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UTILIZATION OF PACIFIC HALIBUT STOCKS: ESTIMATION OF MAXIMUM SUSTAINABLE YIELD, 1960

BY

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FOREWORD

The Convention of 1953 between Canada and the United States for the Preservation of the Halibut Fishery of the Northern Pacific Ocean and Bering Sea continued the conservation and development objectives of the three conventions which preceded it, but set a more specific goal by requiring that the stocks of halibut be developed to levels which will permit maximum sustainable yield and be maintained at those levels.

Research into the dynamics of the Pacific halibut stocks was begun under the first halibut convention and was continued intermittently under the second and third. It has been intensified under the current convention.

This report presents estimates of the maximum sustainable yield of halibut for Pacific waters south and west of Cape Spencer, Alaska for the environmental conditions prevailing between 1951 and 1960 inclusive.

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INTRODUCTION

The joint investigation and management of the Pacific halibut fishery by Canada and the United States has been carried out under the conventions of 1923, 1930, 1937 and 1953. Following the initial convention, the fishery has been under continuous observation and study to guide the course of management. The most recent convention states specifically that the objectives are to develop the stocks of halibut to levels which will permit maximum sustained yield and to maintain the stocks at these levels.

Regulation during the past three decades has consisted primarily of limiting the amount of fishing each year chiefly by means of catch limits in accord with the prevailing stock conditions, as indicated by the catch and age composition statistics. In order to allow the stocks to rebuild, catches were held slightly below what was being added by growth and recruitment. With the rebuilding, it has become increasingly important to obtain reliable estimates of the maximum sustainable yield as a check on the condition of the stocks in relation to the objectives of the management program.

This report is a sequel to that of the Commission upon Utilization of Pacific Halibut Stocks: Yield Per Recruitment (IPHC, 1960), and presents estimates of the present sustainable yield curves and of maximum sustainable yields under prevailing environmental conditions. To use such estimates in the practical management of the fishery it is helpful to possess knowledge of their variability. An attempt is made to estimate the variability of the estimate of maximum sustainable yield for Area 2.

The terms as well as certain conventions that will be used in this report are defined in the following:

1. Stock — the halibut 5 years of age or older in any designated region.

2. Recruitment — the total weight of new individuals which are added to a stock each year. In this report it is the weight of the five-year-olds.

3. Age of entry — the age at which the members of the year class are, on the average, 50 percent as vulnerable to fishing as are the older, fully-vulnerable ages.

4. Skate — the unit of gear used in the halibut fishery, i.e. the standardized unit-of-effort statistic (Thompson, Dunlop, Bell, 1931).

5. Yield — weight of halibut catch taken by the fishery, synonymous with "catch" in this paper, as measured by landings.

6. Potential or sustainable yield — net difference between the increase in stock due to recruitment and growth and the decrease due to removals by natural causes at any stock level assuming equilibrium conditions. This is the "normal" yield of Thompson (1950) and the "equilibrium" catch of Schafer (1954).

7. Theoretical yield — the calculated weight of the catch according to the method of Thompson and Bell (1934).

8. Optimum stock size — that stock size which will provide the greatest sustainable yield.

9. Maximum sustainable yield — the sustainable yield obtainable from the optimum-sized stock.

10. The waters between Willapa Bay, Washington and Cape Spencer, Alaska will be referred to as Area 2, as in the Pacific Halibut Fishery Regulations, 1960.

11. The waters off the coast of Alaska lying between Cape Spencer and the Shumagin Islands and the waters west of the Shumagin Islands and south of the Aleutian Islands, which are Areas 3A and 3B South respectively in the 1961 Pacific Halibut Fishery Regulations, will be referred to as Area 3. This does not include the Bering Sea for reasons given later.

12. While Ricker's notation of p and q for fishing and natural mortality coefficients was used in IPHC (1960), the standard notation, Holt, *et al* (1959), of F and M for these coefficients has been adopted here. Also, following the standard notation the term "coefficient" has been used for instantaneous rates.

The term "recruits" is not used in the analysis though it may be helpful to mention it to clarify the concepts of recruitment and age of entry. The recruits are the individuals that move onto the fishing grounds each year from nursery areas. The weight of these recruits is the recruitment, though on the grounds these younger and small fish may not be taken by the gear. Thus, a distinction is made between recruitment into the fishing area and entry into the fishery as did Beverton and Holt (1957, p. 35). They symbolized the ages at which these occurred as t_p and $t_{p'}$ respectively. These symbols are not used but note that age of entry is here synonymous with their $t_{p'}$.

The terms "potential yield" or "sustainable yield" will be used interchangeably. If the population is in equilibrium and the yield is equal to the potential or sustainable yield, the population level remains unchanged. But where reference is made to historical events in the actual fishery, it is obvious that equilibrium conditions have not been realized most of the time so yield has not equalled the potential or sustainable yield.

One further clarification needs to be made here. Equilibrium implies that the removals from the stock are balanced by additions to it so that there is no net gain or loss. It is in this gross sense that the term is used here. A more precise definition would also include a stable age and size structure as part of equilibrium conditions. Since the analysis given here is based primarily on catch and effort statistics it is not possible to consider such refinements. For this and other reasons our analysis represents a first order approximation only. A yield curve is obtained by plotting potential yield against the corresponding stock size.

As mentioned, this analysis like the halibut management program is based for the most part upon interpretations of catch and effort statistics. The assumption that catch per unit effort is proportional to abundance is basic to most fisheries management, (Beverton and Holt 1957, p. 41); yet it is also recognized from anomalies in the data that departures from this simple relationship may occur. For example, in Area 2 in the years 1953 to 1956 the availability of fish on the Goose Islands grounds in particular, but also throughout the area in general, was higher than expected from the long-term trend of catch per skate. Increased availability locally could be due to migration from other areas. However, not only was the increased availability in these years widespread, but the relative abundance of the fully recruited year classes appeared to increase rather than show the natural decline (cf. the age composition data IPHC, 1960, Table 3). Moreover, this apparent increase in relation to relative abundance is independent of the strength of the year class. As these anomalies suggest a change in the availability of the fish to the gear, the data must be analyzed with care. Where possible, long-term averages have been used to minimize such disturbances, as in connection with mortality estimates.

The geographical distribution of fishing in Area 2 has been relatively constant since at least 1921, and the statistics from that date are analyzed. However, in Area 3 the fishery continued to expand geographically during the 1920's, and only the data from 1931 on are used. While the fishery has been relatively stable since 1930 over what is defined as Area 3 in this report, the Bering Sea fishery presents a complication. Only occasional trips were made into the Bering Sea prior to 1958, but in 1958 and subsequently there has been an intensive fishery in the area. It is known from tagging results that Bering Sea halibut contribute to the stocks south and east of the Aleutians through emigration and it is also apparent that the total amount of emigration will vary inversely according to the level of the Bering Sea fishery. While the role of the Bering Sea halibut in the North Pacific halibut fishery will be discussed elsewhere (cf. Investigation, Utilization and Regulation of the Halibut in the Bering Sea – IPHC Ms.), it is necessary to point out here that estimates of maximum sustainable yield for Area 3 (with Bering Sea excluded) based on data prior to 1958 will tend to be overestimates if the Bering Sea catch continues at the present or a higher level. Consequently, decisive conclusions as to the effect of the contribution of Bering Sea halibut to the Area 3 stocks or to that fishery are impossible at this time. A limited examination is given in later discussion.

In this report as in the management of the halibut fishery it is assumed that recruitment is relatively independent of population size at the population levels that have occurred throughout the study period. This is a common assumption in studies of marine fisheries (Beverton and Holt, 1957, p. 44). Recruitment must depend on population size at the very lowest population levels, but there is no known basis in the life history of the halibut for such dependence at intermediate levels. The possibility of a parent-progeny relationship has long been studied by the Commission and the results have so far been inconclusive. A report of these studies is in preparation by members of the Commission's staff.

In estimating the maximum sustainable yields it must be recognized that there has been a continuous increase in the growth coefficient of the Area 3 stock since about 1915; the increase in the coefficient has been pronounced since the early 1930's. This increase was not discernible in the data of the shorter period covered by Thompson and Bell (1934), nor is it discernible in the data for Goose Islands grounds which are here taken to represent Area 2.

The basis of the halibut management program has been described in part by Thompson and Bell (1934), and by Thompson (1937, 1950). Thompson and Bell employed an analytical model to demonstrate that observed fishing intensities were sufficient in themselves to have caused most of the major changes observed in the fishery up to that time. Their model was satisfactory for its intended purpose, but it does not provide estimates of maximum sustainable yield.

For estimates of maximum sustainable yield it is necessary to turn to more recent models, e.g. that of Schaefer (1954) or that of IPHC (1960). Schaefer fitted a parabola to catch and effort statistics of Pacific halibut and proposed that the yield corresponding to the maximum of the parabola was the maximum sustainable yield. The yield per recruitment model used in IPHC (1960) provided estimates of sustainable yield for Pacific halibut using given parameters. There are limitations to both models and hence it is encouraging to find that certain conclusions are supported by both.*

* After this paper was in preparation the authors learned that Schaefer (1961) has also compared the equilibrium yield fishing effort curve and a yield per recruit model for yellowfin tuna.

In estimating the maximum sustainable yield for Areas 2 and 3, two methods have been used: estimates have been made from regressions of five-year averages of the catch and catch per skate data and from a potential yield curve derived from the yield per recruitment model. In addition, for Area 2, an estimate has been obtained by fitting a parabola to estimated potential yields. These estimates will show that the catches taken between 1951 and 1960 in Area 2 were close to, and may have slightly exceeded, the maximum sustainable yield; whereas the 1951 to 1960 catches in Area 3 were slightly below the maximum sustainable yield for that region.

METHODS

Estimation of the Potential Yield by Catch and Effort Statistics

It is generally agreed that the sustainable yield of a stock increases from near zero in a very small stock to some maximum point and then decreases again to near zero in a stock approaching its maximum size. This relationship has been described by a number of authors, some of whom are: Hjort, Jahn and Ottestad (1933), Baerends (1947), Graham (1935,1939), and Schaefer (1954). While the simplest mathematical form for such a sustained yield curve against stock size is a parabola, the actual relationship may not take this simple symmetric form and probably does not.

From the foregoing, *with a fixed catch,* it follows that in general an increasing catch per unit effort implies catch is less than the sustainable yield; a decreasing catch per unit effort implies catch is greater than the sustainable yield; and a constant catch per unit effort indicates that catch equals sustainable yield.

It further follows with a fixed catch, that if catch per unit effort is *increasing* at an *increasing rate* the stock size is less than that which will produce maximum sustainable yield. If catch per unit effort is *increasing* at a *decreasing rate* then the stock size is greater than that which will produce the maximum sustainable yield. Further, the rate at which catch per unit effort increases depends upon the rate of change in slope of the sustained-yield curve.

These relationships are illustrated in Figure 1 where N_o is the optimum stock size.

