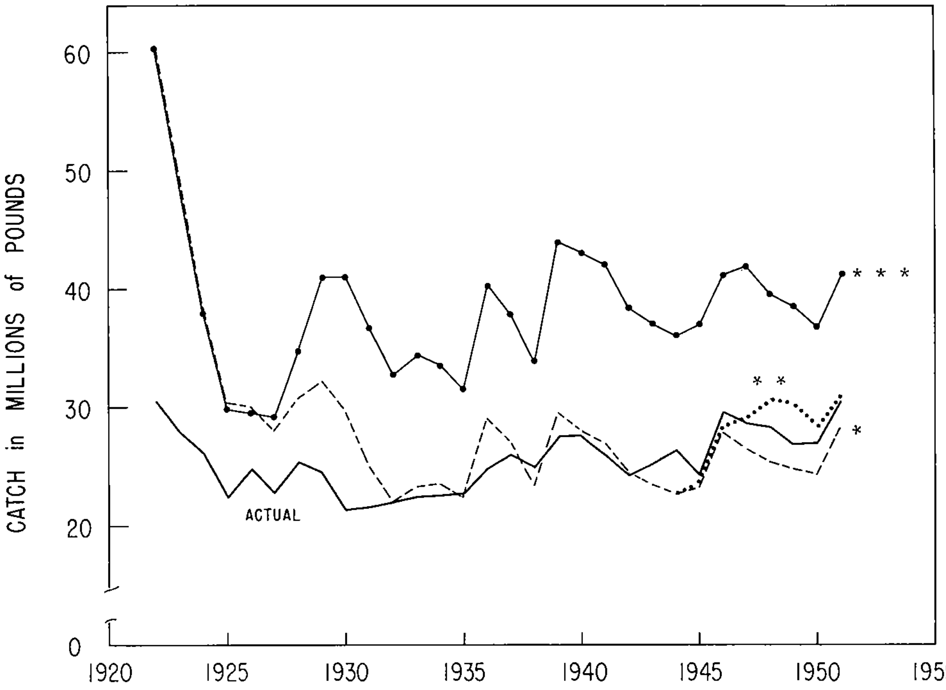

Figure 12. Actual and computed catches for regulatory Area 2 by year for the period 1922 to 1951.
*Recruitment computed by an asymptotic spawner recruit relationship.
**Recruitment doubled in 1941 only.
***Recruitment computed by a dome-shaped spawner recruit relationship.

The dome-shaped spawner-recruit relationship fitted to the data for the years 1935 to 1951 was next assumed. As mentioned on page 17 the data given in Figure 6 were fitted by (7). The fitted spawner-recruit curve rose rapidly, reaching a maximum of about 10.5 million age 4 fish at a parent stock size of 8000 spawners. The declining right-hand limb approaches zero not greatly beyond the largest observed stock size. The catches derived from this curve are also shown in Figure 12. They are represented by the upper line, the level of which is about one-third above the actual catches. When the details of this run were examined, most of the stock sizes encountered were in the area of the maximum of the spawner-recruit curve. It is obvious that recruitment of this magnitude results in more yield than actually was taken. When the two most recent data points are included the shape of the spawner-recruit curve is radically affected; the maximum is shifted considerably to the right. It is apparent that the estimation of the dome-shaped spawner-recruit function is not adequate to describe halibut recruitment and no additional study is made using such a function.

Equilibrium Yield

In determining yield under equilibrium conditions, natural mortality was set at 20 percent annually, growth was as described above, the asymptotic spawner-recruit relationship (see page 18) was assumed and fishing effort and length of season were initially set at 263,000 units and seven weeks, respectively, the averages of the ten-year period 1951 to 1960. Effort was subsequently changed to 50, 70, 150, 200, 300 and 400 per cent of the 10-year average and the model was run for a twenty-five year period in each case. Equilibrium was usually reached in about fifteen years. The units of effort, catch, catch per unit effort, biomass and the instantaneous fishing mortality coefficient under equilibrium conditions for the twenty-five year period are given in Table 9.

Maximum equilibrium yield for the conditions outlined was 30.8 million pounds. In spite of great changes in fishing mortality (approximately 5 times increase from the 131,000 unit level of effort to the 525,000 unit level), the equilibrium yield changed only 15 percent. The population biomass-yield curve is almost symmetrical, with a fairly flat top (Figure 13). While there is a slight tendency for the curve to be skewed to the left, this may be caused by the estimate of biomass when fishing mortality is zero, which is perhaps a doubtful extrapolation. The values shown in

Table 9. Effort, catch, catch per unit effort, biomass and fishing mortality coefficient determined under equilibrium conditions.

Fishing Mortality		Equilibrium	Catch per	Biomass	
Thousand of skates	Coefficient F	Catch in millions of pounds	unit effort in pounds	Total ages 4-20	Commercial stock ages 8-20
0	0	0	0	378	0
131	0.15	26.8	204	252	170
187	0.22	29.7	159	212	131
263	0.31	30.8	117	174	94
394	0.46	30.6	78	138	60
525	0.62	28.3	54	112	40
788	0.93	5.0	6	15	4
1,053	1.24	0.03	0.02	0.05	0.02

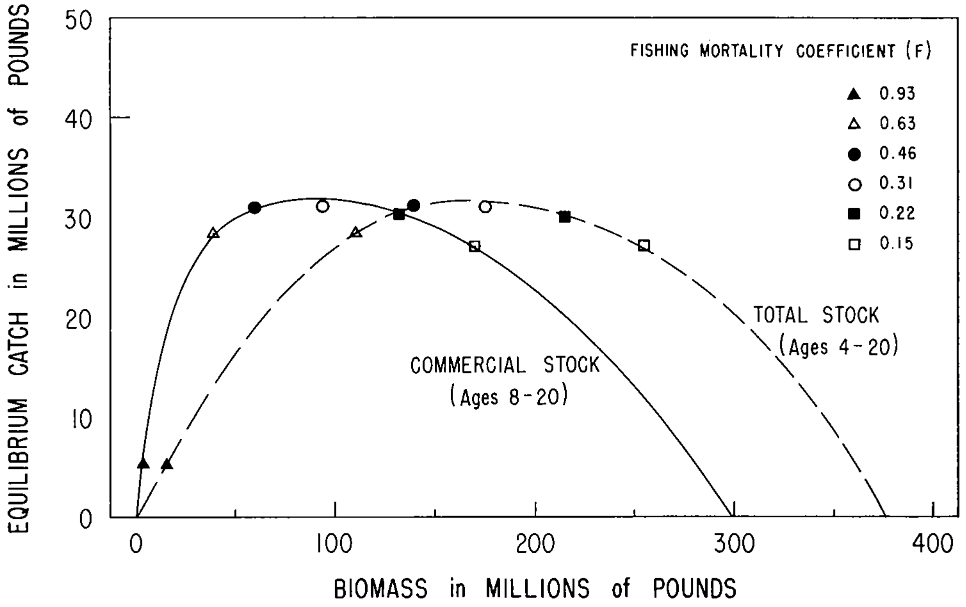

Figure 13. Relationship between catch and biomass under equilibrium conditions.

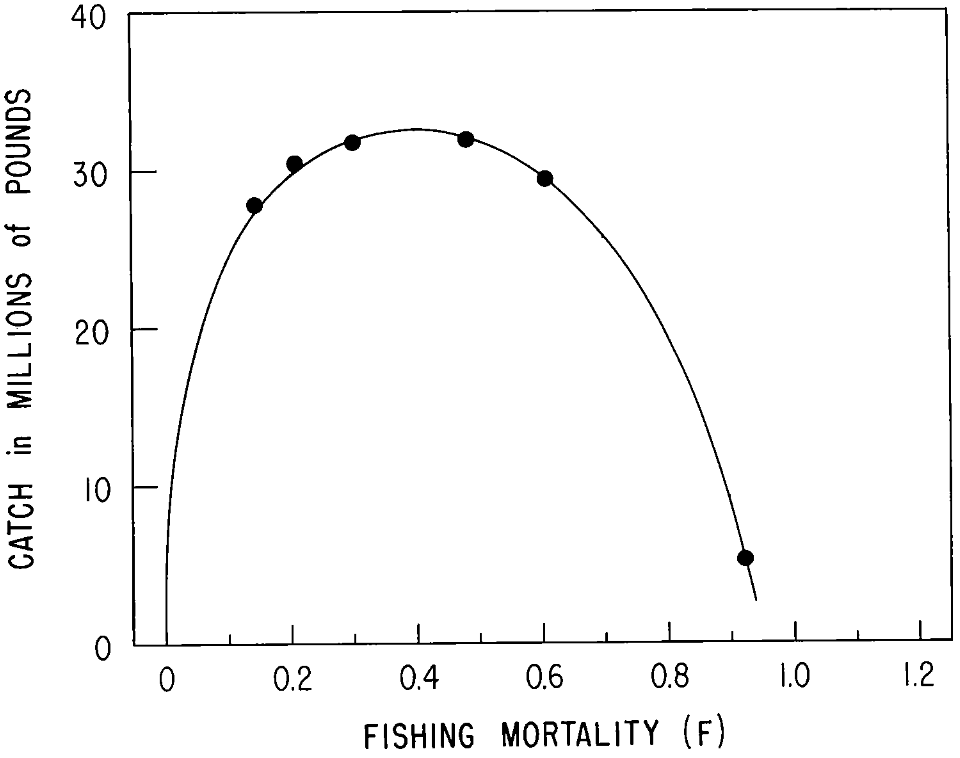

Figure 14. Catch as a function of fishing mortality under equilibrium conditions.

Figure 13 were computed on the assumption of a density-dependent natural mortality relationship. Since natural mortality of an unfished stock of halibut has never been estimated, the postulated relationship may not adequately describe the change in natural mortality with the change in stock size; the effect of such could be overestimation of the biomass, which would result in a skewed population curve.

The skewing becomes more pronounced as the definition of total biomass becomes smaller, that is, if the biomass of the commercial ages is considered the curve is severely skewed. The reason for this is not immediately apparent, but undoubtedly it is related to changes in age composition of the catch. When only the commercial ages are considered, the relative contribution of the younger ages in the catch is more important at high levels of fishing intensity.

Equilibrium catch is shown in Figure 14 as a function of instantaneous fishing mortality, F. At values of F ranging from 0.31 to 0.62, the yield changed less than ten percent.

Catch per unit effort is linearly related to biomass over most of the range of biomass (Figure 15). Somewhere below a biomass of 112 million pounds the relationship ceases to be linear; however until additional points lying between 15 and 112 million pounds are computed, the lower limit of the linear relationship between catch per unit effort and stock size cannot be ascertained.

Catch per unit effort is shown as a function of effort in Figure 16; the decline is not linear even though the first three points (effort ranging from 131 to 263 thousand units) appear to lie on a straight line. In addition, between values of effort

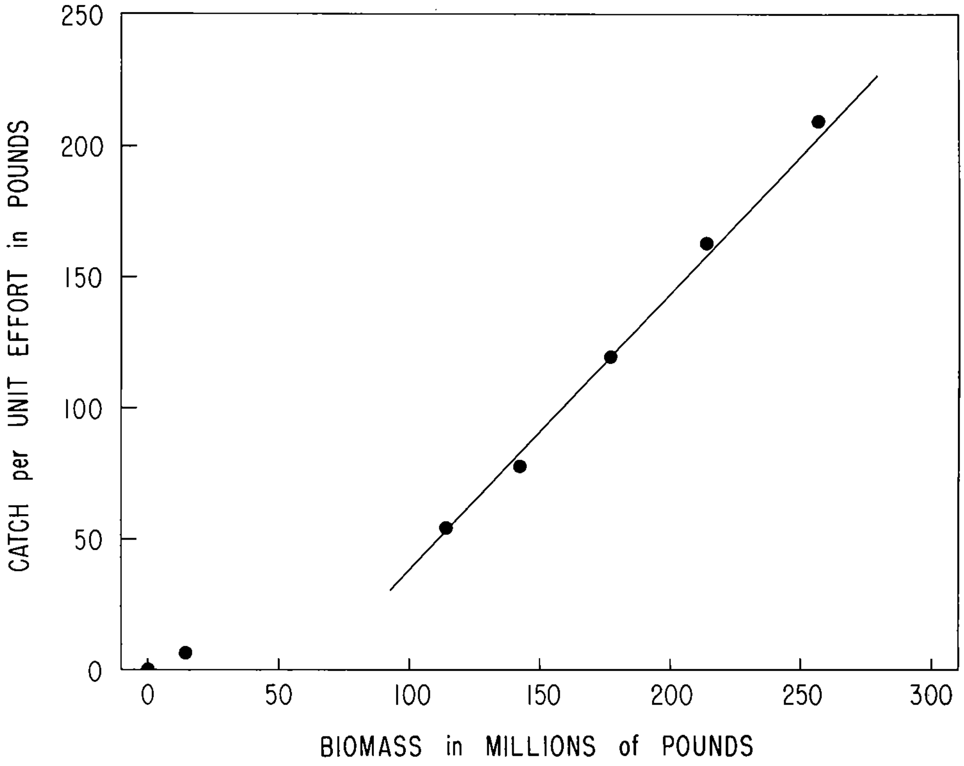

Figure 15. Catch per unit effort as a function of biomass.

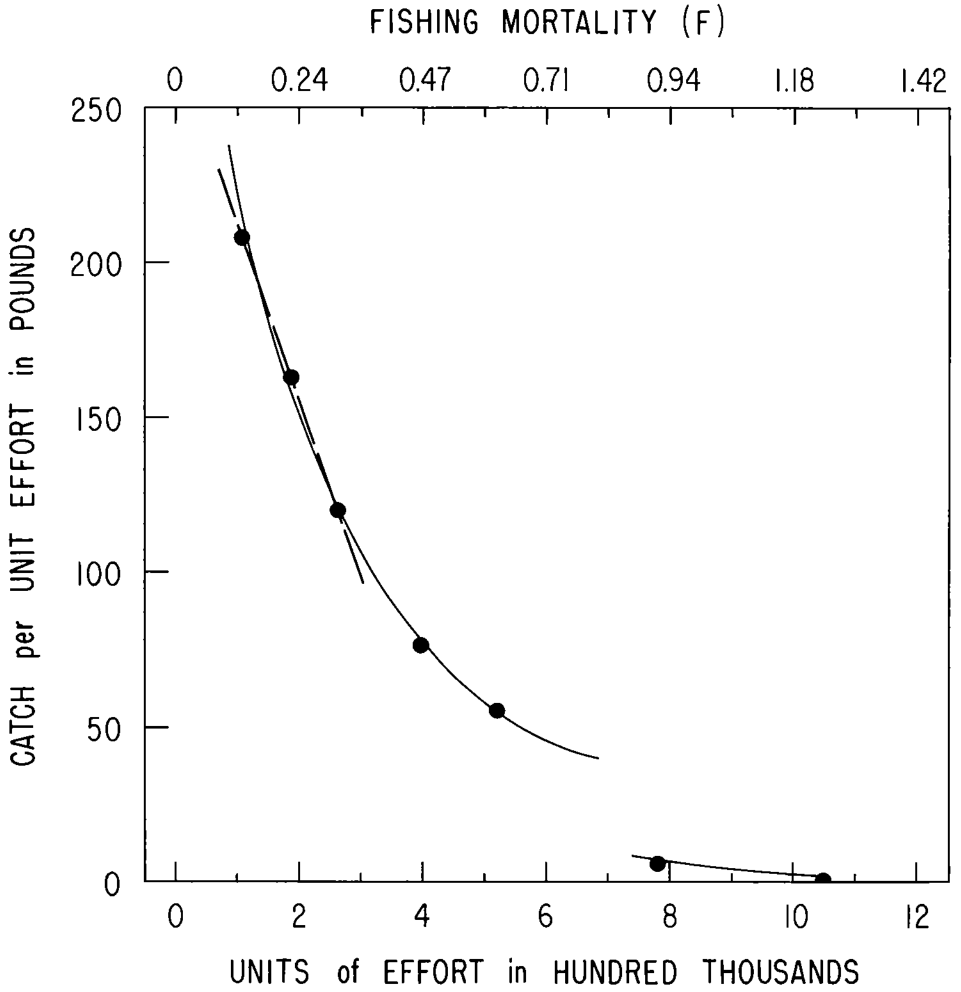

Figure 16. Catch per unit effort as a function of effort under equilibrium conditions.

ranging from 525 to 788 thousand units there appears to be a discontinuity in the relationship, or at least a sharp change.

The final form of the model management decision rule* depends on an exponential equilibrium yield function. It was developed from knowledge that the true-yield curve, that is, potential yield as a function of population size, was skewed. Over most of the range of population sizes the true-yield curve, including the tendency to be skewed, should be described reasonably well by the exponential model of the decision rule. However, at large population sizes the description will not be adequate. In the true-yield curve, yields to the right of the maximum declined as the population increased and ultimately reached zero. This is not the case with decision Rule 2; the yields approach zero asymptotically as the population becomes infinitely large. The asymptotic behavior of the function represents a deficiency in the theory, but should not appreciably affect the results over the population sizes encountered.

*See Rule 2, page 45.

Management Decision Schemes

The 1953 Halibut Convention stipulates that the International Pacific Halibut Commission regulate the Pacific halibut fishery so as ". . . to develop the stocks of halibut in the Convention waters to those levels which will permit the maximum sustained yield and to maintain the stocks at those levels . . .". In order to accomplish the purposes stipulated in the Convention, the Halibut Commission has designed its regulations so that the fishery begins essentially on the first week of May and lasts into early September. This is the period of highest productivity historically for grounds in Area 2 (IPHC, 1951, page 14). From time to time the season is opened as late as mid-May. Because a large part of the fleet leaves the halibut fishery near the first of July to participate in the salmon and tuna fisheries, the fishing power of the fleets would be increased by opening the season earlier than the first of May, but would probably close earlier than the optimum time of early September.

The fishermen's unions and the vessel owners' associations in the various ports have agreed to a voluntary program whereby the vessels remain in port for a period of 8 days following the completion of a trip. The purpose of this program is ". . . to provide for some extension of the fishing season by establishing rest periods or lay ups and for more orderly delivery of the overall halibut production." While the Commission is not involved in any manner with the "lay-in" program, the result has contributed without doubt to the lengthening of the season and has furthered the objective of the Commission to secure a May to early September fishing season.

The management decisions in the simulation model have been written to simulate the above design of regulation. Initially, in the model the season opens on the first week in May, the catch is checked on the first week in September and if the catch limit has not been attained the opening of the season in the following year is moved one week earlier. Conversely, if the limit is attained prior to the first week of September the opening date is set back in steps of one week until the first week in May is reached again. No attempt has been made to simulate the details of the real opening date since the date in Area 2 is related to that in other regulatory areas, a procedure necessary for a balanced fishery.

Empirical Analysis

The empirical analysis of the simulation model adjusts the catch limit yearly depending upon a combination of 5-year trends in catch per unit effort, in number of fish less than 12 years of age and in number of fish 12 years old and older. In the initial formulation of the decision rule the effects of varying the contribution of each of the trends were considered, but in the final form of the rule the three trends were given equal weight in the decision process. In addition to stipulations about the data the rule includes a feature which, by testing removals when stabilization is suspected, prevents the stock from stabilizing at an optimum level.

The empirical management scheme, as is the case with the other schemes, was applied to a fishery operating initially on three different stock sizes; the number of fish of each age on the grounds in 1950 computed from a virtual population calculation, one-half and twice this number. Usually regulation is not initiated until a stock has been reduced to a low level; however, it is possible that in some cases regulation might be initiated when the stock is at a high level. Under these conditions a given management strategy might cause the fishery to stabilize at a stock size larger or smaller than the optimum. Thus the three different initial stock levels are analogous to situations where the stock is below the optimum size, near the optimum and above it. The stock was brought to equilibrium irrespective of initial stock size. In the case of the smallest initial stock size, the catch oscillated about the maximum sustainable yield essentially from the beginning of regulation. The catch exceeded the maximum sustainable yield by approximately 3 million pounds and fell below it by about 5 million pounds. When the initial stock was approximately at the optimum size, the catch stabilized in about 10 years, oscillating about the 25-million-pound level. After the initial period the catch ranged from a low of 21 million to a high of 30 million pounds. And, when the initial stock was four times the size in the first case, the catch stabilized in about 15 years, oscillating about the 23-million-pound level. In this case, the catch ranged from a low of 19 million to a high of 27 million pounds after the intial period.

The requirement of age composition data in management analysis increases the cost and complexity of collecting data from the fishery. Field crews must be stationed at landing ports along the coast to sample the landing of halibut for otoliths and trained personnel are required to read the ages. From an administrative standpoint it would be less expensive and more expedient if only catch and effort statistics were required in management analysis of the fishery. To test the feasibility of using only catch statistics the basic empirical analysis was modified so that the trend in catch per unit effort was used alone in the decision rule. The fishery did stabilize as previously; however, it stabilized near the optimum stock size only with the smallest initial stock size. When the initial stock size was intermediate, the fishery stabilized at approximately 25 million pounds and in the case of the largest initial stock size at approximately 21 million pounds.