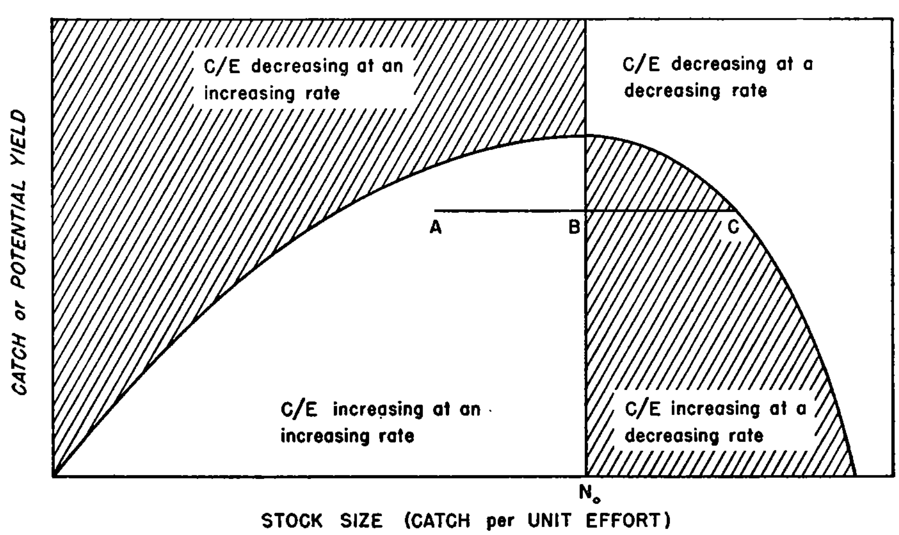

Figure 1. Theoretical relationship between catch or yield and stock size as indicated by catch per unit effort (C/E) with fixed catch along line ABC.

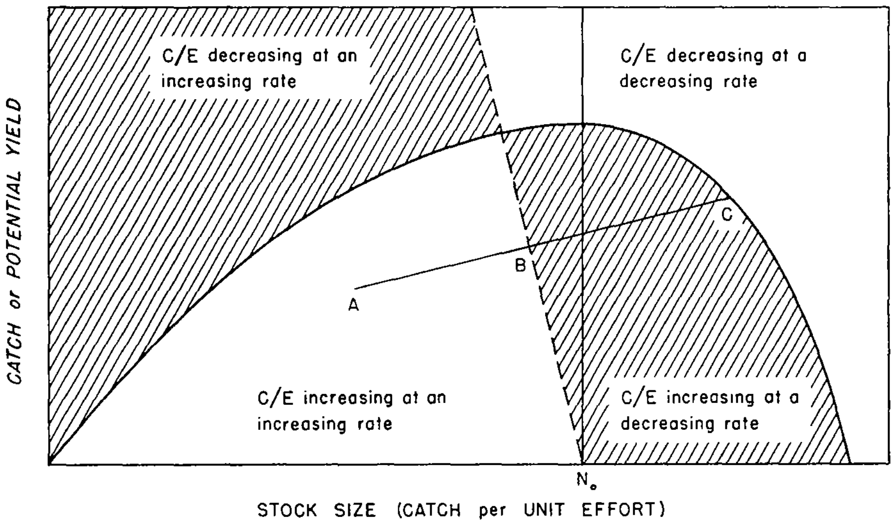

Figure 2. Theoretical relationship between catch and stock size as indicated by catch per unit effort with changing catch as indicated by the line ABC.

If the catch and stock size were at point A, and the catch were held at this level, then the stock would increase through B to C and equilibrium would be attained at C. While the stock moved from A to C, the catch per unit effort would increase at an increasing rate from A to B, and at a decreasing rate from B to C.

If the catch is not fixed but is changing, the reaction is more complicated. One such pattern is shown in Figure 2 where the catch is increasing. In this case, with the initial catch and stock at A and with catch rising from the level of A to the level of C, catch per unit effort will increase at an increasing rate from A to B and at a decreasing rate from B to C. Note that B is to the left of N_0.

Figures 1 and 2 are divided into four areas according to the direction of change in catch per unit effort and the rate at which the change is occurring. This division provides a qualitative determination of stock size relative to the optimum level. Such determinations are difficult because in practice most changes have been small and because catch per unit effort will be subject to sampling errors and random fluctuations.

To illustrate the magnitude of the changes, arbitrary numerical examples are shown in Figure 3. Here the relationship between stock and potential yield is assumed

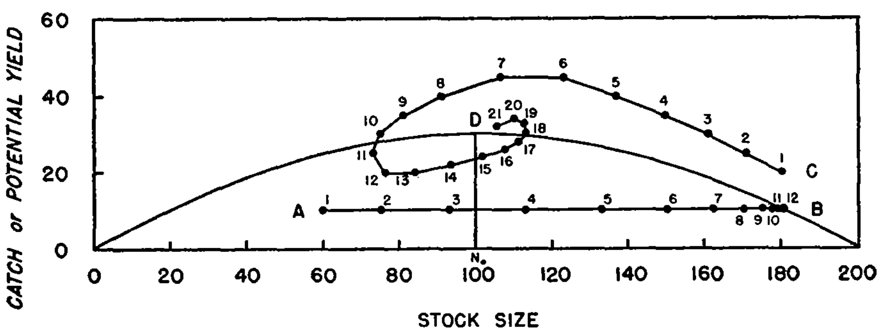

Figure 3. Two examples of effect of catch on stock size (1) fixed catch line AB, (2) changing catch, curve CD.

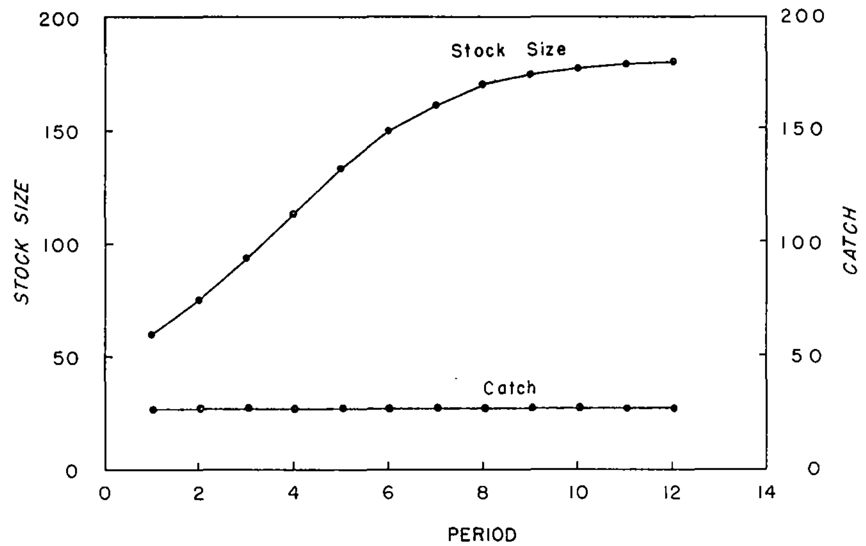

Figure 4. Change in stock size with fixed catch plotted against time from example 1.

to be parabolic with the maximum sustainable yield equal to 30 at the optimum stock size of 100. The line AB corresponds to the line segment ABC of Figure 1, the simple case of fixed catch. The observations that would be derived from this, i.e. change in stock size with time, are shown in Figure 4. The curve is sigmoid with an inflection at about period 3.

The second example with a changing catch, line CD of Figure 3, is more complicated. The catch is increased from 20 to 45 by steps of 5 in periods 1 to 6, remains at 45 in period 7, then is reduced by steps of 5 to 20 in periods 8 to 12, and is increased by steps of 5 to 35 in periods 13 to 20. A subsequent drop of 5 is made in period 21. The plot of catch and of resulting stock size against time are shown in Figure 5. The

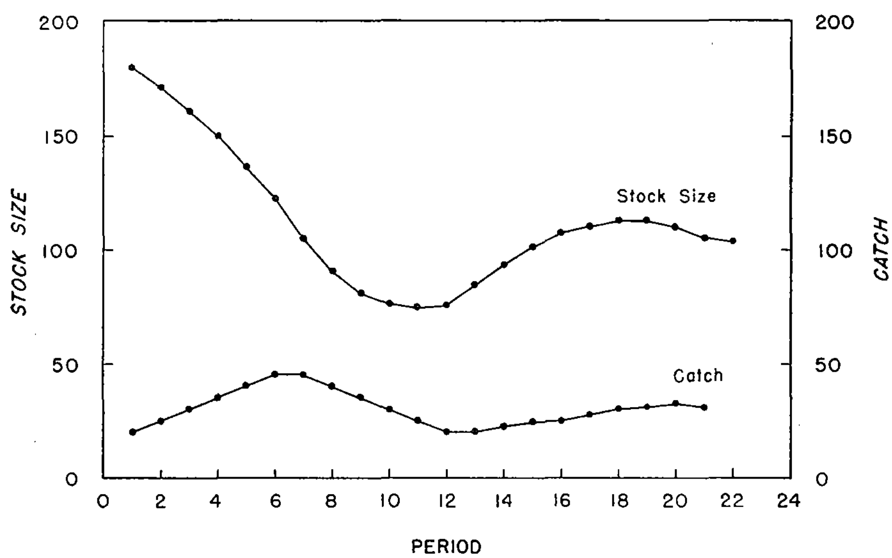

Figure 5. Change in stock size and catch plotted against time from example 2.

reactions of stock size, viz. decreases prior to period 11, increases between periods 11 and 18 and decreases thereafter, indicate that catches taken prior to period 11 and after period 18 exceeded the potential yield at those stock sizes, while potential yield exceeded catches during the interval from periods 11 through 18. The increase in stock size between periods 11 and 18 concurrent with an increasing catch indicates that potential yield is increasing at a rate sufficient to exceed the increased catch until period 18. These conditions can only occur to the left of the optimum stock size. The decrease in stock size after period 18 indicates that the catch exceeds the potential yield.

As pointed out earlier, the subtle changes in the curve of catch per unit effort plotted against time may be masked by random fluctuations and by possible lags in responses, though "smoothing" by means of averages or trends may be used to partially offset the former. In this report five-year periods are used to which regression lines are fitted.

A quantitative determination of the levels of potential yield is also made by estimating the relationship between catch per unit effort and stock size. Each catch per unit effort can then be transformed into a corresponding stock size. This can be done since the average stock size during a fishing season is related to the catch by the formula

$$\text{Average stock size} = \frac{\text{Catch}}{\text{Fishing mortality rate}} \qquad (1)$$

This is similar to the procedure followed by Schaefer (1954) but here no specific mathematical form is assumed for the potential yield curve. Finally, the potential yield in any year can be estimated by adding to the catch the change in the stock.

Extension of the Yield Per Recruitment Model with Variable Age of Entry

The yield per recruitment model of IPHC (1960)* provides estimates of yield in arbitrary units under a given set of parameters. It provides a basis for predicting changes in yield that would result from changes in the parameters. Its utility can be extended by equating a fishing mortality coefficient to a specific amount of fishing effort and by allowing for changes in age of entry as fishing effort varies.

The age of entry in any fishery is a function of selection by the gear and the method of fishing. In the case of halibut the primary factor of selection is the fishery which naturally operates to maximize the catch per skate. Halibut tend to segregate on the grounds by size which enables the fishermen to alter the size composition of their catch by slightly altering the location of their fishing. When older and hence larger fish are relatively abundant the fishermen maximize their catch per skate by fishing on such older fish, which causes an older age of entry. On the other hand, if the fishing mortality rate is increased, the number of older and larger fish is gradually reduced. If the decline in numbers of such older fish is great, it becomes more profitable to fish on younger fish. A simple example of this is shown in Appendix B where the total weight of ages 5-8 ("chickens" and small "mediums") is compared to the total weight of the fish aged 9 and older for a fishing mortality coefficient of 0.20 and 0.40. It is seen that this doubling of effort would change these two groups from approximately equal relative abundance to a 2:1 abundance in favor of the younger ages if the fishery made no

* An error in tabulation of the instantaneous growth coefficient (g), Table 5, of IPHC (1960) was discovered in the preparation of this report. The necessary corrected yields were computed and used for the subsequent calculations of this report. Errata sheets for IPHC (1960) are being prepared.

change in its operating pattern. It would then become advantageous to concentrate more heavily on younger fish which would lower the age of entry.