It is apparent from these trials that, if management of a fishery begins at moderate or fairly large stock sizes and uses some combination of catch statistics and age composition data in the management analysis, the fishery will stabilize at some point on the population curve to the right of the point of maximum sustainable yield, 30.8 million pounds, that is, at stock sizes in excess of the optimum. Since it is desirable to have a management scheme which causes the stock to stabilize at the point of maximum sustainable yield regardless of the initial stock size, the basic management policy was modified further so that if the catch limit remained the same for a period of three years it was automatically increased one million pounds. Such a procedure has the effect of forcing the stock size to change from the stabilized situation because more than the equilibrium yield would be removed. If stabilization occurs to the right of the point of maximum sustainable yield the increased removals would reduce the stock, bringing it closer to the optimum; if stabilization occurs to the left of the optimum, the added removals would initiate regulation to reduce the removals in subsequent years, allowing the stock to increase in size, again bringing it closer to the optimum.

In the final analysis which incorporated the above modification of arbitrarily increasing removals, the catch limit stabilized at approximately 30.8 million pounds, the true maximum equilibrium yield. The range of the oscillations about the maximum sustainable yield was less in the cases of larger initial stock sizes than with the smallest initial stock size. Possibly the greater number of older fish in the cases beginning with the larger stock sizes gives a certain stability to the populations.

Potential-Yield Curve Analysis

Schaefer (1954) estimated the maximum sustainable yield for the Area 2 halibut population from a series of data (Thompson, 1950, Table 2) for the period 1915 to 1947. Chapman, Myhre and Southward (1962) also used this method of estimating potential yield for the Area 2 population for the period 1921 to 1960; however, they modified the procedure of estimating the change in stock size from the difference between average stock to the difference between initial stock size to account for the variable length of the fishing season. Both of the estimates of maximum sustainable yield, approximately 28 million pounds and 32 million pounds, respectively, seem reasonable. However, when this approach was used in the simulation model as a management rule, unrealistic results were obtained after a period of time. At the end of a 50-year period the catch limit was at the 75-million-pound level and the catch had stabilized at the 25-million-pound level. In effect, the fishery had become unregulated and the catch and effort statistics were those for a fishery operating without limit. The season lasted the entire year with the exception of the winter closed season. During the 50-year period the estimate of maximum stock size oscillated greatly; it rose initially, fell to a relatively low level, rose moderately and decreased again, becoming negative after 30 years (Table 10).

An assumption basic to this model is that the entry of vessels into the fishery in any given year is determined by the catch of the previous year. Estimates of catch per unit effort in the early years were considerably higher than the optimum catch per unit effort. Thus the catch limit was increased immediately, and this increase

Table 10. Maximum stock in millions of pounds as estimated from Schaefer's 1954 logistic population model.

Year	Estimated maximum stock in millions of pounds	Year	Estimated maximum stock in millions of pounds
1	83.1	26	9.7
2	20.5	27	7.8
3	11.9	28	6.1
4	12.8	29	4.4
5	15.9	30	4.4
6	17.5	31	0.5
7	17.9	32	-0.5
8	17.5	33	-0.8
9	16.0	34	-1.0
10	13.6	35	-1.6
11	44.0	36	-2.0
12	41.0	37	-2.0
13	36.7	38	-2.1
14	30.6	39	-1.8
15	35.9	40	-1.8
16	31.6	41	-2.1
17	24.0	42	8.9
18	28.4	43	17.0
19	25.3	44	23.5
20	22.2	45	28.1
21	17.5	46	32.6
22	14.2	47	36.3
23	11.5	48	38.1
24	10.2	49	40.1
25	11.5	50	42.0

allowed a greater removal in the following year. The catch remained high for a period of about 10-12 years. By the end of this period the stock had been reduced to such a low level that the catch limit was not taken by early September. The opening date was set progressively earlier, and the amount of effort entering the fishery was increased. The added length of time the effort was in the fishery because of the early opening effectively removed any increase in stock resulting from a low entry of vessels.

The feature of oscillating estimates of maximum stock size was displayed for all initial stock sizes tested, the same three levels used in the empirical analysis. This oscillation indicates that the slope and intercept of the regression of equilibrium yield per unit effort on effort are changing rapidly in spite of a seemingly good estimate of the average change in stock size. As can be seen in Table 11, the computed change in biomass and the actual change are ostensibly of the same magnitude; however, the relative differences are large in some years.

Table 11. Computed and true change in size of stock in the simulation model.

Year	Computed change in population	True change in population	Year	Computed change in population	True change in population
1	+12.9		16	— 2.1	—2.9
2	+17.7	—4.4	17	— 2.4	—3.2
3	+ 1.1	—3.0	18	— 2.8	—3.6
4	— 5.5	—9.1	19	— 2.9	—3.0
5	— 8.3	—4.6	20	— 2.6	—2.3
6	— 7.2	—7.9	21	— 2.0	—1.5
7	— 6.7	—7.5	22	— 1.4	—1.0
8	— 7.5	—8.9	23	— 1.0	—0.9
9	— 7.9	—8.3	24	— .9	—1.4
10	— 7.7	—8.0	25	— 1.1	—2.2
11	— 6.8	—5.3	26	— 1.0	—0.7
12	— 5.3	—4.3	27	— .4	—0.6
13	— 3.8	—2.4	28	+ 0.1	+0.9
14	— 2.6	—2.3	29	+ 0.5	+0.5
15	— 2.1	—2.1	30	+ 0.5	+0.1

A run of fifty years' duration was made with the exponential form described on page 29, of the potential-yield curve method of estimating maximum equilibrium yield. Twenty years of historical data were entered as initial conditions. It is noteworthy that the true maximum of the model was 30.8 million pounds and that the initial estimate of the simulation run was 29.1 million pounds. In subsequent years, however, estimates of maximum equilibrium yield ranged from approximately 32 to 25 million pounds.

In each case of initial stock size, the catch oscillated about the true maximum sustainable yield, varying from about 5 million pounds under to about 3 million pounds over the maximum. Because of large changes in stock size in the run where the initial stock was twice the number of fish computed to be on the grounds in 1950, negative values of equilibrium yield were produced after the seventh year. Since the estimating procedure requires the logarithm of the equilibrium yield, and the model was not programmed to adjust for negative yields, computations for the balance of the run were terminated.

Yield-Per-Recruitment Analysis

The fishery was simulated for a 50-year period using a yield-per-recruitment management scheme at the three initial stock levels. This strategy manages the fishery so that the catch oscillates around the maximum equilibrium yield of 30.8 million pounds regardless of the initial stock size. The larger initial stock sizes merely delay the time when the catch approaches the equilibrium level. With the smaller initial size the initial catches are near the equilibrium; when the 1950 stock size is used, 6 or 7 years are required for the catch to approach the equilibrium catch; and when the initial stock size is twice the 1950 stock, equilibrium is not approached for approximately 13 years. The yield-per-recruitment strategy produces the largest average catch per unit effort of the three schemes, with the average being approximately 130 pounds compared with 123 and 122 of empirical and the potential yield schemes respectively.

In arriving at a management decision, five estimates of equilibrium yield are made based on the observed growth rate, a constant value of natural mortality and five different constant values of instantaneous rates of fishing mortality: 0.1, 0.2, 0.3, 0.4 and 0.6. These equilibrium yields for the various fishing mortality rates were subtracted successively; the sign of the difference is noted in each case. Maximum equilibrium yield is taken as the average yield between the two equilibrium yields when the sign changed.* This procedure is analogous to plotting yield against fishing mortality, F, and determining the maximum graphically. Since it is impossible to plot the yearly results and visually examine the data during a simulation run, the usual procedure has to be duplicated arithmetically.

Successive estimates of theoretical maximum potential yield are converted to pounds of yield by the ratio of the initially determined maximum yield to the average catch during the period 1951 to 1960, when the Area 2 catches were close to or may have exceeded the maximum sustainable yield for the area (Chapman, Myhre and Southward, 1962, page 8). The theoretical potential maximum is also expressed in terms of optimum catch per unit effort by the ratio of the theoretical population size to the average catch per unit effort for the above period. To obtain the latter ratio, the theoretical maximum potential yield is divided by the instantaneous fishing mortality coefficient producing it; successive estimates of the maximum sustainable yield are multiplied by this ratio (Chapman, Myhre and Southward, 1962, page 21). Even though the true recruitment to the population is available in the model, it is assumed that the management agency using a yield-per-recruitment strategy would not have this information and would be required to use the estimating procedures given in Chapman et al.

Stochastic Version of the Simulation Model

Recently, particularly since high-speed digital computers have become available, completely stochastic models in which the underlying probability distribution is known and stochastic forms of earlier deterministic population dynamic models, in which error terms are added to the parameters, have been studied. In most instances the stochastic version is considered to be an improvement of the deterministic model. Chapman (1967) maintains that stochastic models must be examined ". . . to provide error bounds on the deterministic models, to provide answers where the deterministic

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*This interpolation is gross; in any further work with this model a finer interpolation would be in order.

models fail [and] to shed light on the true underlying nature of the natural phenomenon being studied." Chapman reviews the work accomplished to date on stochastic models in ecology tracing the development of the generalized birth-and-death process, the logistic model and a predator model (Barlett, 1960; Leslie, 1958; and Leslie and Gower, 1958).

The Monte Carlo technique, that is, the generation of an artificial series of data with random numbers which conform to a prescribed model, offers a means of experimentally determining the statistical error in a complex ecological system. By varying the initial values of the parameters of the model over a known range, the average variability of these parameters can be determined. This variability represents the random variability of the parameters induced by different combinations of initial conditions (Hammersley, 1960; Hammersley and Handscomb, 1964). This method has been used in studies of relatively simple models, such as the logistic, but more can be done with it in simulation studies.

The simulation model used in this study was written so that by use of the proper controls it can be made either deterministic or stochastic. As described earlier, page 12, error terms have been added to the system in the appropriate places. The stochastic version was used:

1. to estimate the average maximum equilibrium yield and the average variability of the estimate empirically by making a number of runs under equilibrium conditions, the so-called Monte Carlo technique;
2. to compare the three management schemes under the conditions of no variability and when the greatest observed range of the variable in question — recruitment, growth, catchability coefficient — was set equal to 2.4 standard deviations. The limitation was that imposed by the truncation of the normal random deviate function.

In general, the observed range includes the inherent variability of the population as well as the sampling error originating from the collection of the information.

Maximum Equilibrium Yield as Estimated by the Monte Carlo Technique

The maximum equilibrium yield was determined in the stochastic model for a total of 263,000 units of effort fished over a seven-week period, the conditions under which maximum equilibrium yield was produced in the deterministic model. The ranges of variability in recruitment, survival, growth and the catchability coefficient observed in the Area 2 fishery and halibut stocks were introduced as random error in the system. Ten runs were made, in which only the starting point of the random number function for each run was changed. The initial population size remained at one-half the estimated 1950 stock size.

The average yield in the ten 50-year periods was 30.3 million pounds for an average stock of 167.8 million pounds; the average catch per unit effort was 116.3 pounds. Unlike the deterministic model where the catch reached and maintained absolute stability after 18 years, in the stochastic model the catch did not stabilize at a fixed value. Rather, beginning after approximately 12 years, it fluctuated for the remainder of the 50-year period within a relatively fixed interval of approximately 7 million pounds.

One of the advantages of the Monte Carlo technique is that the variance of the system can be determined empirically by making successive runs of the model. In this case, the individual 50-year averages varied between 29.9 million and 31.5 million pounds. This amount of variability is considerably less than the 11 to 14 million pounds estimated by Chapman, Myhre and Southward (1962) as the standard deviation of their estimated maximum sustainable yield. It must be noted, however, that their estimate was not based on a 50-year average and, as they pointed out in that report, ". . . due to the complexity of the estimation procedure . . ." they found it difficult to obtain confidence intervals of the estimates. These authors further noted that "the agreement of the several estimates of the maximum sustainable yield obtained by essentially different methods shows that there is more reliability in the overall estimate than these standard errors suggest." While the three methods used by Chapman et al, gave estimates of 31.4, 32.0 and 32.6 million pounds, these estimates are not, strictly speaking, comparable to independent estimates of maximum sustainable yield, which would reflect sampling variability about the same expected value. However, their estimates varied over a range of 1.2 million pounds, which closely approximates the range of 1.6 million pounds obtained in ten Monte Carlo runs of the simulation model.

Management Decision Schemes

The three management decision schemes, the empirical analysis, the potential-yield curve analysis (the modified Schaefer model) and a yield-per-recruitment analysis, of the deterministic version were each run once under the stochastic version of the model. It was assumed that regulation would begin when the stocks were at a low level; consequently, in the subsequent testing the initial stock size was one-half the 1950 stock. The performance of the stochastic versions of the management schemes at high initial stock sizes is considered at the end of this section. With the exception of variability on recruitment, survival, growth, the catchability coefficient and the number of vessels entering the fishery, conditions were the same in all runs. As mentioned earlier, *sigma* for the random error term was obtained by setting the observed range of the variables equal to 2.4 times the standard deviation.

In each instance, the management scheme brought the stock size approximately to the level which produced the maximum sustainable yield of 30 million pounds. The removals oscillated about the maximum, as was the case with the deterministic version of the model. Because of the introduced variability, the curves were not as smooth as before, although the major trends were reproduced. The catches generally increased and then decreased, the periodicity being about 20 years. Superimposed on this long-term fluctuation was what appeared to be random fluctuation. The long-term periodicity was most pronounced with the yield-per-recruitment analysis and was barely perceptible with the empirical and potential yield analysis.

Comparison of Management Schemes

The significance of variability in the basic relationships in the simulation model – recruitment, growth, efficiency of the gear, survival and number of vessels entering the fleet – is important to an understanding of a given management strategy and to a decision of which rule to choose if more than one is available. Hereafter, the empirical analysis is termed Rule 1, the potential-yield curve analysis, Rule 2 and the yield-per-recruitment analysis, Rule 3.

The administrative costs of implementing the three management rules considered here vary considerably. Rule 2, which requires only catch and effort statistics, is the least expensive and Rule 3, which requires estimates of growth as well as natural and fishing mortality for the various ages in the population, is the most costly; Rule 1 is intermediate in cost. However, the cost of a more expensive rule would be justified if it results in a policy of management which keeps the population closer to the optimum size with a smaller variance, that is, if it maintains the stock at the level that produces the maximum equilibrium catch. The entire cost of estimating rates of growth and mortality cannot be assigned to the management rule alone. In order to more completely understand the effects of regulation, there must be some basis for interpreting changes in catch statistics in terms of stock reactions. A detailed knowledge of stock composition is needed.

Because of the complexity of the simulation model and the large number of ways in which variability can be introduced, an objective judgment of the merits of one management decision rule over another is a difficult task unless the changes in the system are introduced systematically. The general form of the output and the ease with which conditions can be altered experimentally lead to an analysis of variance design for evaluating the effects of systematic changes in the various parameters.

Recruitment, growth and the catchability coefficient of the gear probably influence the size of the catch or the resulting catch per unit effort as much as or more than any other factor. However, this is not to say that these are the only factors that would affect a management scheme; rather they represent factors which prior knowledge indicate to be of greatest importance.

Deterministic models are simpler and require less initial information than stochastic models; consequently, it was desirable, again from an administrative standpoint, to ascertain if the deterministic simulation model would differ significantly from the stochastic model. In the subsequent experimentation the analysis of variance designs were constructed so that the deterministic and stochastic features of the simulation model were examined.

The analysis of variance variables were average catch and average catch per unit effort for a 50-year period, and the standard deviation of the catch and catch per unit effort. A constant amount of random variability was applied in all runs to the natural mortality term and the calculation of the number of vessels entering the fishery. The amount of variation in recruitment, growth and the catchability coefficient was systematically changed in a factorial analysis of variance experiment. The levels of variability examined were (1) no variability, and (2) the observed variability in the real data. The latter was determined by setting the range of the greatest observed variation equal to 2.4 times the standard deviation, the limit of the normal random deviate function.

	R_0				R_s			
	G_0		G_s		G_0		G_s	
	q_0	q_s	q_0	q_s	q_0	q_s	q_0	q_s
Rule 1								
Rule 2								
Rule 3								

The initial analysis of variance was a 3 x 2 x 2 x 2 factorial design, in which the effects tested were management rule, recruitment, growth and catchability coefficient. Each of the last three was examined at the two levels of variability discussed above. Each combination was replicated twice. The design is shown schematically on page 46, where R is recruitment, G is growth and q the catchability coefficient. The subscripts o and *s* refer to no variability and variability in the model, respectively. Average catch and catch per unit effort for the 24 different combinations of variability are given in Tables 12 and 13.

When catch was the variable analysed (Table 14), the effects due to the management rules, the catchability coefficient and the management rule-recruitment interaction were significant. On the other hand, when catch per unit effort was the variable analysed (Table 15), none of the factors was significant. In other words, the variability in the estimate of population size (catch per unit effort) is great enough that differences between the management decision rules cannot be detected. However,

Table 12. Average catch* in millions of pounds resulting from different combinations of variability in recruitment, growth and catchability coefficient for each management decision rule.

Management decision	R_0				R_s				Average
	G_0		G_s		G_0		G_s		
	q_0	q_s	q_0	q_s	q_0	q_s	q_0	q_s	
Rule 1									
Replicate 1	30.5	30.1	29.9	30.4	30.7	30.2	30.9	29.9	30.3
Replicate 2	30.6	29.9	29.7	29.8	31.1	30.5	30.6	31.2	30.4
Rule 2									
Replicate 1	30.4	30.4	30.2	30.0	30.6	29.7	30.2	29.7	30.2
Replicate 2	30.4	30.5	30.5	30.1	30.8	30.1	30.6	31.2	30.5
Rule 3									
Replicate 1	30.1	30.0	30.0	30.1	30.6	29.5	29.0	29.8	29.9
Replicate 2	29.7	29.8	30.5	30.0	30.1	29.5	30.2	29.4	29.9
Average	30.2	30.1	30.1	30.1	30.7	29.9	30.2	30.2	

*Average over a 50-year period.

Table 13. Average catch per unit effort* in pounds derived from combinations of variability in recruitment, growth, and catchability coefficient for each management decision rule.

Management decision	R_0				R_s				Average
	G_0		G_s		G_0		G_s		
	q_0	q_s	q_0	q_s	q_0	q_s	q_0	q_s	
Rule 1									
Replicate 1	126.7	132.0	130.2	119.5	124.8	125.1	115.2	111.7	123.2
Replicate 2	121.1	130.5	130.2	137.6	123.8	140.6	126.7	130.2	130.1
Rule 2									
Replicate 1	122.2	128.4	125.4	127.4	122.2	129.1	129.5	126.1	126.3
Replicate 2	122.8	123.8	123.5	134.3	125.2	144.0	126.4	122.1	127.8
Rule 3									
Replicate 1	128.2	130.9	130.0	128.3	132.3	129.3	128.4	128.4	129.5
Replicate 2	133.5	129.1	129.8	129.7	129.9	134.3	129.3	132.2	131.0
Average	125.8	129.1	128.2	129.5	126.4	133.7	125.9	125.1	

*Average over a 50-year period.

in terms of the total catch over the 50-year period, the differences among the rules can be detected.

The significant management rule-recruitment interaction complicates interpretation of the differences among decision rules and requires additional analysis. A plot of the average catch for the two levels of recruitment, no variability and variability, given in Figure 17, graphically shows the relationship between the decision rules and recruitment.

Table 14. Results* of analysis of variance of annual catch derived from different combinations of variability in recruitment, growth and catchability coefficient.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	2.29292	1.14646	7.100**
Recruitment	1	0.13021	0.13021	0.806
Growth	1	0.07521	0.07521	0.466
Catchability coefficient, q	1	0.77521	0.77521	4.801*
MR	2	1.25792	0.62896	3.895*
MG	2	0.03042	0.01521	0.094
Mq	2	0.00042	0.00021	0.0013
RG	1	0.00521	0.00521	0.0322
Rq	1	0.22687	0.22687	1.405
Gq	1	0.46021	0.46021	2.850
MRG	2	0.57042	0.28521	1.766
MRq	2	0.00375	0.00187	0.012
MGq	2	0.06792	0.03396	0.210
RGq	1	0.25521	0.25521	1.581
MRGq	2	0.44292	0.22146	1.415
Within replicates	24	3.75500	0.15646	
Total	47	10.34979		
(Pooled within error)	26		0.16146	

Table 15. Results of analysis of variance of annual catch per unit effort derived from different combinations of variability in recruitment, growth and catchability coefficient.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	129.43287	64.71644	2.083
Recruitment	1	1.80187	1.80187	0.058
Growth	1	28.06021	28.06021	0.903
Catchability coefficient, q	1	97.18519	97.18519	3.128
MR	2	71.77877	35.88938	1.155
MG	2	13.19292	6.59646	0.212
Mq	2	44.03291	22.01645	0.709
RG	1	108.30019	108.30019	3.486
Rq	1	2.29688	2.29688	0.074
Gq	1	76.25522	76.25522	2.454
MRG	2	37.35540	18.67770	0.601
MRq	2	2.87375	1.43688	0.046
MGq	2	49.40792	24.70396	0.795
RGq	1	29.29687	29.29687	0.943
MRGq	2	65.83864	32.91932	1.065
Within replicates	24	741.93488	30.91395	
Total	47	1499.04444		
(Pooled within error)	26		31.06821	

*Throughout this study significance is indicated with the usual notation of an * at the 5 percent level and ** at the one percent level.