It is thus reasonable to expect that age of entry will vary directly with the abundance of older age groups which in turn varies inversely with the fishing mortality to which these older groups have been exposed. This fishing mortality is proportional to the cumulative fishing effort of the appropriate previous years. To demonstrate that this theoretical argument is valid, age of entry has been estimated for year classes where market sampling data were available and a linear relationship fitted to age of entry against cumulative effort. The basic data are given in Table 1 and the regressions are shown in Figure 6. For Goose Islands grounds a five-year cumulative effort has been used as the independent variable while for Portlock-Albatross grounds, where recruitment takes place over a longer span of years and the number of age classes in the fishery is greater, an eight-year cumulative effort has been used. The method used to estimate age of entry is shown in Appendix C. It is interesting to note that if the independent variables are put on a comparable basis, say average gear fished, the regression coefficients are almost identical, 0.03900 and 0.03856. Other methods of estimating age of entry were tried with similar results.

The relationships between fishing mortality coefficients, fishing effort and age of entry can be substituted in the yield per recruitment model to reconstruct the potential yield throughout the history of the halibut fishery in Areas 2 and 3. Reconstructions based on some sets of parameters will result in better agreement between yield and

Table 1. Amount of gear fished on Goose Islands* and Portlock-Albatross grounds and estimated ages of entry into the fishery for each year class.**

GOOSE ISLANDS

Year Class	5-Year Accumulation of Gear Fished, in 1000's of Skates	Years Represented in the 5-Year Accumulation of Gear Fished	Estimated Age of Entry
1931	250	1933 - 1937	6.8
1932	248	1934 - 1938	7.2
1933	286	1935 - 1939	7.4
1934	309	1936 - 1940	7.1
1935	322	1937 - 1941	7.6
1936	342	1938 - 1942	6.9
1937	345	1939 - 1943	7.6
1938	302	1940 - 1944	6.9
1939	276	1941 - 1945	7.5
1940	252	1942 - 1946	8.4
1941	229	1943 - 1947	8.5
1942	213	1944 - 1948	9.9
1943	204	1945 - 1949	9.1
1944	188	1946 - 1950	8.5
1945	188	1947 - 1951	7.5
1946	175	1948 - 1952	7.4

PORTLOCK-ALBATROSS

Year Class	8-Year Accumulation of Gear Fished in 1000's of Skates	Years Represented in the 8-Year Accumulation of Gear Fished	Estimated Age of Entry
1927	1083	1929 - 1936	9.6
1928	982	1930 - 1937	10.7
1929	960	1931 - 1938	9.8
1930	956	1932 - 1939	9.7
1941	1025	1943 - 1950	10.2
1942	1044	1944 - 1951	9.1
1943	1020	1945 - 1952	8.7

* IPHC statistical area 10B.

**IPHC statistical areas 24-28. Note in Appendix Table 16 of IPHC (1960) data given under heading Portlock-Albatross are for IPHC statistical areas 26-28.

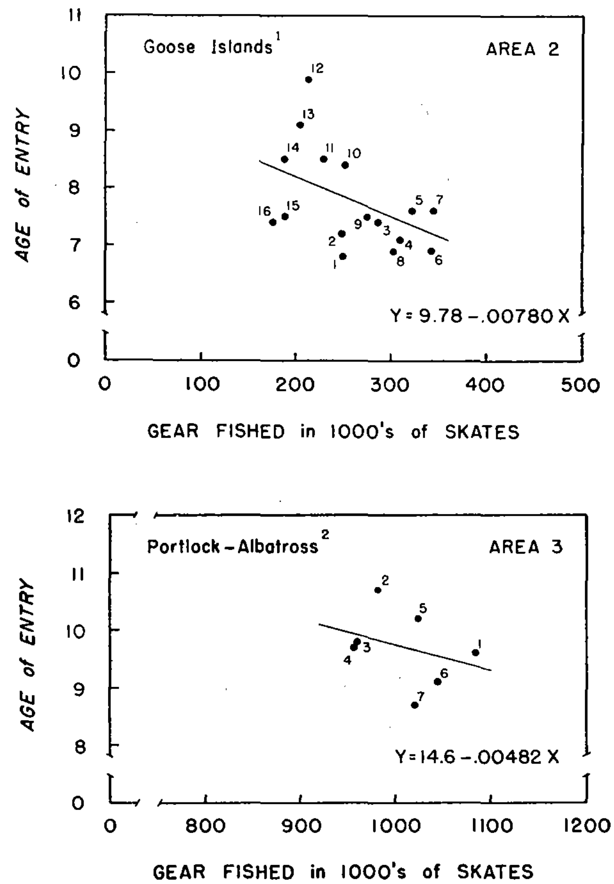

Figure 6. Relationship for Goose Islands and Portlock-Albatross grounds between average age of entry of several year classes and the total amount of gear fished during the 5-year period prior to age 7 and during the 8-year period prior to age 9 respectively.

(1) The numbers 1 through 16 refer to the year classes 1931 through 1946, respectively.

(2) The numbers 1 through 4 correspond to the year classes 1927 to 1930 and the numbers 5 to 7 correspond to the years 1941 to 1943, respectively.

potential yield than will others. The values of the parameters which provide best agreement can be regarded in some sense as best estimates of these parameters.

The relationship between potential yield and fishing mortality coefficients shown by the yield per recruitment model with fixed age of entry is not realistic over an extended range of effort. By letting age of entry vary with the fishing mortality coefficient a more reasonable relationship is obtained. By the conversion process outlined above, this relationship can be extended to show potential yield in millions of pounds in relation to catch per skate in pounds.

As will be shown later, when applied to halibut data this procedure provides reasonable potential yield curves. Nevertheless, caution is always required in using any theoretical model, both because of the assumptions involved and because estimates are used for the parameters. Credibility of the assumptions and also of the model is strengthened if there is substantial agreement of actual and theoretical data over a considerable range of time and conditions.

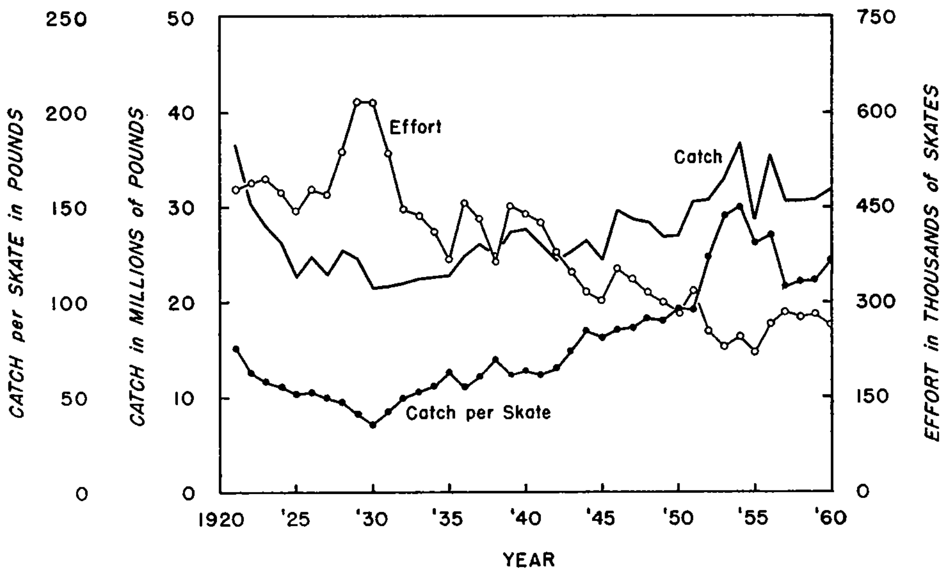

Figure 7. Catch, effort, and catch per skate data for Area 2 for the period 1921-1960.

ESTIMATION OF MAXIMUM SUSTAINABLE YIELD — AREA 2

Analysis of Catch and Catch Per Skate Data

The catch and effort statistics for Area 2 from 1921 to 1960 are given in Figure 7 and Appendix A. Age and size composition data have been collected since 1934. Changes in growth coefficient, based upon back-calculated length measurements from Goose Islands halibut samples which for the most part do not exceed age 12, have been small (IPHC, 1960) and for the purpose of this report are disregarded.

Table 2 and Figure 8 show for Area 2 the average catch and average catch per skate by five-year periods and the trends in catch per skate within these periods.

During the 1920's the removals were in excess of the potential yield. This does not indicate, however, whether the stock size was above or below the optimum stock size. The determination of this is complicated by the fact that catches were falling rapidly during this period: the trend in annual catches from 1921 to 1930 was –1.4 million pounds per year.

It is seen from Table 2 that at stock sizes corresponding to a catch per skate in excess of 100 pounds, the catch, and hence the potential yield, is in excess of 30 million

Table 2. Average five-year catch, catch per skate and trend in catch per skate, 1921-1960, Area 2.

Time Period	Average Catch (million pounds)	Average Catch per Skate	Trend in Catch per Skate*
1921 - 1925	28.8	60.2	–5.7
1926 - 1930	23.8	45.6	–4.3
1931 - 1935	22.3	51.8	4.8
1936 - 1940	26.2	61.4	1.9
1941 - 1945	24.9	72.6	6.0
1946 - 1950	28.1	89.6	2.6
1951 - 1955	32.0	129.0	9.7
1956 - 1960	31.8	117.2	–2.5

*Slope of regression lines in pounds per year.

pounds. If the optimum stock size were below the observed 1926-1930 stock level (45.6 pounds per skate) then the potential yield at the levels between 45 and 100 pounds per skate would be greater than 30 million pounds. This is at variance with the observed sharp decline in stock size as measured by catch per skate in the 1920's with catches averaging 26.3 million pounds.

It is evident from the above that stock size in the 1920's was less than the optimum. The reduced catch fell below the potential yield by 1930 and since that time there has been an increase in stock size. The trend in the change of catch per skate has been downward but only very slightly i.e. less than 0.5 pounds per skate per five-year period. In view of the increasing catch, this suggests that the pattern in Area 2 in this period is similar to that portrayed in Figure 2 with the indicated catch line closer to the potential yield curve.

Following the procedure outlined in the section on methods, it is possible to estimate how close catches have been to the potential yield curve. Substituting the estimate of fishing mortality in Area 2 of 0.30 as given in IPHC (1960) for the period 1953-1958 as that pertaining to the period 1951-1960 and using the average catch for the period of 31.9 million pounds in Formula (1), the estimated stock size for this period is 106.3 million pounds. The average catch per skate for this period was 123.1 so that the relationship between catch per skate and stock size is

$$\overline{S} = 0.8635\,U$$

where $\overline{S}$ equals average stock size in millions of pounds and U equals catch per skate. Using this conversion factor, values for average change in stock size and average yield by five-year periods between 1921 and 1960 are shown in Table 3.