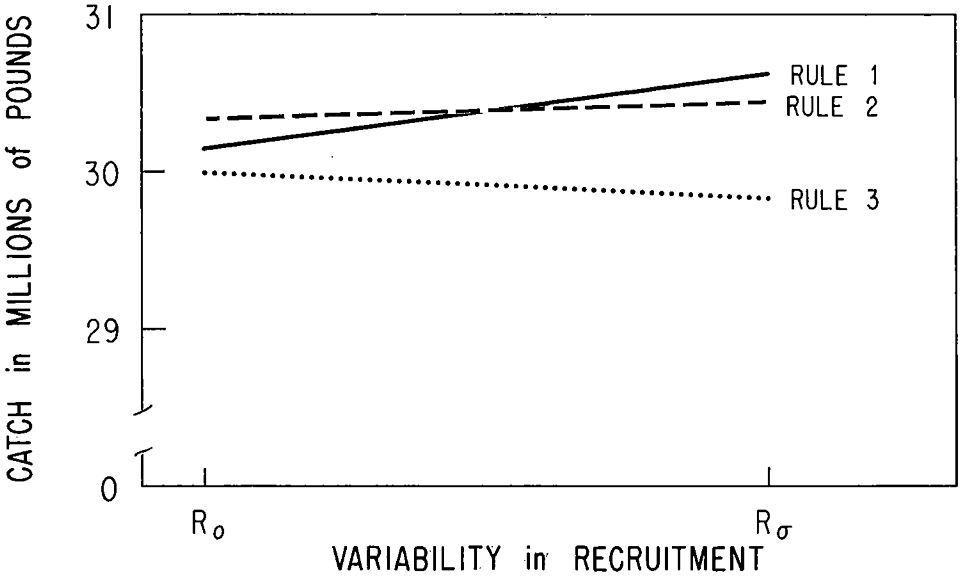

Figure 17. Average catch as a function of levels of variability in recruitment.

The difference between the deterministic and stochastic versions for each management scheme was tested with a t-test (Table 16). The difference between the catches of the deterministic and stochastic models for management Rule 1 was significant, but differences between the other two rules were nonsignificant.

Table 16. Results of t-test of difference in catch due to variability in recruitment for three management decision rules.

Management decision	Model type Deterministic	 Stochastic	t
Rule 1	30.1	30.6	—3.19**
Rule 2	30.3	30.4	—1.93
Rule 3	30.3	29.8	1.28

The differences among the three management rules at the first level of recruitment, no variability, were found to be nonsignificant when examined by analysis of variance. Similar testing of the differences among the decision rules for the second level of recruitment, with variability, indicated that the differences were significant (Tables 17 and 18).

Table 17. Results of analysis of variance of catch resulting from different management rules under the condition of no variability in recruitment.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Among rules	2	0.34	0.170	2.41
Within rules	21	1.48	0.070	
Total		1.82		

Table 18. Results of analysis of variance of catch resulting from different management rules under the condition of variability in recruitment.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Among rules	2	3.20	1.600	6.45**
Within rules	21	5.20	0.248	
Total		8.40		

In summary, examination of the management rule-recruitment interaction indicated that the differences among the management rules are non-significant if a deterministic model is assumed, but are significant if a stochastic model is used to describe the data. Further, that only in the case of management Rule 1, the empirical analysis, is there a significant difference between the deterministic and stochastic models; the variability in average catch for the other two rules is well within the residual variation.

The effect over the short term of increasing or decreasing trends in recruitment as opposed to random variability is important to the management of the halibut stock. Such trends were not considered in this study, and in view of their importance to the Commission they should be considered in any future study of management of the halibut resource since they are indicative of the interaction of a competing fishery. The effect of a long-term decreasing trend in recruitment was considered and will be discussed later in this section.

To further examine the effects of variability in the simulation model, the standard deviations of the catch and catch per unit effort for each management decision rule were next considered as random variables (Tables 19 and 20).

The standard deviation was computed in the usual manner from the 50-year time series of catches. The oscillatory pattern of the catches about the constant maximum sustainable yield line of 30.8 million pounds suggests that the standard deviation of the catches for the 50-year period proceeds from variability of two sources; first, the true random variability and, second, part of the true change in maximum equilibrium yield. This is analogous to the concepts of noise and signal in

Table 19. Standard deviation of the annual catch in pounds derived from combinations of variability in recruitment, growth and catchability coefficient for each management decision rule.

	R_0				R_s				
	G_0		G_s		G_0		G_s		
Management decision	q_0	q_s	q_0	q_s	q_0	q_s	q_0	q_s	Average
Rule 1									
Replicate 1	1.84	1.99	1.88	2.12	1.55	2.02	1.93	1.39	1.84
Replicate 2	1.57	1.93	1.91	1.88	1.48	2.53	2.07	1.82	1.90
Rule 2									
Replicate 1	2.07	2.65	2.96	2.44	2.29	2.37	3.10	3.04	2.62
Replicate 2	2.50	2.91	2.61	2.71	2.37	3.30	2.38	1.83	2.58
Rule 3									
Replicate 1	1.99	2.79	2.20	2.64	3.25	1.95	3.34	2.51	2.58
Replicate 2	2.58	2.37	2.09	2.78	2.90	3.90	2.55	3.79	2.87
Average	2.09	2.44	2.28	2.43	2.31	2.68	2.56	2.40	

Table 20. Standard deviation of the annual catch per uit effort derived from combination of variability in recruitment, growth and catchability coefficient for each management decision rule.

Management decision	R_0				R_s				Average
	G_0		G_s		G_0		G_s		
	q_0	q_s	q_0	q_s	q_0	q_s	q_0	q_s	
Rule 1									
Replicate 1	20.8	24.0	16.5	21.9	23.5	30.9	27.9	25.2	23.8
Replicate 2	22.2	19.4	16.2	21.4	21.9	27.2	19.6	25.0	21.6
Rule 2									
Replicate 1	19.4	26.7	20.8	21.5	19.1	20.9	24.4	29.2	22.8
Replicate 2	19.6	24.4	19.5	28.9	19.0	27.5	18.1	23.5	22.6
Rule 3									
Replicate 1	17.7	21.1	18.1	21.1	24.0	20.5	22.2	21.9	20.8
Replicate 2	18.5	20.0	17.3	21.1	23.3	30.6	19.7	26.1	22.1
Average	19.7	22.6	18.1	22.6	21.8	26.3	22.0	25.2	

the usual time series analysis. It is possible that a Fourier series analysis would remove the portion of the variance attributable to the change in maximum equilibrium yield and give a more precise measurement of the random variability. If in fact the point of maximum equilibrium yield does follow some periodic function, then management of the fishery could be made more precise by analysing the catch data in this manner. However, in this study, and until more evidence is available to suggest that the maximum equilibrium yield is described by a periodic function, it is assumed that the maximum yield is constant and that the standard deviation represents random variability.

The standard deviations resulting from the systematically changed variability in recruitment, growth and the catchability coefficient were examined with the same factorial design used previously. Because the management rule effect on catch was significant, it is not unexpected that the management rule effect on the standard deviation of the catch was significant.* All other effects, both main and interaction, were nonsignificant (Table 21). Tukey's test (Steel and Torrie, 1960), the results of which are shown below (Table 22), indicated that the differences among the average standard deviation of management Rule 1 and Rule 2, and Rule 1 and Rule 3 were significantly different from one another. The main effects due to recruitment and the catchability coefficient were significant in the analysis of the standard deviation of the catch per unit effort (Table 23); all other effects were nonsignificant.

Thus, the hypothesis that the different management decision rules introduce equal variance into the catch was rejected, as were the hypotheses that the deterministic form of the recruitment and catchability coefficient introduce variances equal to those generated by the stochastic version.

Examination of the output of the model up to this point has indicated that the management rules were sensitive to the amount of variability in the recruitment function. To examine further the effects of variable recruitment, differences in catch were examined by analysis of variance for three levels of variability of recruitment:

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*The standard deviations in this experiment and in subsequent experiments were transformed with natural logarithms, the transformation usually applied to variances (Scheffe, 1959, p. 83), and the analysis of variance was computed. Results were the same as with the nontransformed data, due no doubt to the large number of degrees of freedom. Consequently, the nontransformed data are presented.

Table 21. Results of analysis of variance of the standard deviation of the catch derived from different combinations of variability in recruitment, growth and catchability coefficient.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	6.82652	3.41326	16.152**
Recruitment	1	0.37630	0.37630	1.781
Growth	1	0.01577	0.01577	0.075
Catchability coefficient, q	1	0.37630	0.37630	1.781
MR	2	1.04247	0.52123	2.467
MG	2	0.00980	0.00490	0.023
Mq	2	0.02322	0.01161	0.055
RG	1	0.02950	0.02950	0.140
Rq	1	0.06527	0.06527	0.309
Gq	1	0.40150	0.40150	1.900
MRG	2	0.03662	0.01831	0.087
MRq	2	0.09855	0.04928	0.233
MGq	2	0.69572	0.34786	1.646
RGq	1	0.08755	0.08755	0.414
MRGq	2	0.16952	0.08476	0.382
Within replicates	24	5.32495	0.22187	
Total	47	15.57955		
(Pooled within error)	26		0.21132	

Table 22. Results of Tukey's test of differences among average standard deviations of the catch resulting from different combinations of variability in recruitment, growth and catchability coefficient for each management decision rule.

Rule 1	Rule 2	Rule 3
1.87	2.60	2.73

Table 23. Results of analysis of variance of the standard deviation of the catch per unit effort derived from different combinations of variability in recruitment, growth and catchability coefficient.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	16.45541	8.22771	.942
Recruitment	1	111.32521	111.32521	12.748**
Growth	1	4.75021	4.75021	.458
Catchability coefficient, q	1	171.38521	171.38521	19.626**
MR	2	52.53791	26.26896	3.008
MG	2	21.26042	10.63021	1.217
Mq	2	15.29041	7.64521	.875
RG	1	0.31687	0.31687	.036
Rq	1	0.01688	0.01688	.001
Gq	1	0.11021	0.11021	.013
MRG	2	9.94625	4.97312	.569
MRq	2	1.57624	0.78812	.090
MGq	2	1.27042	0.63521	.073
RGq	1	6.67521	6.67521	.764
MRGq	2	19.06289	9.53144	1.102
Within replicates	24	207.52497	8.64687	
Total	47	639.50469		
(Pooled within error)	26		8.73269	

Table 24. Average catch and catch per unit effort resulting from different amounts of variability in recruitment for each management decision rule.