The estimated average potential yield for 1951-1955 must be interpreted with care because of changes in availability during the period. Such changes in availability are shown by the catch per skate of the age classes for the successive years 1953-1956 (IPHC, 1960). The foregoing treatment has utilized five-year periods working back-

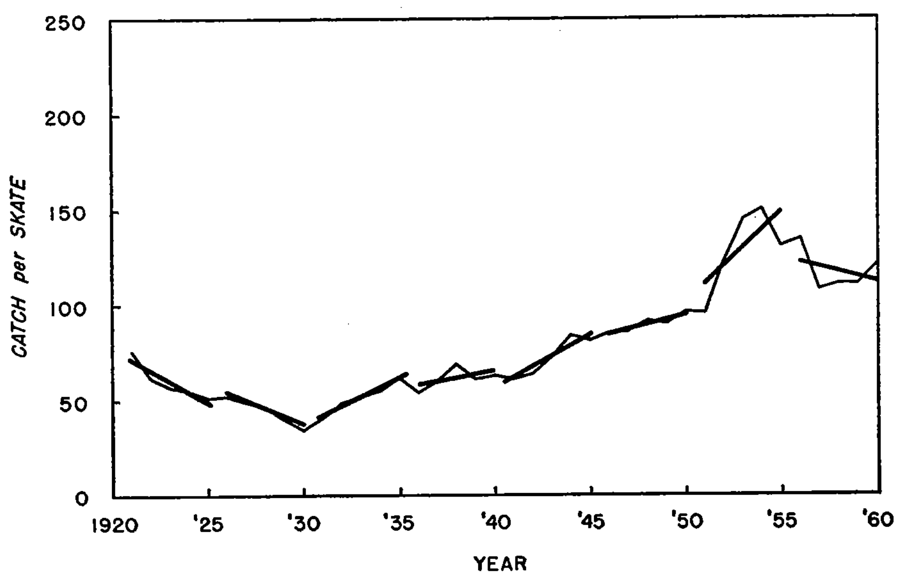

Figure 8. Trends in catch per skate by 5-year periods for Area 2 compared with the annual catches per skate.

Table 3. Estimated change in average stock size and average potential yield, 1921-1960 for Area 2.

Time Period	Average Change in Stock Size from Table 2 (million pounds per year)	Average Potential Yield* (million pounds)
1921 - 1925	−4.9	23.9
1926 - 1930	−3.7	20.1
1931 - 1935	4.1	26.4
1936 - 1940	1.6	27.8
1941 - 1945	5.2	30.1
1946 - 1950	2.2	30.3
1951 - 1955	8.4	40.4
1956 - 1960	−2.2	29.6

* This is derived by adding column 2 of this table and column 2 of Table 2.

wards from 1960. Slight variations would result from a different choice of time periods but the essential conclusions would be unchanged.

It should be noted (Appendix A) that the average catch taken in 1951-1955 was only exceeded in the period covered in this report in 1921 and 1922, and in both cases such high catches were followed by a sharp decline in catch per skate.

The indicated average potential yield for the four five-year periods 1941-1960 is 32.6 million pounds. Since this analysis shows that the present stock size is near the optimum, the maximum sustainable yield is thus estimated to be in the neighborhood of 33 million pounds.

The assumption that the sustained yield curve is parabolic has been deliberately avoided in the above analysis since it is desirable to determine what conclusions can be reached without making this strong assumption. Nevertheless, it is of interest to consider how the results of fitting a parabola compare to the results of the non-parametric approach.

Schaefer (1954, 1957) has discussed in detail the procedure for fitting a parabola to estimated yields. The method outlined in the 1954 paper depends on the transformation of catch per unit data to absolute units on the basis of fishing mortalities estimated from tagging data. In the 1957 paper a new method is developed to estimate both the parameters of the yield equation and the factor to transform catch per unit data to absolute units. As was mentioned in IPHC, (1960, p. 13) this method has proved unsatisfactory when applied to halibut data. Consequently, the earlier (1954) approach is used here, though with a modified method of estimating yields and population sizes, as outlined in the following paragraph.

The average stock size, $\bar{S}$, is computed for each year from 1921 to 1961 by the equation given earlier, viz.

$$\bar{S} = 0.8635\,U.$$

To determine initial stock size S_o at the start of any fishing season the following equation is used

$$\bar{S} = S_o(1 - e^{-(F+M)r}).$$

This equation may be derived from equation 2.10 of Chapman (1961) where

F = fishing mortality coefficient
M = natural mortality coefficient
r = fraction of the year that fishing took place.

With M set equal to 0.15, F can be calculated for each year from equation (1) on page 11. The actual value of r and the calculated F and initial stock size at year t, $S_{o,t}$ are shown in Table 4.

Table 4. Estimated initial stock sizes and potential yield in millions of pounds for Area 2, 1921-1960, M of 0.15.

Year	Fishing Mortality Coefficient (F)	Fraction of Year Fished (r)	Initial Stock Size ($S_{o,t}$)	Potential Yield (Y_t)
1921	.56	.90	91.1	20.1
1922	.57	.90	74.6	24.5
1923	.57	.90	68.6	24.9
1924	.55	.87	65.5	19.9
1925	.51	.75	59.2	24.8
1926	.55	.75	61.4	20.9
1927	.54	.75	57.6	22.8
1928	.63	.75	57.5	18.6
1929	.71	.75	50.7	18.3
1930	.71	.75	44.3	26.6
1931	.61	.70	49.6	28.8
1932	.52	.67	56.8	24.5
1933	.50	.56	59.3	24.6
1934	.47	.46	61.4	29.4
1935	.43	.51	68.2	16.3
1936	.53	.40	61.7	30.7
1937	.50	.37	67.6	33.1
1938	.42	.33	74.7	19.4
1939	.52	.33	69.1	29.1
1940	.51	.28	70.8	24.6
1941	.49	.25	67.8	27.5
1942	.44	.20	69.3	32.5
1943	.40	.18	77.5	36.0
1944	.37	.23	88.2	22.0
1945	.35	.13	83.7	30.5
1946	.40	.11	89.8	30.3
1947	.39	.11	90.4	32.5
1948	.36	.09	94.2	26.9
1949	.35	.09	92.7	32.2
1950	.33	.09	98.0	28.9
1951	.37	.10	99.9	54.0
1952	.29	.10	123.3	50.7
1953	.26	.09	143.2	39.3
1954	.28	.09	149.5	16.0
1955	.25	.09	128.7	36.4
1956	.30	.15	136.4	9.7
1957	.33	.15	110.7	33.4
1958	.32	.18	113.5	30.8
1959	.32	.21	113.7	40.4
1960	.30	.27	123.3	—

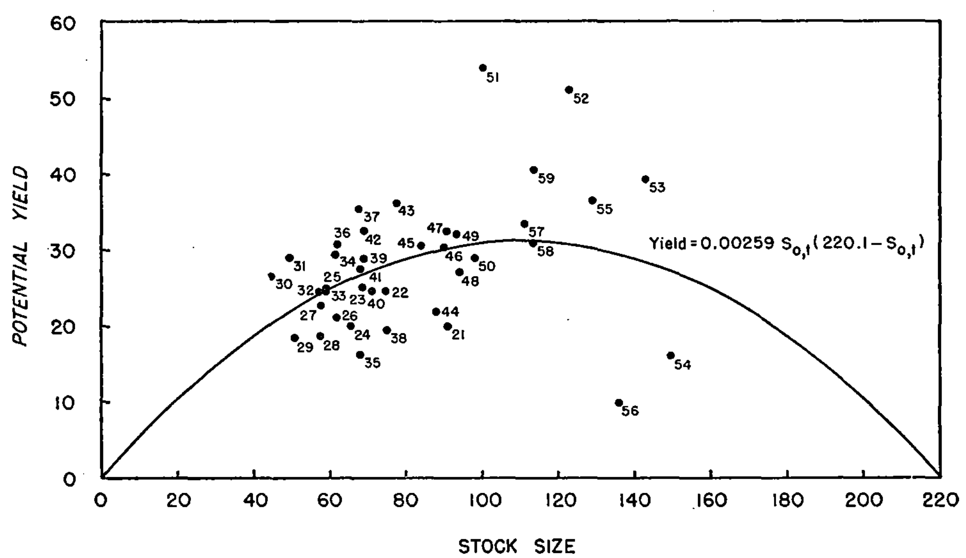

Figure 9. Estimated potential yields and stock sizes for Area 2, 1921-1960, and the fitted parabola for M of 0.15.

The estimated potential yield at year t, or Y_t is found from the equation

$$Y_t = C_t + (S_{o,t+1} - S_{o,t}).$$

In words this says that potential yield equals catch plus initial stock size in the following year minus the initial stock size in each year. These estimated stock sizes in each year are also shown in Table 4.

The parabolic relationship

$$Y_t = k\,S_{o,t}(L - S_{o,t})$$

is now assumed and the parameters k, L are estimated by the method of least squares. While there is some question as to the suitability of least squares technique because the errors of Y_t and $S_{o,t}$ are obviously correlated, other methods of estimation (cf. Schaefer, 1957) yield substantially similar results. Also there is the drawback that no lag is being assumed. In other words, it is assumed that the potential yield responds in a single season to changes in the size of population.

These estimated yields are computed by the equation

$$Y_t = 0.00259\,S_{o,t}(220.1 - S_{o,t})$$

and are shown in Figure 9. This parabolic treatment gives a maximum potential yield of 31.4 million pounds at a stock size of 110.6 million pounds at the beginning of the fishing season. The estimated average initial stock size for 1956-1960 is 110.0 million pounds with an implied potential yield of 31.1 million pounds. The same analysis with M of 0.20 gives an estimated maximum sustainable yield of 31.9 million pounds at an initial stock size of 111.6 million pounds. Despite the limitations inherent in this method, namely the restrictive equation assumed, the presence of correlated errors and the assumpton of no lag, the results are in excellent agreement with each other and with those obtained by the non-parametric treatment.

Analysis of Yield Per Recruitment Data

Several models have been constructed for the halibut fishery. The first of these referred to in the introduction was used by Thompson and Bell (1934) to show that if recruitment were constant and fishing intensities were allowed to vary as observed in the fishery much of the observed changes in the stock could be explained. Their method, with some modification noted below, has been used to construct the theoretical yield since 1930 for Area 2 shown in Figure 10. This modified model supports the theory of fishing as applied to halibut (Thompson, 1937) and the estimates of population parameters used herein.

The yield per recruitment model is used here for estimates of the maximum sustainable yield. It is also necessary to show that this model explains the observed changes reasonably well, which is done in two ways. First, the trends in catch statistics are approximated by the model as was done by Thompson and Bell, though the comparison here is less appropriate because the comparison is between actual and potential yields. A second comparison is made by deriving a yield curve and showing that the estimated potential yields also explain population changes.

It has been suggested by W. E. Ricker and K. S. Ketchen, (personal communication) that —

> "The names used for the two models tend to conceal both their resemblances and their differences. The Thompson and Bell model is just as much a 'yield per recruitment' model as the one to which the latter name is applied here. A distinctive name for the latter would perhaps be 'Equilibrium yield per recruitment model with variable age of entry' (EYV) — the key words being 'equilibrium' and 'variable entry'. By contrast, the Thompson and Bell model (as they used it) is an 'actual yield per recruitment model with fixed age at entry', or AYF model".