	Catch			
Management Decision Rule	Normal Variability	2.5 x Normal Variability	Uniform Variability	Average
Rule 1				
Replicate 1	30.7	31.5	30.2	30.8
Replicate 2	31.1	31.1	31.8	31.3
Rule 2				
Replicate 1	30.6	30.1	31.2	30.6
Replicate 2	30.8	31.0	30.5	30.8
Rule 3				
Replicate 1	30.6	29.2	30.6	30.1
Replicate 2	30.1	31.2	29.4	30.2
Average	30.6	30.7	30.6	

	Catch per unit effort			
Management Decision Rule	Normal Variability	2.5 x Normal Variability	Uniform Variability	Average
Rule 1				
Replicate 1	124.8	117.5	137.4	126.6
Replicate 2	123.8	105.8	108.8	112.8
Rule 2				
Replicate 1	122.2	126.9	129.3	126.1
Replicate 2	125.2	127.7	138.5	130.5
Rule 3				
Replicate 1	132.3	127.7	133.5	131.2
Replicate 2	129.9	126.4	138.9	131.7
Average	126.4	122.0	131.1	

Table 25. Standard deviation of catch and catch per unit effort resulting from different amounts of variability in recruitment for each management decision rule.

	Catch			
Management Decision Rule	Normal Variability	2.5 x Normal Variability	Uniform Variability	Average
Rule 1				
Replicate 1	1.55	2.23	2.61	2.13
Replicate 2	1.48	3.10	2.31	2.30
Rule 2				
Replicate 1	2.29	2.86	2.29	2.48
Replicate 2	2.37	3.39	4.27	3.34
Rule 3				
Replicate 1	3.25	2.94	3.35	3.18
Replicate 2	2.90	2.93	5.61	3.81
Average	2.31	2.91	3.41	

	Catch per unit effort			
Management Decision Rule	Normal Variability	2.5 x Normal Variability	Uniform Variability	Average
Rule 1				
Replicate 1	23.5	32.3	32.9	29.6
Replicate 2	21.9	35.4	25.6	27.6
Rule 2				
Replicate 1	19.1	21.3	20.7	20.4
Replicate 2	19.0	19.1	26.9	21.7
Rule 3				
Replicate 1	24.0	22.7	25.7	24.1
Replicate 2	23.3	24.8	32.0	26.7
Average	21.8	25.9	27.3	

normal variability, the same as before, 2.5 times the normal random variability and a uniformly distributed error. In the case of the uniformly distributed error, the observed minimum and maximum values of recruitment were designated as the lower and upper limits of the uniform distribution function available in DYNAMO. In this manner more weight was given to the extreme tail values than would be given in a normal distribution. The variability of growth and catchability coefficient were held constant.

The second level, 2.5 times the normal random variability, introduced the largest error; for all practical purposes certain year classes were complete failures. The average catch per unit effort and the standard deviation (the catch and catch per unit effort) for the three levels of variability are given in Tables 24 and 25 respectively.

The analysis of variance of the catch is given in Table 26; there are no significant differences indicated. The analysis of variance of the catch per unit effort is given in Table 27. In this, there is a significant difference among management rules but there are no significant differences among the other variables. The lower catch per unit effort of Rule 1 is significantly different as indicated by Tukey's test (Table 28) from the catch per unit effort of Rules 2 and 3; no difference in catch per unit effort is indicated between Rules 2 and 3.

A significant difference in standard deviation of the catch among management rules was indicated by the analysis of variance (Table 29). A Tukey test (Table 30) showed Rule 1, for which the standard deviation is lower than for the other rules, different from Rule 2 or Rule 3. In the analysis of variance of the standard deviation of the catch per unit effort (Table 31), the management rules were again significantly different, as was the effect due to the levels of variability of the recruitment. All management rules were significantly different from each other when tested by a Tukey test (Table 32). In this instance the standard deviation of catch per unit effort of Rule 2, the potential-yield curve analysis, was the smallest and that of Rule 1,

Table 26. Results of analysis of variance of the catch resulting from different amounts of variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	2.36333	1.18167	2.146
Recruitment variability	2	0.01333	0.00667	0.012
MR	4	0.37333	0.09333	0.170
Within replicates	9	4.95500	0.55056	
Total	17	7.70500		

Table 27. Results of analysis of variance of the catch per unit effort resulting from different amounts of variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	445.24774	222.62387	4.238*
Recruitment variability	2	246.72444	123.36222	2.349
MR	4	139.44889	34.86222	.577
Within replicates	9	543.36997	60.37444	
Total	17	1374.79104		
(Pooled within error)	13		52.52453	

the empirical analysis, was the largest. Even though the recruitment term was significant, additional testing was not done, since it was fairly obvious that with differences in variability as great as those introduced, significant differences would be obtained.

The behavior of the system under the three management rules when recruitment decreases is of great interest. There is some evidence to suggest that recruitment to the Area 2 halibut stock may have decreased during the past 20 years. As discussed earlier in the treatment of the spawner-recruit relationship, the calculated number of age 4 halibut has been progressively smaller since about 1945. It appears that the

Table 28. Results of Tukey's test of differences in catch per unit effort due to management decision rule with three levels of variability in recruitment.

Rule 1	Rule 2	Rule 3
119.6	128.3	131.5

Table 29. Results of analysis of variance of the standard deviation of the catch resulting from different amounts of variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	4.94123	2.47062	4.600**
Recruitment variability	2	3.63003	1.81502	3.379
MR	4	1.81483	0.45371	0.790
Within replicates	9	5.16750	0.57417	
Total	17	15.55360		
(Pooled within error)	13		0.53710	

Table 30. Results of Tukey's test of differences in standard deviations of catch due to management rule with three levels of variability in recruitment.

Rule 1	Rule 2	Rule 3
2.21	2.91	3.50

Table 31. Results of analysis of variance of the standard deviation of the catch per unit effort resulting from different amounts of variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	174.00110	87.00055	10.213**
Recruitment variability	2	98.40444	49.20222	5.776*
MR	4	87.12222	21.78055	2.557
Within replicates	9	76.66999	8.51889	
Total	17	436.19774		

Table 32. Results of Tukey's test of differences in standard deviation of catch per unit effort due to management rule with three levels of variability in recruitment.

Rule 2	Rule 3	Rule 1
21.0	25.4	28.6

growth rate of halibut in Area 2 has increased, beginning by the early 1950's. It can be implied that the increase in growth rate has been density dependent. By the late 1950's the landings of the United States and Canadian trawl fishery from Queen Charlotte Sound and Hecate Strait had increased to the 30-million-pound level from the 5-million-pound level in the early 1940's.* It could be argued that since the trawl fisheries are on the same grounds as the setline fisheries an added mortality of young halibut has resulted from the increase in trawling. In addition, in the past few years, a foreign trawl fishery has developed in the Gulf of Alaska. The effect, if any, of this fishery on recruitment of young halibut into the Area 2 fishery is not known. As noted above, all of this is circumstantial evidence and does not demonstrate a reduction in recruitment. However, the possibility of a decrease in recruitment is of utmost concern to the Commission.

In order to study experimentally the effects of a decrease in recruitment on the three management rules, four trends in recruitment operating for a 20-year period were considered in a further simulation experiment. These were — no trend, the constant recruitment considered previously; one-half of one percent reduction per year; one percent reduction per year; and two percent reduction per year. The simulation model was set so that constant recruitment occurred for the first 10 years, the trends in recruitment occurred during the period from the 11th to the 30th year, and from the 31st to the 50th year recruitment was at the level of the 30th year. The average catch per unit effort and the standard deviation under these conditions were obtained for the 50-year period and are given in Tables 33, 34, 35 and 36 respectively.

As might be expected, the effect on catch due to trends was significant (Table 37); also, the management rules were significantly different from one another. These hypotheses were examined further with Tukey's test (Table 38) and Rule 1 was found to be different from Rule 3 and Rule 2. The catch from Rule 1, the empirical

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*The Canadian trawl fishery has increased steadily since 1943 when the landings from Queen Charlotte Sound and Hecate Strait were approximately 2 million pounds; by 1956 the landings had reached the 15-million-pound level. The United States fishery in the same area increased sharply in 1945 to a 17-million-pound level from a 3-million-pound level in 1944 and has remained reasonably constant since.

Table 33. Catch resulting from four trends* in recruitment at two levels of variability in recruitment.

	Recruitment no variability				Recruitment variability				
Management decision	NOTR	TR1	TR2	TR3	NOTR	TR1	TR2	TR3	Average
Rule 1									
Replicate 1	30.5	29.7	27.8	25.1	30.7	29.4	28.7	25.8	28.5
Replicate 2	30.6	29.6	28.2	25.3	31.1	29.6	28.8	25.6	28.6
Rule 2									
Replicate 1	30.4	29.3	28.1	24.8	30.6	28.8	28.2	24.7	28.1
Replicate 2	30.4	29.3	27.6	24.4	30.8	29.1	28.3	23.9	28.0
Rule 3									
Replicate 1	30.1	28.2	27.8	24.4	30.6	29.1	26.7	23.7	27.6
Replicate 2	29.7	29.2	27.5	24.2	30.1	28.8	27.9	24.9	27.8
Average	30.3	29.2	27.8	24.7	30.6	29.1	28.1	24.8	

*NOTR is no trend.
TR1 is one-half of one per cent reduction per year for 20 years.
TR2 is one per cent reduction per year for 20 years.
TR3 is two per cent reduction per year for 20 years.

Table 34. Catch per unit effort resulting from four trends in recruitment at two levels of variability in recruitment.

	Recruitment no variability				Recruitment variability				
Management decision	NOTR	TR1	TR2	TR3	NOTR	TR1	TR2	TR3	Average
Rule 1									
Replicate 1	126.7	119.8	126.7	115.7	124.8	109.1	94.2	89.4	113.3
Replicate 2	121.1	120.8	122.3	118.5	123.8	105.7	109.4	89.9	113.9
Rule 2									
Replicate 1	122.2	119.3	116.7	117.8	122.2	123.3	119.3	118.6	119.9
Replicate 2	122.8	121.7	121.1	118.3	125.2	121.7	121.0	113.7	120.7
Rule 3									
Replicate 1	128.2	129.2	127.7	127.1	132.3	126.7	122.4	125.5	127.4
Replicate 2	133.5	127.0	122.2	120.8	129.9	129.6	124.2	106.4	124.2
Average	125.6	123.0	122.8	119.7	126.4	119.4	115.1	107.2	

Table 35. Standard deviation of catch resulting from four trends in recruitment at two levels of variability in recruitment.

	Recruitment no variability				Recruitment variability				
Management decision	NOTR	TR1	TR2	TR3	NOTR	TR1	TR2	TR3	Average
Rule 1									
Replicate 1	1.84	1.97	2.89	5.19	1.55	2.73	3.19	5.89	3.16
Replicate 2	1.57	2.09	2.29	4.51	1.48	1.90	2.35	5.53	2.72
Rule 2									
Replicate 1	2.07	2.37	3.58	6.24	2.29	3.40	3.70	6.37	3.75
Replicate 2	2.50	2.75	4.21	6.91	2.37	3.25	3.30	7.44	4.09
Rule 3									
Replicate 1	1.99	3.47	3.37	5.89	3.25	2.99	4.61	7.71	4.16
Replicate 2	2.58	2.60	4.26	6.41	2.90	3.34	3.01	6.49	3.95
Average	2.09	2.54	3.43	5.86	2.31	2.94	3.36	6.57	

Table 36. Standard deviation of catch per unit effort from four trends in recruitment at two levels of variability in recruitment.

	Recruitment no variability				Recruitment variability				
Management decision	NOTR	TR1	TR2	TR3	NOTR	TR1	TR2	TR3	Average
Rule 1									
Replicate 1	20.8	28.1	20.0	26.2	23.5	30.0	42.6	41.4	29.1
Replicate 2	22.2	24.7	24.4	26.2	21.9	39.1	31.6	42.4	29.1
Rule 2									
Replicate 1	19.4	19.7	21.9	22.1	19.1	22.4	21.0	22.7	21.0
Replicate 2	19.6	19.3	22.9	25.1	19.0	22.3	21.0	28.9	22.2
Rule 3									
Replicate 1	17.7	22.8	17.9	21.7	24.0	22.5	24.5	24.4	21.9
Replicate 2	18.5	18.4	25.4	21.5	23.3	18.1	19.7	31.0	22.0
Average	19.7	22.2	22.1	23.8	21.8	25.7	26.7	33.3	

Table 37. Results of analysis of variance of the catch for each management rule under the conditions of two levels of variability of recruitment and four trends in recruitment within each level of recruitment.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Management rule	2	5.82167	2.91083	21.997**
Recruitment variability	1	0.28521	0.28521	2.155
Recruitment trend	3	217.29729	72.43243	547.362**
MV	2	0.27167	0.13583	1.026
MT	6	0.86833	0.14472	1.094
VT	3	0.36562	0.12187	.921
MVT	6	0.88498	0.14750	1.148
Within replicates	24	3.08500	0.12854	
Total	47	228.87977		
(Pooled within error)	30		.13233	

Table 38. Results of Tukey's test of difference in catch due to management rule under conditions of trends in recruitment.

Rule 3	Rule 2	Rule 1
27.7	28.0	28.5

Table 39. Results of Tukey's test of difference in catch due to trends in recruitment.

TR3	TR2	TR1	NOTR
24.7	28.0	29.2	30.5

analysis, was larger than from the other two rules. All of the differences among the trends were significant (Table 39).

Catch per unit effort was affected considerably more by trends in recruitment than was the catch. The management-rule-recruitment interaction (deterministic vs. stochastic versions) was significant; the main effects due to trends, type of model for recruitment and the management rule were all significant (Table 40). This indicates that if a decrease in recruitment occurs in the real population then the choice of a management rule will critically affect the measure of stock size.

Table 40. Results of analysis of variance of the catch per unit effort for each management rule under the conditions of two levels of recruitment and four trends in recruitment within each level of recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	563.66098	281.83049	30.063**
Recruitment variability	1	252.26670	252.26670	26.909**
Recruitment trends	3	298.14294	99.38098	10.601**
MV	2	185.00004	92.50002	9.867**
MT	6	108.03654	18.00609	1.921
VT	3	57.10494	19.03498	2.030
MVT	6	127.03557	21.17259	2.258+
Within replicates	24	224.99407	9.37475	
Total	47	1816.24174		

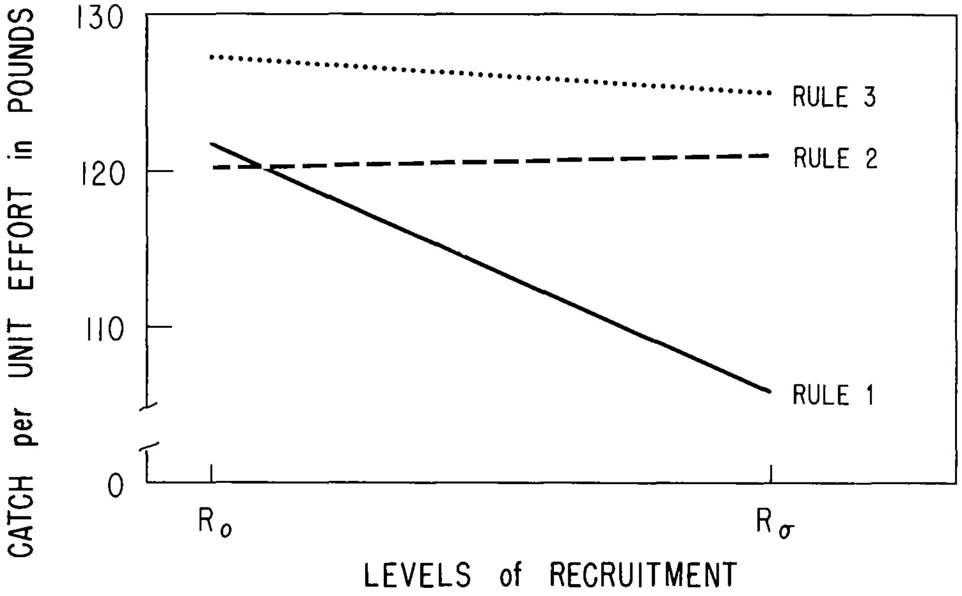

Figure 18. Catch per unit effort by management rule for two levels of variability in recruitment.

The averages involved in the management rule-recruitment interaction are plotted in Figure 18. A significant difference between the deterministic and stochastic versions of the simulation model was shown only for the empirical analysis, Rule 1 (Table 41). Within each model type, the differences among the average catches per unit effort of both the deterministic and the stochastic versions were significant (Tables 42 and 43). In the deterministic model, the average catch per unit effort of Rule 1 and Rule 2 were nearly the same and both were less than that of Rule 3. Within the stochastic model the catch per unit effort of Rule 1 was considerably less than those of Rules 2 or 3. The latter two changed little because of the form of the model. The stochastic model caused a considerable decrease in the average catch per unit effort of Rule 1.

Table. 41. Results of t-test of differences in catch per unit effort due to variability in recruitment for three management decision rules.

	Catch per unit effort		
	No variability	Variability	t
Rule 1	121.4	105.8	15.6*
Rule 2	120.0	120.6	-0.6
Rule 3	127.0	124.6	2.4

Table 42. Results of analysis of variance of catch per unit effort from each management rule under conditions of trends in recruitment with no variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Among rules	2	108.24	54.12	6.359*
Within rules	9	76.59	8.51	
Total	11	184.83		

An analysis of variance of the standard deviation of the catches (Table 44) indicated that the three main effects — management rule, variability in recruitment and trends in recruitment — were significant. In isolating the factor which caused the significance, Rule 1 was found to be different from Rules 2 and 3, but Rule 2 was found not different from Rule 3 (Table 45). Rule 1 had the smallest standard deviation. There was no difference between the situation of constant recruitment and trend 1, one-half of one per cent reduction per year; otherwise all of the differences among the trends were significant (Table 46).

As for the trends, the only nonsignificant difference was between the case of constant recruitment and that of one-half of one per cent reduction per year.

Table 43. Results of analysis of variance of catch per unit effort from each management rule under conditions of trends in recruitment with variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Among rules	2	788.00	394.00	4.51*
Within rules	9	786.26	87.36	
Total	11	1574.21		

Table 44. Results of analysis of variance of the standard deviations of the catch for each management rule under the conditions of two levels of recruitment and four trends in recruitment within each level of recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean squares	F
Management rule	2	11.95652	5.97826	26.840**
Recruitment variability	1	1.16875	1.16875	5.247*
Recruitment trends	3	114.94572	38.31524	172.018**
MV	2	0.16162	0.08081	0.363
MT	6	0.75424	0.12571	0.564
VT	3	0.97672	0.32557	1.462
MVT	6	0.97792	0.16299	0.686
Within replicates	24	5.70425	0.23768	
Total	47	136.64573		
(Pooled within error)	30		0.22274	

Table 45. Results of Tukey's test of differences in standard deviations of catch by management rules under conditions of trends in recruitment.

Rule 1	Rule 2	Rule 3
2.9	3.9	4.1

Table 46. Results of Tukey's test of differences in standard deviations of catch by trends in recruitment.

NOTR	TR1	TR2	TR3
2.2	2.7	3.4	6.2

The main effects and the management rule-recruitment interaction of the standard deviations of the catch per unit effort were highly significant (Table 47). Subsequent examination of the interaction (Figure 19 and Tables 48 and 49) indicated that differences among the three management rules were nonsignificant in the deterministic version but significant in the stochastic version. However, as had been the case previously, there was a significant difference between the models with regard to Rule 1 (Table 50). Regarding the difference in standard deviation in catch per unit effort due to trends, only the difference between the situation of constant recruitment and that of a two percent reduction per year was significant (Table 51).