The modification made here to the model used by Thompson and Bell is to allow for variable age of entry so it is in this terminology an AYV model.

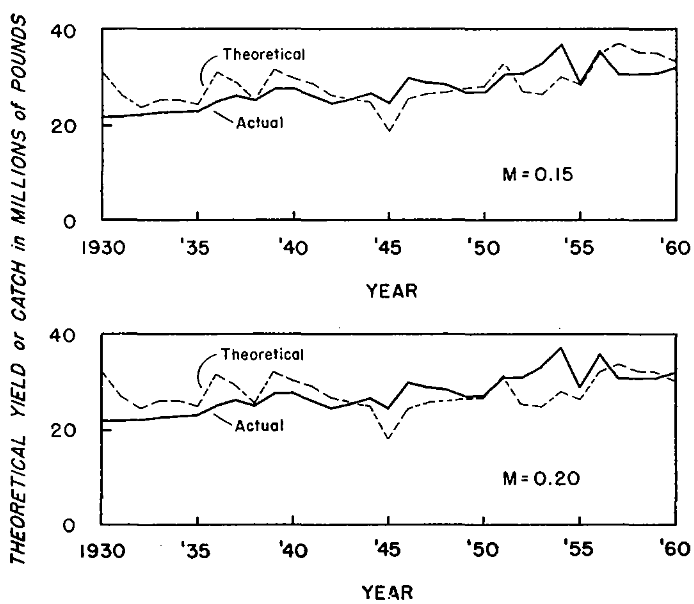

Figure 10. Comparison of theoretical yield (AYV) and actual annual catches for Area 2 for the years 1930 to 1960, using natural mortality coefficients of 0.15 and 0.20.

To use any of these it is necessary to determine the ratio between a skate of gear and the fishing mortality coefficient for fully-vulnerable ages. This ratio is determined from the 1951-1960 data when the average of 263,000 skates generated an estimated fishing mortality coefficient of 0.30, IPHC (1960, p. 13). Then to obtain the theoretical yields and potential yields the fishing mortality coefficients for any other year is estimated from the amount of gear fished on a proportional basis. The equation relating age of entry to the amount of gear fished is based on Goose Islands data. To extend it to all of Area 2 a factor was determined equal to the total effort on the Goose Islands grounds for 1933 to 1948 divided by the total Area 2 effort in the same period. The ratio of these two is 0.14055. This factor is used to adjust the five-year accumulated effort in Area 2 so that age of entry can be determined from the regression of Figure 6.

The Area 2 catch since 1930 and the calculated potential yields for the same period using values of 0.15 and 0.20 for natural mortality coefficients are shown in Figure 11.

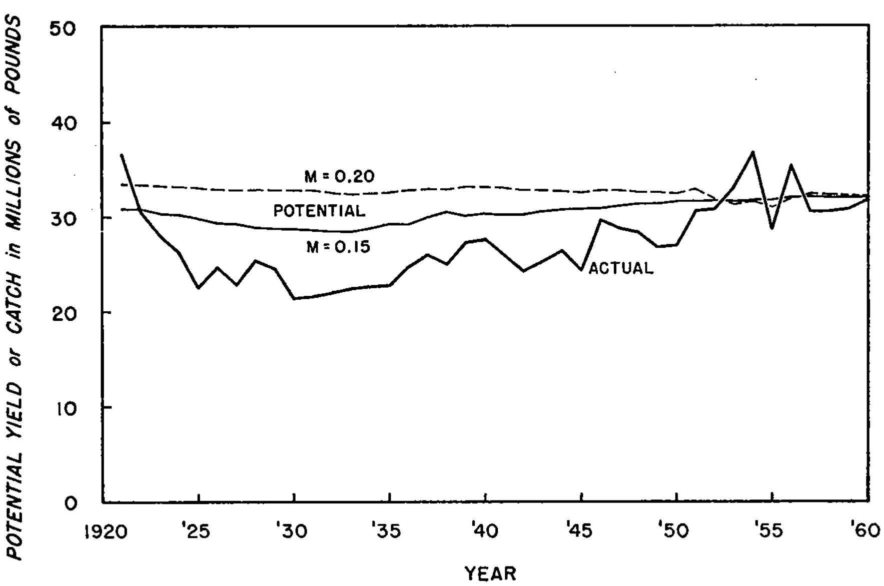

Figure 11. Calculated potential yields and actual catches for Area 2 since 1921 using observed growth coefficients, calculated variable fishing mortality and variable ages of entry.

The potential yield corresponding to the appropriate fishing mortality coefficient and age of entry for each year is then read from Tables 8 and 9 of IPHC (1960). The potential yields are then scaled to the magnitudes of the catch by the factor

$$\frac{\text{sum of calculated potential yield (1951-1960)}}{\text{sum of yields (1951-1960)}}$$

In this fitting two natural mortality coefficients, 0.15 and 0.20, were used. Trial tests with a natural mortality coefficient of 0.25 produced a potential yield curve which deviated more from the actual yield curve, while 0.10 seemed too low. The potential yield curve obtained with a natural mortality coefficient of 0.15 approximates the actual yields more closely than the one with natural mortality coefficient of 0.20 as judged by visual inspection. Also the potential yield calculated with natural mortality coefficient 0.15 shows more reasonable responses to changes in fishing mortality coefficients and ages of entry. A natural mortality coefficient of 0.20 has been accepted in IPHC (1960) as applying to halibut in all areas. However, tagging results indicate a small net migration from Area 3 to Area 2 which may tend to increase the apparent mortality in Area 3 and decrease it in Area 2. Certainly, a value for the natural mortality coefficient between 0.15 and 0.20 is not unreasonable.

If the natural mortality coefficient is 0.15, and the present age of entry is 8.4 years, the maximum sustainable yield in Area 2 as seen by interpolation in Table 8 of IPHC (1960) has been slightly exceeded and from this it follows that additional sustained yield could be obtained only by reducing effort.

If the natural mortality coefficient is 0.20, the best estimate given in IPHC (1960), and allowances are made for the changes in age of entry that accompany changes in fishing effort, it is seen from Table 9 (IPHC, 1960) that increasing effort might increase yield but it would be no more than a two percent increase.

Turning to the second comparison mentioned on page 18 it was pointed out earlier that not only could the yield from the yield per recruitment model (EYV) be converted into absolute units, viz. millions of pounds, but also it could be used to generate a potential yield curve by applying a variable age of entry. To do this it is necessary to convert the potential yield, shown in the appropriate table of IPHC (1960) for any given natural mortality coefficient, growth coefficient, fishing mortality coefficient and ages of entry, into catch per unit effort by dividing each potential yield by the fishing mortality coefficient which generates this yield. For the period 1951-1960 when the age of entry was 8.0 and the fishing mortality coefficient was 0.30, the actual catch per skate was 123.1 (Appendix A). Taking M of 0.15, the catch shown in Table 8 of IPHC (1960) for these parameters is 1892 pounds per 1000 pounds of age 5 fish. Divided by 0.30, this becomes 6307 which is equated to 123.1 pounds per skate. The Table 8 yields are converted into millions of pounds in a similar manner, that is, 1892 pounds is equated to 31.9 million pounds (Appendix A). These two values, the converted Table 8 catch per unit effort and the converted Table 8 potential yields for fixed ages of entry, when plotted form the potential yield curves of Figure 12. Appropriate ages of entry for various levels of effort were derived from the regression shown in Figure 6 and applied to these curves to produce the potential yield curve for variable ages of entry. A natural mortality coefficient of 0.15 is used in all of these calculations though similar results were obtained when a natural mortality coefficient of 0.20 was used. The potential yield curve has a maximum at a catch per skate of 120 pounds. The indicated maximum sustainable yield is 32 million pounds.

This potential yield curve is shown again in Figure 13 together with the five-year averages of catch and catch per skate plotted at their midpoints. Except for the 1923 point, the trend of these five-year averages is similar to that of the estimated potential yield (broken curve). The fact that the observed averages lie below the broken line suggests that up to 1950 the catch was below the sustainable yield; in spite of this,

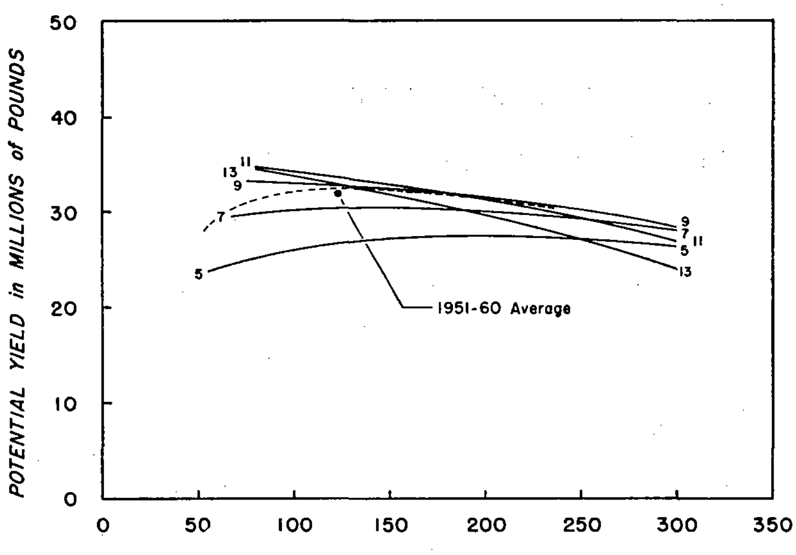

Figure 12. Area 2 potential yield as related to stock size for several fixed ages of entry (——) and the potential yield curve for the variable age of entry (- - -). These yields were computed on the basis F of 0.30 and M of 0.15 for the 1951-1960 period.

during the 1920's stock levels fell off. This may be due to errors in estimation of the parameters, to the effect of lag in the reaction of the stock to the prevailing density, or to error arising from extrapolation of the age of entry regression line. With this exception, the changes in population level are adequately explained by these differences between potential yield and actual catch.

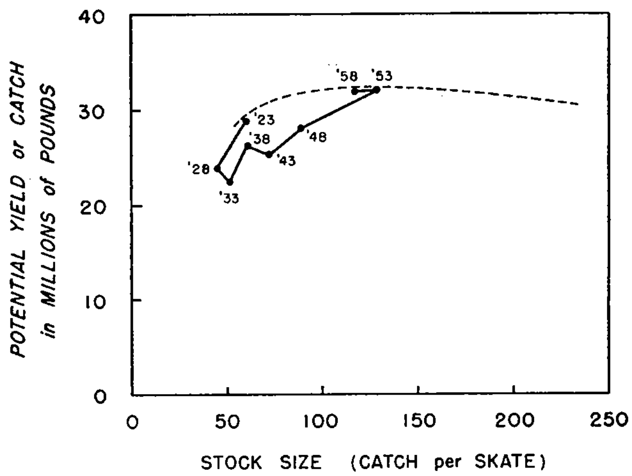

Figure 13. Area 2 potential yield curve (- - -) derived from yield per recruitment model compared with average annual catches by five-year periods.

The three differently based estimates of the maximum sustainable yield are in very good agreement and all point to the conclusion that a more intensive exploitation of the Area 2 stock would provide not an increase but a decrease in the sustainable yield. The best estimate of present maximum sustainable yield is 32 million pounds. This is also the average catch of the 1951-1960 period.

From the standpoint of management it would be useful to have a measure of the statistical variability of this estimate, for example, a confidence interval about the estimated maximum sustainable yield. This is difficult to obtain due to the complexity of the estimation procedure; however, the standard deviation gives an approximation of the interval. The standard deviation of the estimated potential yield for 1950-1959 given in Table 4 is 13.8 million pounds. The standard deviation of the average potential yield for the four five-year periods 1941-1945, 1946-1950, 1951-1955, 1956-1960 from Table 3, adjusted to an annual basis by multiplying by $\sqrt{5}$, becomes 11.4 million pounds. While these standard deviations are nearly the same, they are much too large to be useful to a management agency as estimates of the variability of the maximum sustainable yield. The agreement of the several estimates of the maximum sustainable yield obtained by essentially different methods shows that there is more reliability in the overall estimate than these standard errors suggest.

Some additional information on the variability of the estimate of maximum sustainable yield is obtained by analyzing the fluctuations in the annual catches that occurred between 1951 and 1960 when these catches were near the maximum sustainable yield level. In this period the catches ranged from 28.7 to 36.7 million pounds with a standard deviation of 2.5 million pounds. This is not an estimate of the variability of the maximum sustainable yield. The estimate of the standard deviation of the maximum sustainable yield must surely be larger than this. It thus appears that the standard error of this estimate lies between 2½ and 11½ million pounds, and it is not possible to evaluate it more precisely at this time.

ESTIMATION OF MAXIMUM SUSTAINABLE YIELD — AREA 3

Analysis of Catch and Catch Per Skate Data

The catch and effort statistics for Area 3 for the period 1921-1960 are given in Appendix A and Figure 14. In interpreting these data it is pointed out that the growth coefficient has been increasing during this period, and that in the 1920's the fishery was still expanding geographically. Some of the change in growth coefficient may be delayed response to stock changes following the onset of the fishery; some may be the result of factors independent of the fishery. These limitations must be kept in mind in the following analysis which is similar to that made for Area 2.

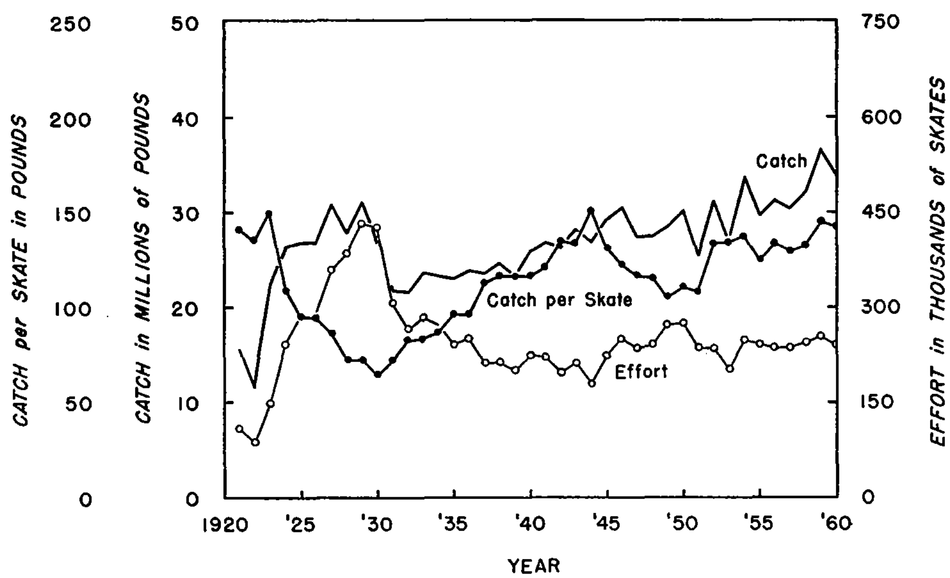

Figure 14. Catch, effort, and catch per skate data for Area 3 for the period 1921-1960.

Table 5 shows for Area 3 from 1931 to 1960 the average catch by five-year periods, the average catch per skate by five-year periods and the trend in catch per skate. These five-year trends in catch per skate are shown in Figure 15.

From 1931 to 1960 the catch per skate has increased an average of 8.3 pounds per five-year period. On the other hand, the rate of increase in catch per skate has declined by 0.57 pounds each five-year period. However, the trend in catch has been upward: 1.96 million pounds per five-year period. While the average catch has been increased 1.4 percent per year, the rate of increase in catch per skate has declined 4.1 percent per year. These changes appear to be similar to those illustrated by the line segment ABC in Figure 2. The very low rate of decrease in the increase in catch per skate indicate that the potential yield curve is flat near the maximum and that the present stock size is near the optimum level. However, this method does not take into account growth changes and their effect on the potential yield curve which cannot be measured directly by this arithmetical treatment.

Table 5. Average five-year catch, catch per skate and trend in catch per skate, 1931-1960, Area 3.

Time Period	Average Catch (million pounds)	Average Catch per Skate	Trend in Catch per Skate*
1931 - 1935	22.6	83.6	5.4
1936 - 1940	24.2	111.2	4.4
1941 - 1945	27.4	133.8	3.6
1946 - 1950	28.8	113.6	−3.5
1951 - 1955	29.4	127.2	3.6
1956 - 1960	32.9	136.0	3.3

*Slope of regression lines in pounds per year.

As was done for Area 2, the relationship between stock size and catch per skate can be used to evaluate the actual change in stock size. This, in turn, permits estimation of the potential yield at any time and of the relationship of potential yield to stock size.

The fishing mortality coefficient for Area 3 is known less precisely than in the case of Area 2. In IPHC (1960) the average fishing coefficient for 1951-1960 was estimated to be 0.20, largely on the basis of age composition data. However, tagging data and preliminary estimates based on gear density (Myhre, Ms.) suggest the 1951-1960 average fishing coefficient was near 0.10, which is consistent with the rates used by Thompson and Bell (1934) for the 1920's. In view of the above differences, an intermediate fishing mortality value of 0.15 is assumed to be generated by 240,000 skates, the rounded 1951-1960 average and is used here in view of the uncertainty of the data, as noted in the footnote in Appendix A. Substituting this coefficient and the average catch from the 1951-1960 period, viz. 31.1 million pounds into equation (1), it is estimated that the average stock in the ten-year period was 207.3 million pounds. With an average catch per skate of 131.6 pounds for the period, the relationship between stock size and catch per skate is

$$\overline{S} = 1.5752\,U$$

where $\overline{S}$ and U are stock size in millions of pounds and catch per skate in pounds. The

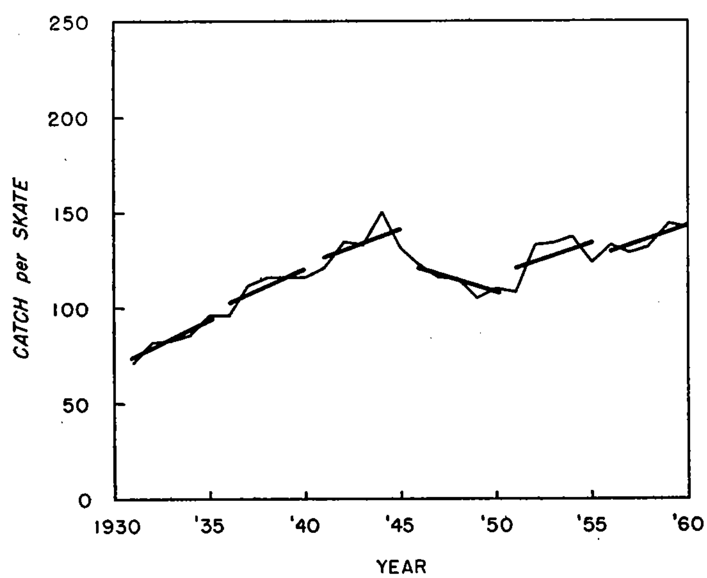

Figure 15. Trends in catch per skate by 5-year periods for Area 3 compared with the annual catch per skate.

average change in stock size and the average yield by five-year periods between 1931 and 1960 are shown in Table 6.

The estimated potential yield has increased by 1.1 million pounds per five-year period since 1931. Some of this change is the result of changes in stock size and some of it is a reflection of changes in growth rate that have occurred in Area 3. However, the average annual catch in 1959-1960 was 35.3 million pounds. Thus, this analysis shows that the catch is close to the estimated potential yield of 38.1 million pounds.*

A parabolic analysis of the Area 3 data has not been attempted because of the change in growth coefficient throughout the period.

Table 6. Estimated change in estimated average stock size and estimated average potential yield, 1931-1960 for Area 3.

Time Period	Average Change in Stock Size from Table 5 (million pounds per year)	Average Potential Yield (million pounds)
1931 - 1935	8.5	31.1
1936 - 1940	6.9	31.1
1941 - 1945	5.7	33.1
1946 - 1950	−5.5	23.3
1951 - 1955	5.7	35.1
1956 - 1960	5.2	38.1

Analysis of Yield Per Recruitment Data

Calculation of theoretical yields and potential yields from a yield per recruitment model for Area 3, similar to that done for Area 2, requires information on the changing growth coefficients in this area. Growth coefficients obtained by back calculations from otoliths for the years 1926, 1931, 1936, 1941, 1946, 1951, 1956, and 1960 are incorporated in a series of yield per recruitment curves. In calculating the annual theoretical yields and the annual potential yields from the yield per recruitment values, growth coefficients are assumed to hold for the five years with midpoints at the above dates.

The relationship between the average number of skates fished during the ten-year period 1951-1960 and the average fishing mortality coefficient is that used above, i.e. 240,000 skates generate a fishing mortality coefficient of 0.15. This relationship and the known amount of gear fished are then used to determine an annual fishing mortality coefficient on a proportionate basis for each year from 1921 to 1960. Further, this annual fishing mortality coefficient is used to determine the potential yield for each year and subsequently the yield per unit effort. These potential yield values are converted, as described earlier, to absolute units. Age of entry is related to the amount of gear fished by an extension of the regression shown in Figure 6. The equation given in Figure 6 is based on Portlock-Albatross data; so to extend it to all of Area 3 a factor is determined equal to the total effort on the Portlock-Albatross grounds for the period 1928 to 1947 divided by the total Area 3 effort for the same period. The ratio of these two is 0.5147 and is used to determine the age of entry for Area 3.

The actual catches are compared with theoretical yield calculated according to the method of Thompson and Bell (AYV) in Figure 16. These theoretical yields are calculated on the basis that 240,000 skates generate a fishing mortality coefficient of 0.15, as noted above. However, if it is assumed that this effort generates an F of 0.10 or 0.20 the differences in the theoretical yields are inconsequential.

* In 1961 as a result of an increase in the catch quota the catch was 36.5 million pounds. The 1962 catch is expected to be again about 36.5 million pounds.