Table 47. Results of analysis of variance of the standard deviations of the catch per unit effort for each management rule under the conditions of two levels in recruitment variation and four trends in recruitment within each level of recruitment variation.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Management rule	2	501.00149	250.50074	21.745**
Recruitment variability	1	283.72687	283.72687	24.629**
Recruitment trends	3	298.43431	99.47810	8.635**
MV	2	222.92872	111.46436	9.676**
MT	6	108.30291	18.05048	1.567
VT	3	60.29897	20.09966	1.745
MVT	6	164.90877	27.48479	2.385+
Within replicates	24	276.47586	11.51983	
Total	47	1916.07789		

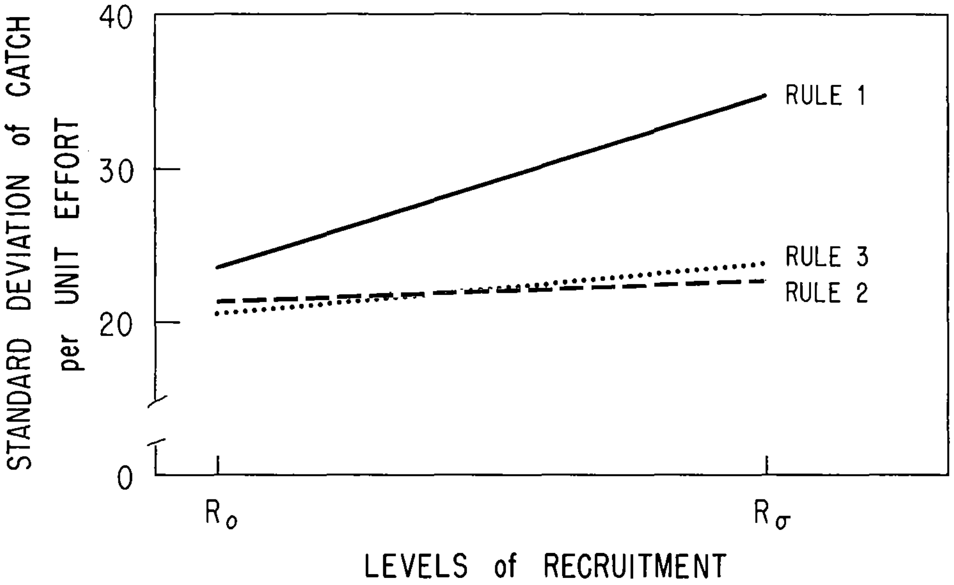

Figure 19. Standard deviation of catch per unit effort by management rule for two levels of variability in recruitment.

Table 48. Results of analysis of variance of differences among standard deviations of catch per unit effort assuming no variability in recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Among rules	2	16.08	8.04	1.17
Within rules	9	61.88	6.87	
Total	11	77.96		

Table 49. Results of analysis of variance of differences among standard deviations of catch per unit effort under variable recruitment.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Among rules	2	345.95	172.98	6.12*
Within rules	9	254.30	28.26	
Total	11	600.25		

Table 50. Results of t-test of differences in standard deviations of catch per unit effort due to variability in recruitment for three management decision rules.

	Standard deviation of catch per unit effort		
	No variability	Variability	t
Rule 1	23.23	34.07	—3.19*
Rule 2	21.24	22.06	—0.24
Rule 3	20.49	23.42	—0.86

Table 51. Results of Tukey's test of differences in standard deviations of catch per unit effort for trends in recruitment.

NOTR	TR1	TR2	TR3
20.8	24.0	23.9	27.8

SUMMARY

The management of the United States and Canadian Pacific halibut fishery in regulatory Area 2, extending from Willapa Bay, Washington to Cape Spencer, Alaska, was simulated on a digital computer. DYNAMO was the simulation language used. Management of the fishery by three different schemes, each involving a limitation of the catch and two of them dependent on an estimation of stock size, was studied in detail.

The simulation model consisted of three sections: the biology of the halibut population, the commercial fishery, and the management of the resource, with feedback among the three sections. The model has two main features: (1) the various logical operations are written as components or blocks and can be modified or replaced when a specified condition is to be studied, and (2) the decision-making strategy of the management section utilizes a feedback of information from the biology and fishery sections.

In the biology section the population of halibut is simulated under the assumption of an exponential survival model and has 17 age groups. Recruitment is introduced through a spawner-recruit function and natural mortality is assumed to be compensatory with population density. Growth of the population is expressed by the Gompertz growth equation.

In the fishery section the entry of vessels to the fleet and the opening and closing of the season is simulated. Fishing effort is calculated from the number of vessels in the fleet and the subsequent fishing mortality, based on the assumption of a constant selection ogive, is applied to the population.

In the management section three different management schemes or strategies are considered, each requiring different types of information. The first is an empirical approach based on 5-year trends in catch per unit effort, number of fish less than 12 years of age and number of fish 12 years old and older. The second, based on catch and effort statistics, estimates maximum sustainable yield and optimum stock size from a potential-yield curve of the population. The third, a yield-per-recruitment analysis, requires age-specific rates of growth and mortality and also estimates maximum sustainable yield and optimum stock size. The simulated management agency collects data from the fishery and makes biological observations of the population. From these data a decision is made as to the opening date and catch limit for the coming year.

The model used in such simulation was validated by hindcasting the catches from 1921 to 1950 on the basis of known effort data. Constant recruitment was assumed over the period of the hindcast.

The maximum equilibrium yield of the population in the model is approximately 30 million pounds (30.8 in the deterministic version and 30.3 in the stochastic version) and is obtained at a catch per unit effort of about 116 pounds (117 in the deterministic version and 116 in the stochastic version). The variance in equilibrium yield was determined for the stochastic version by Monte Carlo techniques at 1.6 million pounds.

The simulated fishery was managed for a 50-year period by each of the three management schemes: the empirical analysis, Rule 1; the potential yield analysis, Rule 2; and by the yield-per-recruitment analysis, Rule 3. Each of the schemes

stabilized the population and produced a catch approximately equal to the maximum sustainable yield. There is some indication that the yield-per-recruitment scheme produced a slightly lower catch than the other schemes. The catch per unit effort of each scheme, however, was higher than the indicated optimum stock size. As might be expected from the lower catch of the yield-per-recruitment analysis the associated catch per unit effort was higher than with the other strategies.

Differences among the three management schemes due to different combinations of variability in the catchability coefficient, the growth rate and the amount of recruitment were examined by analysis of variance. Average catch, average catch per unit effort and the standard deviation of each for the 50-year periods were the variables analyzed.

Regarding the catch, there were significant differences attributable to the management decision rules, the catchability coefficient and the management-rule-recruitment interaction term. Subsequent testing indicated that the catch resulting from Rule 1 is significantly different from those of the other two rules when variability in recruitment is introduced. Also there is a significant difference within Rule 1 between the deterministic and stochastic versions of the model. There were no significant differences in the analysis based on average catch per unit effort.

In the analysis based on the standard deviation of the catch there were significant differences among the management rules and Rule 1 resulted in a significantly lower standard deviation than the other two rules. There were significant differences in the standard deviations of the catch per unit effort attributable to variability in recruitment and the catchability coefficient.

To examine further the effects of variability in recruitment the variation added to the recruitment function was increased 2.5 times over the normal random variability and it was also considered to be uniformly distributed. The effects of a systematic decrease in recruitment were studied by considering three different rates of decrease. The analysis of variance of the catch in which the variability was increased indicated no significant differences among any of the variables. There were significant differences among the management rules when the catch per unit effort was analyzed. The catch per unit effort of Rule 1 is significantly lower than the catch per unit effort of the other two rules. The standard deviation of the catch produced by Rule 1 is significantly lower than those produced by the other rules. The standard deviations of the catch per unit effort of all management rules were significantly different. In this instance the standard deviation of catch per unit effort of Rule 2, the potential-yield analysis, was the smallest and that of Rule 1, the empirical analysis, was the largest. Also the effects on the standard deviation of catch per unit effort due to variability in recruitment were significant.

Significant differences in catch due to management rules were indicated when decreasing trends in recruitment were tested. In this case the catch resulting from each rule was different from the others with Rule 1 resulting in the highest catch. Significant differences in catch per unit effort due to management rules, variability in recruitment, trends in recruitment and in the management rule-recruitment variability interaction were evident. Subsequent testing indicates that there is a signifiicant difference in Rule 1 between the deterministic and stochastic versions of the model. The standard deviation of the catch resulting from Rule 1 was significantly lower than those for the other rules; however, the standard deviation of the catch per unit effort was higher with Rule 1 and significant differences between the deterministic and stochastic versions of the model were indicated.

CONCLUSIONS

Simulation studies offer a means of investigating different strategies that might be applied to the management of the Pacific halibut resource. Since this study was concentrated on long range aspects, it was necessary to forego investigation of several interesting short-term questions that arose. However, simulation does provide a way of formulating hypotheses about relationships existing in the population such as density-dependent growth responses to fishing, and offers a means of determining the type and amount of field sampling necessary to study the relationship in the population itself.

In general it appears that each of the three management schemes would in the long run stabilize the stock of halibut at a level where the maximum sustainable yield would be obtained. However, Rule 1, the empirical analysis, seems to be the preferable scheme from the standpoint of small variance, in most of the situations studied, the exception being the situation where the recruitment is highly variable. In this case, Rule 2, the potential yield analysis, results in a lower standard deviation of the catch per unit effort.

The empirical scheme of management is a mathematical approximation to the management policy actually followed by the Commission from the beginning of regulation in 1932 to the early or mid-1950's. During this period the main emphasis was on rebuilding the stock and this analysis has also shown that such a policy of utilizing catch per unit effort and some gross age data would, when accompanied by orderly increases in permitted removals, lead to the optimum stock size.

By the early or mid-1950's it was apparent that the stock was either at or near the optimum size and that regulation should be concerned with maintaining the stock at this level. Since the empirical scheme neither requires nor estimates the maximum sustainable yield or optimum stock size it was necessary to develop a scheme of management which would measure changes in stock size relative to an optimum base. Since the mid-1950's the Commission has formulated a scheme of management based on maximum sustainable yield and optimum catch per skate. These have been estimated in a variety of ways.

This study has shown that maximum sustainable yield and optimum stock size as indicated by optimum catch per unit effort can be estimated satisfactorily from catch and effort statistics assuming an exponential relationship between equilibrium yield and stock size; and further, that this formulation will in the long run stabilize the stock at the optimum level. It has the feature that it will rebuild the stocks if the stock level is low, as did the empirical scheme, and also provides a base for detecting when the catch and catch per unit effort are significantly different from the maximum sustainable yield and the optimum stock size.

This study has demonstrated that if the assumptions made herein on recruitment relationship and other effects of the fishery are correct, then the population equilibrium yield curve has a broad, flattopped shape. Because of this, from a practical standpoint, essentially the same yield can be taken from a stock which varies greatly in size. Such a response brings into sharp focus the view that the objectives of management of the Pacific halibut resource should be the maintenance of the level of stock which will produce the maximum sustainable yield, rather than to attempt to produce that yield per se. As such, this conclusion is consistent with the dictates of the 1953 Halibut Convention which stipulates that the Commission through regulation shall ". . . develop the stocks of halibut in the Convention waters to those levels which will permit the maximum sustained yield and to maintain the stocks at those levels . . ."

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