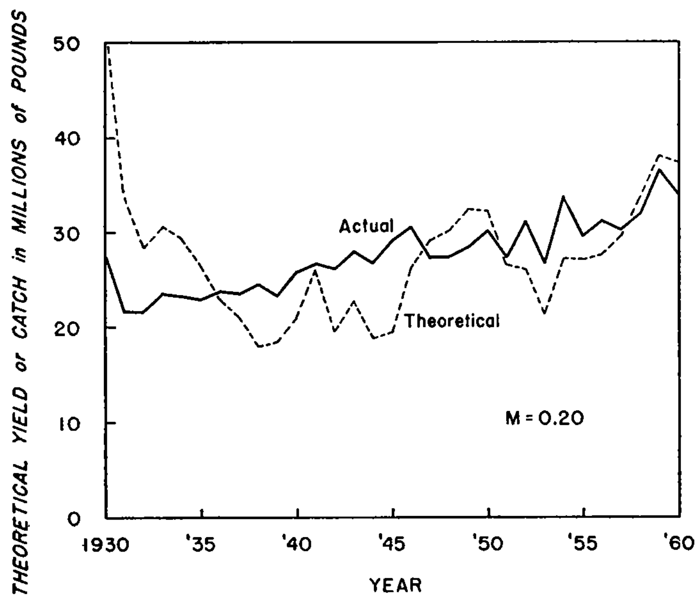

Figure 16. Comparison of theoretical yield (AYV) and annual catches for Area 3 for the years 1930-1960 using a natural mortality coefficient of 0.20.

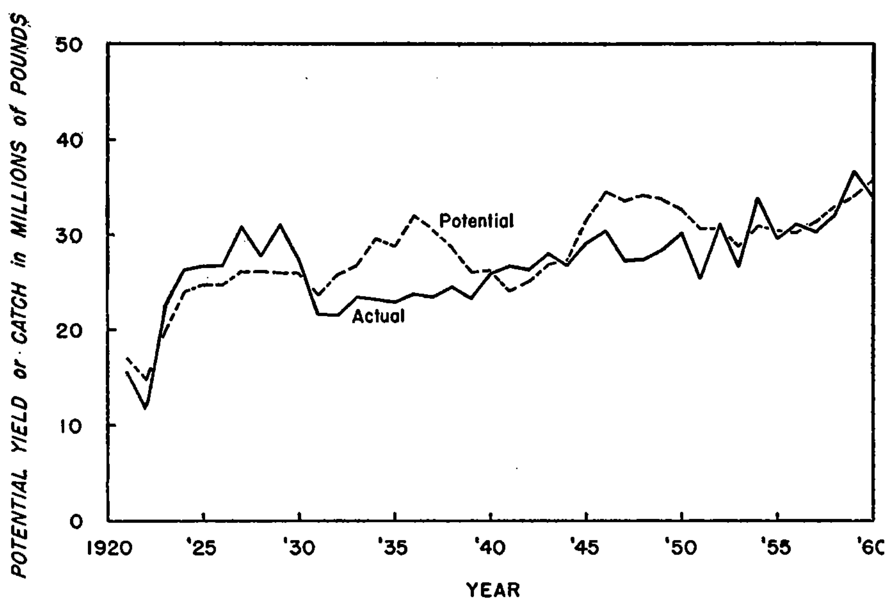

Figure 17. Calculated potential yields and actual catches for Area 3 since 1921 using observed growth coefficients, calculated variable fishing mortality and variable ages of entry.

The calculation of these yields requires the use of effort data extending to 1912, which includes gear fished from dories as well as longliners and includes the period of initial exploitation of Area 3. Consequently, the points corresponding to the first 4 or 5 years in Figure 16 are subject to an unknown amount of error and must be viewed with caution.

The observed catches and estimated potential yields calculated with the same parameters as above are shown in Figure 17. The approximation is good with some suggestion of a possible under-estimation of growth coefficient in 1946. Calculations were also made setting F for 1951-1960 equal to 0.10 and 0.20. In this case the best fit judged by visual examination and as determined by the usual correlation coefficient occurs with F equal to 0.10. Even here the differences are not great.

It is seen from Figure 6 that doubling effort at the present time would reduce age of entry to age five. Assuming the present fishing mortality coefficient is 0.15, the result of these two changes would be an increase in sustainable yield of 4 percent (IPHC, 1960, Table 14). If in fact the 1951-1960 fishing mortality coefficient is 0.10 the same change in effort and age of entry would mean a 14 percent increase in sustainable yield. On the other hand, if the fishing mortality coefficient for 1951-1960 is 0.20 this change of effort and age of entry brings about a decrease of 6 percent in sustainable yield.

To construct a potential yield curve from data of the yield per recruitment (EYV) model, the same procedure as in Area 2 is followed. Age of entry is taken as the dependent variable and the natural mortality coefficient as 0.20. Due to the increase in growth coefficient in Area 3 two potential yield curves are shown: one for the fishing mortality coefficient of 0.15 for 1951-1960 and the 1956 growth coefficient, and one for the fishing mortality coefficient prevailing in 1926 on the assumption of the 1951-1960 coefficient being 0.15 and the 1926 growth coefficient (Figures 18 and 19). These

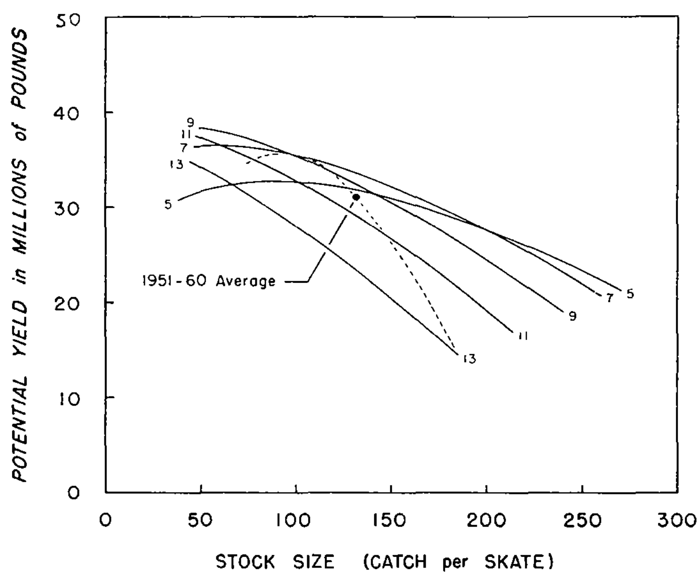

Figure 18. Area 3 potential yield as related to stock size for several fixed ages of entry (——) and the potential yield curve for the variable age of entry (- - -). These yields were computed using the 1956 growth coefficient, an F of 0.15 and an M of 0.20 for the 1951-1960 period.

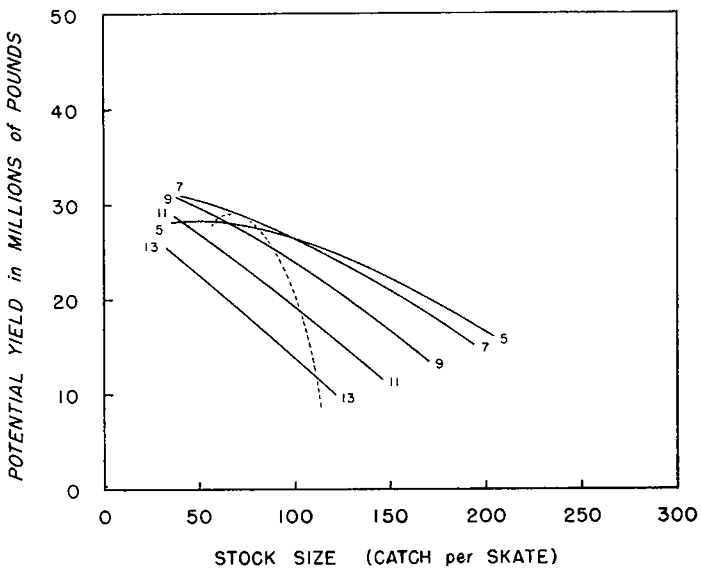

Figure 19. Area 3 potential yield as related to stock size for several fixed ages of entry and the potential yield curve (——) for a variable age of entry (- - -). These yields were computed using the 1926 growth coefficient, an F of 0.18 and an M of 0.20.

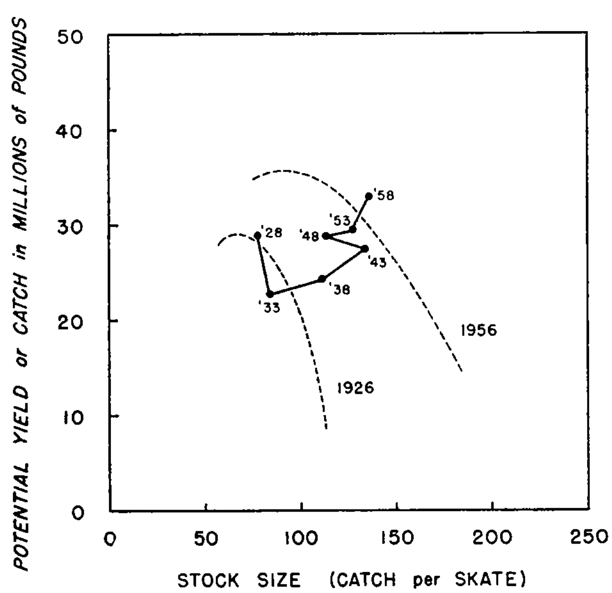

Figure 20. Area 3 stock potential yield curves (- - -) compared with average annual catches by five-year periods.

two potential yield curves together with the five-year average catch and catch per skate data are shown in Figure 20. The sharpness of these curves in contrast to the flat maximum of the Area 2 yield curve is partly explained by the lower fishing mortality coefficient in Area 3. The derived potential yield curves again provide an explanation of the changes that have actually occurred when note is taken of the changes in the curves themselves. From Figure 20, the maximum sustainable yield is estimated to be 36 million pounds, which would be attained at a catch per skate of 95 pounds.

The analysis based on the catch and effort statistics suggested the maximum sustainable yield is at or above 38 million pounds, not far from the estimate of 36 million pounds from the potential yield curve. This reinforces the conclusion that the 1951-1960 yield is close to, but slightly below the maximum sustainable yield.

As was pointed out earlier, the size of the stock and hence the level of maximum sustainable yield in Area 3 must be reduced to some degree by a fishery in Bering Sea. Between 1958 and 1960 the Bering Sea catch was in the neighborhood of 4 million pounds annually. Although all of these fish would not necessarily have migrated to Area 3 and be taken by that fishery, it is clear that this level of Bering Sea catch does represent a reduction in the maximum sustainable yield of Area 3. However, the actual reduction is no doubt less than the statistical variability in the present estimate of maximum sustainable yield for Area 3.

It must be emphasized that the present estimates of both the size of the Bering Sea stock and the emigration rate from the Bering Sea are still tentative. Also it is not known what effect a Bering Sea fishery may have on the stock of juvenile halibut in the Bering Sea nor what is the contribution of such young halibut to the Area 3 stock.

In view of the uncertainties in the basic parameters for Area 3 and the complications introduced by a Bering Sea fishery it seems premature to discuss the statistical variability of the Area 3 estimates.

SUMMARY

This report estimates the maximum sustainable yield for Areas 2 and 3 as of 1960 under prevailing conditions of the longline fishery and environment. The theory concerning the behavior of catch per unit effort in relation to a potential yield curve is discussed briefly. The existence of an inverse relationship between age of entry and fishing mortality is shown and the role of such a relationship in interpreting yield per recruitment data is discussed. A procedure for estimating average stock size, catch and effort statistics is given as well as a procedure for estimating a potential yield curve from yield per recruitment data.

The trends in catch and fishing effort for Area 2 are analyzed by five-year periods. The behavior of the trends of catch per unit effort for various stages in the history of the fishery are discussed with respect to the theory involved. Theoretical yields using the method of Thompson and Bell are computed and compared with the annual catches from 1930 to 1960. The annual potential yields for the years 1921-1960 are calculated from yield per recruitment data and compared with the annual catches for the years 1921-1960. Maximum sustainable yield is estimated in three ways: (1) estimates of potential yield obtained from the catches and catches per unit effort in five-year periods suggest the maximum sustainable yield is near 33 million pounds under present environmental conditions, (2) maximum sustainable yield is estimated to be 31.4 million pounds by fitting a parabola to the potential yields, (3) a potential yield curve is derived from the yield per recruitment data of IPHC (1960) and an estimate of maximum sustainable yield of 32 million pounds is obtained. Since these three independent estimates of maximum sustainable yield agree, a "best" estimate of maximum sustainable yield is taken to be 32 million pounds, which is also an average of the last ten years' catches.

In Area 3 the increase in growth coefficients makes difficult the comparison of annual catches with theoretical yields computed by the method of Thompson and Bell for the years 1930 to 1960. Using yield per recruitment data, the annual potential yields for the years 1921-1960 are calculated and compared with the annual catches. The estimation of maximum sustainable yield by fitting a parabola to potential yields is not attempted.

Maximum sustainable yield for Area 3 is estimated in two ways. Analysis of catch and fishing effort statistics by five-year periods indicates that the average potential yield for the period 1956-1960 is near 38 million pounds. Due to the conflicting evidence between tagging and catch statistics regarding the current estimate of fishing mortality coefficient in Area 3, values between 0.10 and 0.20 must be considered as possibilities. To simplify the presentation of the analysis an intermediate value of 0.15 was chosen and used in the calculations. Using yield per recruitment data and observed changes in growth coefficients the annual potential yields for the period 1921-1960 of the history of the Area 3 fishery were calculated and compared with the annual catches. A potential yield curve is derived as in Area 2 and from this curve a second estimate of maximum sustainable yield is found to be 36 million pounds. The average catch in 1959-1960 was 35.3 million pounds.

Since these two independent estimates of maximum sustainable yield — one based on catch and fishing effort statistics, the other on yield per recruitment calculations — are not too divergent, it is concluded that the 1960 stock size in Area 3 was close to, but slightly greater than that which would provide maximum sustainable yield.

LITERATURE CITED

Baerends, G. P.

1947 De rationeele exploitatie van den zeevischstand, in het bijzonder van den vischstand van de Noordzee.
Versl. en Meded. Afd. Visscherijen, (36): 99 pp.
(Published in English by U.S. Fish and Wildlife Service in Spec. Sci. Rept. – Fisheries (13): 102 pp.)

Beverton, R. J. H. and S. J. Holt

1957 On the dynamics of exploited fish populations.
Gt. Brit. Min. of Agric., Fish. and Food, Fish. Invest. Ser. II, vol. 19: 533 pp. London.

Chapman, D. G.

1961 Statistical problems in dynamics of exploited fisheries populations. Proceedings of the Fourth Berkeley Symposium on Math. Stat. and Prob. vol. 4: 153-168.

Graham, Michael

1935 Modern theory of exploiting a fishery, and application to North Sea trawling. Jour. Cons. Int. Explor. Mer., **10**: 264-274.

1939 The sigmoid curve and the overfishing problem. Cons. Int. Explor. Mer, Rapp. et Proc. – Verb, vol. 110: 15-20.

Hjort, J., G. Jahn and P. Ottestad

1933 The optimum catch. Essays on population. Hvalrad. Skd. (7): 92-127.

Holt, S. J., J. A. Gulland, C. Taylor and S. Kurita

1959 A standard terminology and notation for fishery dynamics. J. Cons. Int. Explor. Mer. vol. 24: 239-242.

International Pacific Halibut Commission

1960 Utilization of Pacific halibut stocks: yield per recruitment, Rept. Int. Pac. Hal. Comm., (28): 52 pp.

Ms. Investigation, utilization and regulation of the halibut in the Bering Sea. Rept. Int. Pac. Hal. Comm.

Myhre, Richard J.

Ms. A study of errors inherent in tagging data on Pacific halibut. (To appear in ICNAF, Spec. Public. Ser.)

Schaefer, Milner B.

1954 Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. Inter-Amer. Trop. Tuna Comm. Bull., vol. 1 (2): 27-56.

1957 A study of the dynamics of the fishery for yellowfin tuna in the eastern tropical Pacific Ocean. Inter-Amer. Trop. Tuna Comm., Bull. vol. 2 (6): 247-268.

1961 Report on the investigations of the Inter-American Tropical Tuna Commission for the year 1960. Inter-Amer. Trop. Tuna Comm., Annual Rept. 1960, Append. A: 40-107.

Thompson, William F.

1937 Theory of the effect of fishing on the stock of halibut. Rept. Int. Fish. Comm. (12): 22 pp.

1950 The effect of fishing on stocks of halibut in the Pacific. Fish. Res. Inst., Univ. of Washington, Seattle. 60 pp.

Thompson, William F. and F. Heward Bell

1934 Biological statistics of the Pacific halibut fishery. (2) Effect of changes in intensity upon total yield and yield per unit of gear. Rept. Int. Fish. Comm. (8): 49 pp.

Thompson, William F., Henry A. Dunlop and F. Heward Bell

1931 Biological statistics of the Pacific halibut fishery. (1) Changes in yield of a standardized unit of gear. Rept. Int. Fish. Comm. (6): 108 pp.

APPENDIX A

Catch in Millions of Pounds, Gear Fished in Thousands of Skates, and Catch per Skate for Area 2 and Area 3 for the Period 1921 to 1960.

	Area 2			Area 3		
Year	Catch	Gear Fished	Catch Per Skate	Catch	Gear Fished	Catch Per Skate
1921	36.6	478.7	76	15.5	109.8	141
1922	30.5	488.5	62	11.6	86.7	134
1923	28.0	494.0	57	22.3	148.7	150
1924	26.2	473.0	55	26.3	241.0	109
1925	22.6	441.3	51	26.8	283.2	95
1926	24.7	478.0	52	26.9	287.0	94
1927	22.9	469.0	49	30.8	356.8	86
1928	25.4	537.3	47	27.8	384.3	72
1929	24.6	617.2	40	31.1	431.2	72
1930	21.4	616.3	35	27.3	424.9	64
1931	21.6	534.0	41	21.7	304.8	71
1932	22.0	445.1	49	21.6	264.7	82
1933	22.5	437.5	52	23.5	284.1	83
1934	22.6	410.9	55	23.3	272.1	86
1935	22.8	365.6	62	23.0	241.0	96
1936	24.9	458.8	54	23.8	249.7	96
1937	26.0	430.9	60	23.5	210.6	112
1938	25.0	363.0	69	24.6	212.1	116
1939	27.4	452.1	61	23.4	201.2	116
1940	27.6	440.4	63	25.9	223.6	116
1941	26.0	425.6	61	26.7	220.3	121
1942	24.3	378.2	64	26.2	195.3	134
1943	25.3	345.8	73	28.1	212.0	133
1944	26.5	314.2	84	26.8	178.9	150
1945	24.4	302.8	81	29.2	222.0	131
1946	29.7	351.2	85	30.5	249.8	122
1947	28.7	333.6	86	27.4	235.7	116
1948	28.4	312.2	91	27.4	239.3	115
1949	26.9	299.0	90	28.4	272.0	105
1950	27.0	281.7	96	30.2	275.4	110
1951	30.6	320.8	96	25.4	236.0	108
1952	30.8	251.8	123	31.2	234.6	133
1953	33.0	228.6	145	26.9	200.7	134
1954	36.7	244.2	150	33.8	247.6	137
1955	28.7	219.9	131	29.7	240.4	124
1956	35.4	263.2	135	31.2	234.6	133
1957	30.6	283.6	108	30.3	235.2	129
1958*	30.6	275.5	111	32.1	243.2	132
1959*	30.8	277.5	111	36.6	254.2	144
1960*	31.8	262.9	121	34.1	240.1	142

* Preliminary figures. Since 1958 pronounced changes have been made in the baits used by the fishery. While tentative adjustments have been made for the effects of such changes upon the efficiency of the gear, the above figures since 1958 must be regarded as preliminary.

APPENDIX B

The Effect of the Fishing Mortality Coefficient on Age of Entry

The following table shows the differences in age structure of the population that will follow from two levels of fishing if the selection curve were unchanged. The table shows the weight of the stock by age per 1000 recruits computed with the natural mortality coefficient of 0.20 and the weight by age values taken from IPHC, 1960, Table 5, Goose Islands data. Also, in both cases it is assumed that the selection curve is a straight line starting with zero at age 5, 25 percent at age 6, 50 percent at age 7, 75 percent at age 8, and 100 percent at age 9. In the first column the fishing mortality coefficient is assumed to be 0.20 and in the second column to be 0.40.

Weight by Age for 1000 Recruits

Ages	F = 0.20	F = 0.40
5	4000.0	4000.0
6	4912.2	4912.2
7	5100.8	4852.0
8	5195.3	4471.5
	Total: Ages 5-8 19,208.3 pounds	Total: Ages 5-8 18,235.7 pounds
9	4659.2	3451.0
10	3792.7	2300.1
11	3139.5	1558.2
12	2505.0	1017.5
13	1948.0	646.7
14	1530.0	414.8
15	1177.8	261.3
16	909.0	166.5
17	675.0	100.0
18	504.0	61.6
19	378.0	38.5
20	280.0	23.4
21	207.9	14.2
22	151.2	8.5
23	110.4	5.1
24	80.0	3.0
	Total: Ages 9-24 22,047.7 pounds	Total: Ages 9-24 10,070.4 pounds

It is seen that with the lower fishing mortality coefficient, medium and large fish (ages 9 to 24) are the more abundant group in terms of total weight. However, with the higher fishing mortality coefficient the small fish form a group, by weight, nearly double that of the medium and large. Thus if this factor remains unchanged the fishery will intensify its selection of younger fish in the second case and the result will be a lower age of entry.

APPENDIX C

Estimation of Age of Entry

For the purposes of this analysis, age of entry is defined as the age at which the members of the year class are on the average 50 percent as vulnerable to fishing as are the older fully-vulnerable ages. To estimate this age the logarithms of the catch in numbers per unit effort for successive years are plotted against age. A straight line is then fitted by eye to the right hand side of the curve passing through the maximum point and extending to the youngest age. Successive differences between the curve and the extrapolated line are computed. The antilogarithm of any differences mulitplied by 100 is the estimated percentage selection at that age. Linear interpolation between the percentages is used to determine the age of 50 percent recruitment. Data for the 1947 year class on the Goose Islands grounds are used below as an example of this procedure.

Age	No. Per Unit Effort	Logarithm (No. Per Unit Effort)	Logarithm From Extrapolated Line	Differences	Anti-logarithm of Differ-ences
4	66	4.18965	11.89	−7.70	.001
5	903	6.80572	11.37	−4.56	.011
6	2322	7.75018	10.69	−2.94	.053
7	8110	9.00085	10.10	−1.10	.329
8	13117	9.48166	9.49	− .01	.990
9	6382	8.76124			
10	3993	8.29230			
11	2458	7.80710			
12	1218	7.10496	Age at which 50 percent occurs is 7.2.		
13	693	6.54103			
14	258	5.55296			